incorporated writing-to-learn activities into new course designs for my classes in college algebra, analytic methods, and statistics. I have found writing-to-learn strategies to be equally effective in achieving higher level objectives involving analysis, synthesis, and evaluation. Finally, I would urge all instructors to experiment with innovative teaching techniques even if the results of such experimentation can be evaluated only in qualitative terms. Do not be intimidated by higher education's obsession with quantitative research data that must be subjected to formal statistical analysis. The main objective is to improve the learning environment through improved teaching and teaching techniques. If the only means by which this objective can be accomplished is through qualitative research, then this type of research should be encouraged at all levels of instruction.

REFERENCES

CHAPTER 13
Writing as a Vehicle to Learn Mathematics: A Case Study

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There is a dangerous myth going around that people learn from experience. . . . the best that can be claimed is the possibility of learning from reflecting on experience.

D. Pimm (1987, p. 60, emphasis original)

One learns from reflecting on experiences—a sensible and uncontroversial correction of the well-known adage. Because of its very familiarity, Pimm’s statement is in danger of stimulating only momentary thought rather than enduring, fundamental transformations of our perspectives on learning and teaching. In the prevailing model of mathematics teaching, referred to by some as the “chalk-and-talk” method, there are few, if any, situations in which students are asked explicitly to reflect on the mathematics they are doing, their feelings about mathematics, or themselves in relation to the discipline. Instead, the results of others’ reflections are narrated to students who are simply asked to memorize these results. This method of teaching incorporates assumptions about teaching and learning that Freire (1970, 1973) characterizes as the “banking” method and Gattegno (1971) describes as the traditional method of schooling.

In the prevailing model, the absence of explicit requests for students to reflect suggests that learning occurs as one moves along a linear sequence of experiences. This view is depicted in Figure 13.1. In this model, “experiences” are didactic situations in which mathematics is presented in a pre-cognized, atomized, and rule-bound form. As a consequence, learning is

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Figure 13.1 The prevailing model of mathematics education, which assumes that learning occurs as one is simply transported from one experience to the next largely an intellectually passive activity in which the need to construct meaning is minimized. Furthermore, since within this model mathematics is believed to yield objective answers to quantitative questions, feelings are excluded from consideration.

On the other hand, the reality of learning suggests a more complex, dynamic model, one which captures the interconnections between experiences and reflections. The model in Figure 13.2 portrays dialectical movement between experiences and reflections, as well as among them and another type of reflection—critical reflection. In the didactic moments of this model, “experiences” are situations in which one perceives, feels, and acts on aspects of one's environment. Reflections on experiences are thoughts about ideas, things or objects, and feelings. These reflections are descriptive, comparative, inferential, interpretive, and evaluative. Such reflections also involve awareness of one's affective responses to experiences. Reflections, then, have two components: thoughts and feelings. These components of reflections are interconnected in that affect influences thoughts and thoughts impinge on affect.1

Thoughts and affectivity are also aspects of the third component of this model of learning. Critical reflections are reflections removed from the immediacy of particular experiences. Such reflections are thoughts about thoughts directed toward planning, monitoring, reviewing, and revising. While cognition is a component of reflections, metacognition is a component of critical reflections. The second component of critical reflections is what could be termed “meta-affectivity,” monitoring and controlling one's affective responses to experiences. In this model (refer to Figure 13.2), learning is recognized to be a cognitively and affectively interrelated and active enterprise. This view presupposes the existence of learners capable of directing their cognitive and affective resources to learn from experiences.

Writing is a powerful instrument with which to reflect on experiences and, like mathematics, is a major tool for thought. Two decades ago, Bruner (1968) advised that both writing and mathematics were “devices for ordering thoughts about things and thoughts about thoughts” (p. 112). This instrumental notion of writing and mathematics can be extended to include the ordering of both thoughts about affect and thoughts directed toward monitoring affect. It is, therefore, reasonable that there be pedagogical techniques in which these instruments together function to augment learning.

