I. **Overview**

Over the millennia, humanity has witnessed radical, technological and pedagogical changes in mathematics education. These include corresponding changes in technologies for representing as well as manipulating ideas and in pedagogies for teaching mathematics as well as participating social groups in mathematics. From about 3,000 BC, with the invention of writing in the form of Egyptian hieroglyphs and Near Eastern cuneiform, mathematics teaching has essentially involved the following:

- variations of choral response;
- memorization of rules, facts, and procedures; as well as
- subject-centered curricula in a teacher-centered environment.

However, recent, important developments have dramatically changed the way that educators think about the teaching of school mathematics. Since the beginning of the 1950s, the mathematics education community has progressively focused more pedagogical attention on ways of making school mathematics meaningful as well as encouraging the serious participation in mathematics of increasingly larger and diverse proportions of the country’s students. This focus has prompted pedagogical shifts away from concentrating on algorithms and computations toward emphasizing

- student-centered curricula with teacher as facilitator or coach;
- discovery activities, open-ended investigations; and
- sense making.

Paralleling this pedagogical shift, electronic, information technologies have advanced rapidly as have their use in research mathematics and in mathematics education. This technological advance is widespread and has reached a point where sophisticated educational hardware and software tools are available to mathematics classes even in economically poor, urban schools. These technological tools have simultaneously made obsolete many of the algorithms taught in previous mathematics classes (e.g., calculating square roots by hand and interpolating logarithms) and made possible many previously impractical activities (such as simulations of large samples of empirical probabilities to acquire insight into theoretical probabilities and graphing exponential equations to explore effects of varying parameters).

These twin changes have contributed to the now prominent roles played by meaningfulness and technology in many school mathematics curricula and classrooms. More to the point, mathematics educators are encouraged (even urged) to use technological devices as tools to enhance students’ ability to develop meaningfully their mathematical ideas and ways of reasoning mathematically. For instance, in 2000, the National Council of Teachers of Mathematics (NCTM) published what has become an

In this volume, the NCTM describes principles that all mathematics classrooms should meet and specifically lists the use of technology as one of their six principles and further declares that *all* school mathematics courses should regularly and meaningfully employ technology. The challenge for prospective and practicing teachers alike is to determine how to incorporate technology sensibly and effectively into the learning and teaching of mathematics.

**II. Objectives**

The central goal of this course is to enable you to learn how technology can be used to enhance mathematics learning and acquire facility in using particular technologies as an effective pedagogical tool in the mathematics courses that you will teach. This course is organized around the exploration of concepts in three strands of mathematics: number and algebra; geometry; and data analysis and probability. In these strands, using various technological tools, you will revisit your own learning of mathematics and investigate mathematical concepts through collaboration, problem solving, and mathematical justification.

More specifically, by participating actively and completing assignments and projects, you will accomplish the following objectives:

∇ Revisit, broaden, and deepen your understanding of certain mathematical ideas and forms of mathematical reasoning through the use of technology.

∇ Extend your facility to use various types of electronic technologies available to enhance the teaching of school mathematics.

∇ Enlarge your knowledge of classroom, problem-solving activities that incorporate technology.

∇ Increase your understanding of psychological and educational principles that underpin effective uses of technological tools in mathematics classrooms.

∇ Augment your awareness of the racial, gender, and economic-class imperatives for access to technology-rich, learning environments for learning mathematics.

∇ Develop your ability to design lessons that incorporate technology as well as your facility to evaluate critically lessons that involve technology.

To achieve the goal and objectives of the course, you can expect to work outside of class in a neighborhood of eight to ten hours per week on assignments and projects. In class, you will continually engage in critical discussions and reflections about your mathematical learning and about the learning and teaching of mathematics with technology as well as about issues of equity and access to technology-rich mathematics classrooms.

**III. Relationship to Education Program**

This course is part of the sequence of courses required for elementary-middle school New Jersey State certification.

**IV. Two closely related courses**

21.300.342 Elementary Mathematics and Pedagogy
21.300.343 Elementary Science

**V. Pre- and co-requisites**

By permission

**VI. Learning Outcomes**

By the end of this course, you should be able to perform the following skills with the listed technology:
Microsoft Word

1. Insert a $M \times N$ table
2. Insert an Equation using the Equation Editor that contains a fraction, radical, exponent, parentheses, and at least 3 mathematical operations.
3. Insert from another application a Screenshot or Picture that does NOT float over the document.
4. Insert 2-D and 3-D geometrical figures using the Drawing Tools
5. Create a Hyperlink to: 1) an Internet website, 2) another Word document, 3) a file from either Excel, Sketchpad, TinkerPlots, or Fathom.

