1. (10 points) In the box below, write the definition of $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

2. (10 points) From the definition compute $f'(x)$ if $f(x) = x^2 - 7x + 3$.

$$2x - 7$$

3. (25 points) Find the derivatives of the following functions.

(a) $f(x) = \frac{10}{21\sqrt{x}} - \frac{x^{1000}}{1000}$

(b) $f(x) = \frac{x^2 + 31}{x^2 - x + 12}$

(c) $f(x) = (\ln(3x) - \ln(x))^{10}$

(d) $f(x) = (\ln x)^5 \cdot e^{x^2 - 3x}$

(e) $f(x) = \ln \left(10(4x - 15)^5(x - 1)^{2/3}e^{-7x}\right)$

4. (10 points) Use implicit differentiation to find $\frac{dy}{dx}$ at the point $(2, 1)$ if $5x^2 - x^4y^5 - 3y^2 = 1$.

$$\frac{6}{43}$$
5. (40 points) The entire graph of \( y = f(x) \) is pictured below.

(a) What is the domain of \( f \)? \( [0, 2] \cup (3, 6) \)

(b) What is the range of \( f \)? \( [1, 2) \cup [3, 4] \)

(c) What is \( f(4) \)? \( 1 \)

(d) What are the \( x \)-coordinates of all minima? \( 3.9, 5.1 \)

(e) On the graph, draw the tangent line to \( y = f(x) \) through the point where \( x = 1 \).

(f) Estimate \( f'(1) \). \( 1/3 \)

(g) Give the \( x \)-coordinates of all inflection points. \( 1, 4.2, 4.8 \)

(h) Where is \( f(x) \) increasing? \( (0.6, 1.4) \cup (3.9, 4.5) \cup (5.1, 6) \)
6. (15 points) Use the first and second derivatives to find the x-coordinates of all local maxima and minima of \( f(x) = 7e^{-5x} + 2x \). Be sure to say which of your answers are local minima and which are local maxima.

There is a local minimum when \( x = -\frac{1}{5} \ln \frac{2}{35} \)

7. (10 points) Find the equation of the line tangent to the graph of the function \( f(x) = 9e^{-3x} + 5 \) at the point where \( x = 0 \).

\[ y = -27x + 14 \]

8. (20 points) Graph \( y = -x^3 + 3x^2 - 5 \). Label all local maxima, local minima, and inflection points.

See the above figure.

9. (15 points) The demand equation for a monopolist is \( p(x) = 18 - x \) and the cost function is \( C(x) = 14x - 6 \) where \( x \) is the number of millions of units produced. Find the value of \( x \) that maximizes the profit.

\[ 2 \]

10. (15 points) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence?

625 square feet (when both sides are 25 feet)
11. (15 points) Determine the area of the region between the curves \( y = 2x^2 - 3x + 2 \) and \( y = x^2 + 2x - 4 \).

12. (15 points) Find each of the following integrals.

   (a) \( \int \left( \frac{3}{7} \sqrt{x} - \frac{1}{5x} \right) dx \) 
   \[ \frac{6}{7} x^{1/2} - \frac{1}{5} \ln x + C \]

   (b) \( \int e^{3t/2} dt \) 
   \[ \frac{2}{3} e^{3t/2} + C \]

   (c) \( \int (-2x + 7)^3 dx \) 
   \[ \frac{(-2x+7)^4}{8} + C \]

13. (10 points) Use a Riemann sum to approximate the area under \( y = x^2 \) between \( x = 1 \) and \( x = 2 \). Use 2 subintervals, each of the same length, and use left endpoints.

14. (10 points) Find all points \( (x, y) \) where \( f(x, y) = \frac{1}{2} x^2 + y^2 - 3x + 2y - 5 \) has a possible local minimum or local maximum.

15. (15 points) Use Lagrange multipliers to find all points \( (x, y) \) that minimize \( \frac{1}{2} x^2 - 3xy + y^2 + \frac{1}{2} \) subject to the constraint \( 3x - y - 1 = 0 \).