For a number of years, mathematics educators have explored pedagogical connections between writing and mathematics, specifically, writing as a support to mathematics learning. The rationales and purposes for, as well as implementations of, writing in the curricula have varied. From among this variety, two categories of approaches can be distinguished: product and process-product. In the former approach, writing is used as a way of demonstrating knowledge; in process-product approaches, writing is a way of knowing. In product approaches, mathematics educators engage learners in writing activities for purposes immediately focused on mathematics, rather than on learners. The concern is for what learners know at the moment, not for the evolution of their understanding of mathematical concepts. The reverse is true of approaches in the process-product category, where writing is used first to focus on learners and then as a means to have learners reflect on mathematics. It follows that writing activities in the process-product category tend to have more than one phase or drafting stage. These stages provide learners with opportunities to construct meaning and generate knowledge.

Different approaches require students' writings to have different functions. These functions range between two of three functions of writing categories formulated by Britton et al. (1975), namely expressive and transactional.2 Transactional writing is chiefly the type of writing students are to do in product approaches. Writing activities in product approaches are used for assessment and diagnosis; to have students complete sentences or to write short, well-developed passages in a response to instructor-supplied questions or topics; and for students to record step-by-step arithmetic procedures (Azzolino & Roth, 1987; Geeslin, 1977; Goldberg, 1983; Johnson, 1983; Nahrgang & Petersen, 1986; Pullmann, 1982; Watson, 1980). Because these activities are mainly for evaluation, students are required to
produce writing which is impersonal or transactional, rather than expressive (see also King, 1982). More pedagogically interesting than product-oriented approaches are ones that foster writing that reflects students' independent thinking. Process-product approaches strive to do this, and as a consequence, reflection and critical reflection are pedagogical foci of writing activities. In process-product approaches, writing activities tend to require exploratory, speculative writing in which the writer externalizes some content of her or his mind. Writing is used primarily as a means to learn mathematics and about oneself, not just as a means to measure information acquisition.

Though not necessarily expressed in terms of the theoretical framework provided by Britton et al., process-product activities move students along the expressive-transactional continuum; that is, expressive writing activities often provide starting points since, according to Britton and others, they are "a kind of matrix from which differentiated forms of mature writing [poetic and transactional] are developed" (1975, p. 83). Students write to articulate their beliefs about the nature of mathematical knowledge and their affective responses to the mathematics they are doing; to construct and negotiate meaning; and to reflect on and monitor their learning and affectivity (Buerk, 1982; Countryman, 1985; Frankenstein, 1983; Powell, 1986). From these starting points and through a process including feedback and revision, students move into a mode of writing about mathematics whose function is transactional (Burton, 1985; Gopen & Smith, 1988; Kenyon, 1987; Mett, 1987; Stempien & Borasi, 1985).

Can writing, in fact, be used as a vehicle to learn mathematics? Drawing on studies in cognitive psychology (Bruner, Luria, Vygotsky), neuropsychology (Gardner), sociolinguistics (Hymes), and philosophy (Dewey, Polanyi), Emig (1977) argued theoretically and convincingly that writing is a unique, multirepresentational, and bi-spherical "languaging process" that also corresponds to other powerful learning strategies; therefore, it ought to be incorporated as a central academic process. A number of mathematics educators have asserted that writing facilitates mathematics learning; however, little evidence of students' conceptual development or increased mathematical maturity has been proffered to support the reasonableness of this assertion. Empirical studies designed to measure the effect of writing on students' achievement in, and attitude toward, mathematics were themselves problematic. Among other technical and pedagogical considerations, either the instructional period was too brief (Bell & Bell, 1985) or the statistical instruments yielded confusing and contradictory information (Selje, Petersen, & Nahrgang, 1986). As a first approximation toward inquiring into the reasonableness of the claim, the present case study was initiated with the following question in mind: What changes could be observed in students' understanding of and feelings toward mathematics through their writing? The following three questions also drove this study:

1. Do students' writings display their recognition of patterns, relationships, and attributes in mathematics?
2. Is writing a means by which students can construct or negotiate meaning?
3. Do students use writing to draw conclusions, make conjectures, ask questions, and express their feelings about mathematics?