TI-73 Explorer Calculator

1. Use Constant function with counter-creating number patterns
2. Perform basic computations (including permutations, combinations, factorial)
3. Use List editor to enter categorical and numerical data
4. Use Plot to specify a scatter plot and a histogram
5. Use Graph and Window to display List data plotted in an appropriate viewing window
6. Use Y=, Window, and Graph to display functions using an appropriate viewing window
7. Use the zoom and trace features in the graphing application
8. Create a split screen
9. Create a table of values
10. Attach formulas to list names (formulas should include the use of the sequence command)
11. Compute univariate and bivariate statistics
12. Find an appropriate regression equation for a set of data
13. View a set of data and a regression equation simultaneously

TinkerPlots (for elementary education students)

1. Explore data sets, create graphs: scatter plots, time series graphs, histograms, and box plots.
2. Add a trend line to a scatter plot.
3. Enter Data in either a New Collection or a New Case Table with appropriate Attribute Names.
4. Import raw, case-based data or microdata from the Internet, using Import for URL, from the site http://lib.stat.cmu.edu/DASL/DataArchive.html into New Collection or a New Case Table.
5. Be able to Lock and Unlock a Collection
6. Enter data in Collection or Case Table based on a Formula that uses at least one built in Function
7. Create a Function Plot with a New Plot and Enter a Formula for the Function you wish to graph.
8. Add a New Slider

Fathom (for secondary education students)

1. Enter Data in either a New Collection or a New Case Table with appropriate Attribute Names
2. Import Data from an Internet website into a either a New Collection or a New Case Table
3. Be able to Lock and Unlock a Collection
4. Display Univariate Data in appropriate graphs (e.g., Dot plot, Line Graph, Histogram, Box Plot)
5. Display Bivariate Data in a Scatterplot, Add a Least-Squares line, and Create a Residual Plot
6. Create a Summary Table with the Basic Statistics calculated for a data set
7. Add a formula to the Summary Table to display another Statistic (e.g., variance)
8. Enter data in Collection or Case Table based on a Formula that uses at least one built in Function
9. Create a Function Plot with a New Plot and Enter a Formula for the Function you wish to graph.
10. Add a New Slider and use that Variable (Global Value) in a Formula, especially a formula that controls a function plot.

The Geometer's Sketchpad, version 4

1. Create basic geometrical objects: Point, line segment, line, ray, circle, polygons, ...
2. Construct geometrical objects: Midpoint of a segment, a line passing through a point and perpendicular to a segment, a line passing through a point and parallel to a segment or line, connecting midpoints of segments, ...
3. Particular types of triangles (acute, obtuse, right, scalene, isosceles)
4. Particular types of quadrilaterals (parallelogram, rectangle, rhombus, square, kite)
5. Centers of triangles: incenter, circumcenter, orthocenter, centroid
6. Arcs of circles
7. Perform operations: Apply transformations to geometrical objects (translation, reflection, rotation, and dilation)
8. Measure lengths and angles
9. Perform computations using the calculator tool
10. Hide objects
11. Label objects
12. Create captions and apply formatting options to text
13. Include animation
14. Create custom tools (version 4)
15. Plot points and create graphs
Probability Explorer

1. Run a 1-event experiment with Coins or Dice.
2. Run any experiment over 1000 trials with appropriate displays of the data
3. Save Images of a Bar and Pie Graph in the Notebook and also be able to Copy and Paste these images into a Word document.
4. Change the probability in an experiment using the Weight Tool and Hide the Weight Tool with a Password.
5. Save an experiment and Open an existing file.
6. Design and Run a Marble experiment with at least 3 different colors and at least 12 marbles in the bag.
7. Use the RunUntil tool
8. Be able to Design your own experiment.
9. Use the MakeIt Tool to create all possible outcomes for the experiment.
10. Show the outcomes of a 2-event experiment both Ordered and Unordered in both the Data Table and the Stack Columns.

Microsoft Excel

1. Enter data in columns and rows
2. Import data from Internet website and format into columns
3. Format cells and text with Color and Font Size and, resize rows & columns, and add borders to cells
4. Perform basic operations (e.g., sum, average, random) and create formulas using Excel functions and mathematical operations with cell references
5. Create basic types of graphs with appropriate data, label axis, and change axis scale (e.g., Line graph, bar graph, pie graph, X-Y plot)
6. Insert a scroll bar and link to control a cell’s value. Use the controlled cell in a formula that generates a table of data and/or a graph that will all update when the scrollbar is changed.
7. Add a trendline to an X-Y scatterplot and display formula and r2 on chart

VII. Materials and Reading List


VIII. Evaluation Criteria

Readings, problem solving, and written reflections 20%
Portfolio of selected, completed assignments 30%
Project and showcase presentation 30%
Final Examination 20%

IX. Week-by-week list of topics and tools as well as readings and assignments

<table>
<thead>
<tr>
<th>Date</th>
<th>Mathematics Topics</th>
<th>Technology Tools</th>
<th>Readings and Assignments</th>
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| **Week 1: Introduction** | From Movement to Algebra; Writing as a Technological Tool for Learning Mathematics | Paper and pencil | Task: Leapfrog (or Chip Switch) Situation  
• Write about your experience with the Leapfrog (or Chip Switch) Situation and how you used patterns to develop a general solution.  
• Read Zinsser (1988) and Powell (2001) as well as write an abstract and a reflection paper for each article.  
• Through the class’s electronic list on Blackboard, distribute a brief assessment of your experience writing in connection with learning mathematics. |
| Week 2 | Generalizing and specializing; Writing as Learning; counting activities with Unifix cubes. | Microsoft Word: Table, Equation Editor, and Graphic Tools | • Discussion Questions for Reading (DQR)  
• Generalizing and specializing in the Leapfrog Problem  
• Task: Find a number with exactly \( n \) factors: A discussion of Ian Child’s solution expressed in prose.  
• Writing, reflecting, and learning  
• Find an article about the use of writing in mathematics classes in elementary, middle, or secondary school; write an abstract and a commentary; and distribute them to the class through Blackboard.  
• Using MS Word, create a table containing data from the Leapfrog Situation, including algebraic expressions for the case of \( n \) frogs per side and submit through Blackboard.  
• Read Huinker (2002), write an abstract, a reflection paper, and answer Week 3 DQR. |
| Week 3 | Combinatorics; | TI-73 | • Discuss reading and DQR |
### Weeks 3:

| 3 | Geometric and numerical patterns of multiplication and division; inverse operations, and factors; square, triangular, and prime numbers | Explorer | - Tasks: (1) Five Steps to Zero and (2) Towers Problem (4-tall, 2-colors)
- Problem-solving heuristics: tables and simplifying a problem
- Write a letter to a student who is ill and unable to come to school. Describe all of the different towers that you can build that are three cubes tall, when you have two colors available to work with. Why were you sure that you had made every possible tower and had not left any out? Indicate how your procedure can be used to build towers that are four cubes tall, when you have two colors available to choose from.
- Read Kieran & Guzmán (2005), write an abstract, a reflection paper, and answer Week 4 DQR |

### Weeks 4:

| 4 | Discrete and continuous situations; using data to formulate an equation; linear relations | TI-73 Explorer | - Tasks: (1) Towers Problem (3-tall, 3-colors)
- Discuss reading and DQR Week 4 Task: Write a laboratory report based on your investigations of the problems: “Temperature” and “A Big Moosetake”
- Read Asimov (1982) and complete the worksheet, “Reading Folklore for Mathematical Information.”
- Read Zevenbergen (2000), write an abstract, a reflection paper, and answer Week 5 DQR |

### Weeks 5:

| 5 | Scientific notation; using data to formulate an equation; linear relations | TinkerPlots TI-73 Explorer | - Tasks: (1) Factors and Number Properties and (2) Wai Mui’s quarter collection
- Performance assessment and rubric scoring
- $p$ is a factor of $q$, $p$ is a divisor of $q$, $q$ is a multiple of $p$, and $q$ is divisible by $q$.
- Based on your modifications of a plot of the first one hundred counting numbers, describe as fully as you can patterns that you notice among each of the following sets of numbers: square, triangular, and prime. Based on your pattern, determine what are the next five elements greater than 100 of each set of numbers.
- Re-read Zevenbergen (2000) and revise abstract, reflection paper, and DQR answers. Read Powell (1993), write an abstract, a reflection paper, and DQR |

### Weeks 6-8: Geometry

| Week 6 | Dynamic Geometry and Exploring Triangle Centers | The Geometer’s Sketchpad | - Discuss reading and DQR Week 5
- Tasks: (1) Explore the shapes created when the midpoints of the sides of quadrilaterals are connected and determine whether there are any invariants or other relationships. Based on your exploration, develop a conjecture. (2) Explore the shapes created |
when the midpoints of the sides of triangles are connected and determine whether there are any invariants or other relationships. Based on your exploration, develop a conjecture.
• Read Peressini & Knuth (2005) write an abstract and a reflection paper as well as answer week 7 DQR.

| Week 7 | Construction of geometric figures and Exploring Properties of Quadrilaterals | The Geometer’s Sketchpad | Tasks: (1) How many ways can you come up with to construct a rhombus? Try methods that use the Construct menu, the Transform menu, or combinations of both. Consider how you might use diagonals. Use the drag test on each construction. Write a description of each construction method along with the properties of rhombuses that make that method work. (2) Do the same for trapezoids.
| Week 8 | Exploring and Generalizing the Pythagorean Theorem | The Geometer’s Sketchpad | Task: Create a visual demonstration of the Pythagorean Theorem.
|        |                                                   |                          | Read Bakker & Frederickson (2005) and write an abstract and a reflection paper.

Weeks 9-12: Data Analysis and Probability

| Week 9 | Comparing related sets of data; differences between categorical and numerical data; display data as graphs. | TinkerPlots | Watch TinkerPlots Basics; Tasks: Who Has the Heaviest Backpack; Is Your Backpack Too Heavy for You?
• As a health and safety expert, write a memorandum to your town’s Board of Education that discusses your findings about students carrying backpacks that are too heavy (more than 15% of their body weight). Include what percentage of students in the lower grades (1 and 3) and in the higher grades (5 and 7) carry backpacks that are too heavy. Explain which students tend to carry backpacks that weigh more for their body weight. Include graphs and explain how your graphs substantiate your conclusions.
• Read Shaughnessy (2003) and write an abstract and a reflection paper.

| Week 10 | Determining trends in time series data; representing data to examine connections between two | TinkerPlots & Probability Explorer | Tasks: Men’s 100-Meter Dash at the Olympics; Men and Women at the Olympics
• Write a report for a sports magazine in which you compare the records for men and women in the gold-medal times for the 100-and 200-meter races as well as the gold-
<table>
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<th>Week</th>
<th>Activity Details</th>
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| 11   | **Probability simulations; Sampling with replacement, distribution, variance, and making inferences from data**  
**Probability Explorer**  
- Tasks: Ten Marbles in a Bag (physical enactment and calculator simulation) 100 Marbles in a Bag (computer simulation).
- Write an opinion editorial to your town’s newspaper that details your prediction of the distribution of different marbles in the bag. Justify your prediction with representations of your data.
- Read Jones, Thornton, Langrall, and Tarr (1999) and write an abstract and a reflection paper. |
| 12   | **Sampling with replacement, distribution, variance, and making inferences from data. Using and creating probability experiments**  
**Probability Explorer & TinkerPlots**  
- Read Forman (2003) and write an abstract and a reflection paper.  
**Task: Schoolopoly**  
- Create a poster exhibit for your court testimony that asserts whether the dice sent you by a manufacture is fair or biased. Your poster must also illustrate compelling evidence that supports your assertion.  
- View video clips of 4th and 6th graders working with Probability Explorer on 100 Marbles in a Bag Problem and write a one-page description of the mathematical ideas you observe that they engaged in. |
| 13   | Technology and Pedagogy Test  
- Read Tang and Ginsburg (1999) and write an abstract and a reflection paper. |
| 14   | Project Showcase Archiving Materials onto CD |

**X. Catalog Description**

Explore and analyze technologies available to learn and teach school mathematics in the areas of number and algebra, geometry, as well as data analysis and probability. Through class discussions, problem-solving sessions, readings, writing assignments, student presentations, and team projects, students will deepen and expand their understanding of mathematics in addition to broaden their knowledge of methods and materials for teaching mathematics, particularly in urban kindergarten through 8th-grade classrooms. As with all methods courses in the education sequence, students take this course after their foundations courses and after being formally admitted to the teacher education program.
What is a reaction paper?

The term, “reaction paper,” has many meanings, each depending on how an instructor defines it. In this course, a reaction paper is just what its name suggests—a piece of writing in which you react to some text in a reading—an article or a book chapter—by relating your experience or thinking to the text. Your reaction may involve judgment or evaluation of a main or subsidiary point of a reading; it may be an analysis of a point; or it may raise a question triggered by a passage in the reading. In the paper, you will present a brief, personal reaction rather than a summary of the reading or attempt to provide either definitive judgments or detailed analysis.

Specifically, for each reaction paper, you will select two to three striking pieces of text from the reading and write briefly about your reaction to each. You should interpret the word text broadly. It can be a word, a phrase, a sentence, or a collection of sentences; it can also be a geometric figure, a mathematical expression, a table of values, or an illustration of children using technology to do mathematics. The text you chose will be something that you find striking and, on the topic of the text, your reaction will connect it to your experiential and intellectual life.

Think of your reaction papers as serving three purposes: (1) a record of what you found thought provoking in the reading, (2) an opportunity for you to deepen and extend your pedagogical insights into the teaching of mathematics with technology, and (3) an invitation for you to develop further your personal philosophy of education.

A Reaction Paper

• Tells readers what pieces of text you found striking in the reading.
• Relates your personal reaction to each text.
• Mentions how the text selections affect you.

Qualities of an Interesting Reaction Paper

• Follows an introduction-body-conclusion structure.
• Uses one or more well developed paragraphs: these are unified, coherent, and concise.
• Provides logical connections (or transitions) linking paragraphs.
• Contains your voice.

For this course, include at the top of your reaction paper your abstract with the reference information of the reading and submit your paper to the Digital Dropbox through Blackboard.
What is an abstract?

Broadly speaking, there are two distinct types of abstracts: descriptive and informative. Each is a short summary of a longer piece of writing that highlights the major points covered and concisely describes the content as well as the scope of the writing in abbreviated form.

You are to write descriptive abstracts of course readings (articles and book chapters). Think of your abstracts as serving two purposes: (1) a useful study guide that you will read to recall the important features and points of the reading and (2) a brief text that professionals might read to decide whether it is worthwhile to read the full article or chapter.

A Descriptive Abstract

- Tells readers what information the reading contains.
- Includes the purpose, methods, and scope of the report, article, or paper.
- Mentions the results, conclusions, or recommendations.

Qualities of an Effective Abstract

- Uses one or more well developed paragraphs: these are unified, coherent, and concise.
- Contains an introduction/body/conclusion structure that presents the reading’s purpose, results, conclusions, and recommendations in that order.
- Follows strictly the chronology of the reading.
- Provides logical connections (or transitions) between the information included.
- Adds no new information other than what is contained in the reading.

Four Steps for Writing Effective Abstracts

1. Reread the article, paper, or report with the goal of abstracting in mind.
   - Look specifically for these main parts of the reading: purpose, methods, scope, results, conclusions, and recommendation.
   - Use the headings, outline heads, and table of contents as a guide to writing your abstract.

2. After you’ve finished rereading, write a rough draft without looking back at what you’re abstracting.
   - Don’t merely copy key sentences from the reading: you’ll put in too much or too little information.
   - Don’t rely on the way material was phrased in the reading: summarize information in a new way.

3. Revise your rough draft:
   - Correct weaknesses in organization.
• Improve transitions from point to point.
• Drop unnecessary information.
• Add important information you left out.
• Eliminate wordiness
• Fix errors in grammar, spelling, and punctuation.

4. Print your final copy and read it again to catch any remaining glitches.

Finally, for this course, include before your abstract reference information for the reading in APA format, as in the sample below, and then submit your abstract to Blackboard.

A Sample Abstract


In the context of a computer-based calculus course, the authors describe how students’ lab reports become learning tools. After instructors comment on the first drafts, students revise their reports, applying principles of “reader expectation,” a writing approach that they learn in a university-required writing course. By providing examples of student writings, the authors show that revising forces students to recast and refine their ideas into clearer and more precise form and, thereby, enables them to deepen their understanding of the mathematical concepts and techniques of the course.