**SETTING**

During the fall semester of 1987, this study was conducted in one section of a computation course, Developmental Mathematics I, at Rutgers University's Newark College of Arts and Sciences, whose students are primarily commuters. The course, based on a pedagogical model of Hoffman and Powell (1987) that departs in fundamental ways from the chalk-and-talk model, includes the study of fractions, decimals, percents, word problems, and an introduction to elementary algebra. The course met three times a week for fourteen weeks and had an initial enrollment of twenty-four students, out of which eighteen completed the course. Most were first-year students, and all were placed in the course on the basis of their performance on the New Jersey Test of Basic Skills or, on an in-house instrument, the Mathematics Placement Test. The content of both instruments is arithmetic computation and elementary algebra.

Based on previous scholastic experiences, many students in this course have developed negative feelings and beliefs about mathematics and themselves as mathematics learners. A student expressed one such view as, "Mathematics is something you do, not something you understand." Like students in similar settings (Buerk, 1982) and generally (McKnight et al., 1987, pp. 42-49), most students in this course consider mathematics not only as an abstruse symbol system but also as an arcane and fixed body of knowledge whose secrets will not be revealed. Students have developed an estranged relationship with academic mathematics: This estrangement is manifested in strategies of avoidance which include learning passivity, inappropriate study routines, and reluctance to participate actively in class.

Developmental Mathematics I was chosen for two important reasons:
It is the first of a three-course sequence, the last of which is an intermediate algebra course, College Algebra, for which successful completion satisfies a degree requirement. Approximately 50% of the entering students correctly answer less than 40% of the items on either placement examination. These students are placed into one of the first two courses in the sequence. In any given semester, the failure rate in College Algebra is approximately 40 percent. This figure is as true of students who place directly into that course as for those who are required first to take one or two developmental mathematics courses. Consequently, there is college-wide concern for the retention of students who do not meet the college mathematics proficiency standard upon matriculation.

There was a second, equally compelling reason for selecting this research setting. Students in Developmental Mathematics I come from among the most disadvantaged sectors of our society and are victims of racial, gender, and class oppressions. The aggregate effect of these impact negatively on students' academic performance, generally, and on their mathematics performance, specifically. (For an elaboration of a model that attempts to explain differences in mathematics achievement based on students' race, sex, and socioeconomic status, see Reyes & Stanic, 1988). The prevailing chalk-and-talk method of mathematics teaching contributes to students' estranged relationship with the discipline and, as such, is another element of oppression. As writing requires an active rather than a passive involvement of learners, this project aimed to empower students in two ways: (1) to promote students' awareness of and facility in the use of writing as a vehicle for learning, and (2) to put students at the center and in control of their own learning by engaging them in reflection and critical reflection on mathematical experiences.

**METHOD**

During the second week of the semester, the nature and objectives of this study were discussed with the class verbally and in writing (see Figure 13.3), and research collaborators were solicited. Students were asked to respond in writing, explaining whether they wished to be a research partner and why. From among the affirmative respondents, two students were selected; one, José, is a co-author of this paper and his work is the basis of this case study.

A number of writing activities were used in the course, two of which, freewriting and journal writing, have been analyzed for this study. For five minutes, at the start of each class and each examination, students were

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**Figure 13.3 Letter to Developmental Mathematics I students**

Dear Developmental Mathematics I Student:

This semester, I will conduct a research project for which I am looking for student collaborators. The goal of the research project is to discover whether writing about the mathematics that one is learning and doing can be helpful in learning mathematics. Let me tell what the project is about.

In this course, I will ask each of you to keep a journal about your learning and to do other types of short writing assignments related to the course. Most of the writings that you do I will collect and analyze, and to some writings I will respond. Those who collaborate with me may be asked to do a bit more writing than others. Each week, collaborators and I will meet as a research team to help me analyze their writings.

The central research question that I hope to answer by the end of this research project is: What types of thinking about mathematics are displayed in students' writing? In addition, there are three subquestions that I will be asking about the writing that you do.

1. Do students' writings display their recognition of patterns, relationships, and attributes in mathematics?
2. Is writing a means by which students can construct or negotiate meaning?
3. Do students use writing to draw conclusions, make conjectures, ask questions, and express their feelings about mathematics?

Why do I ask students to write in a mathematics class? I believe that writing can be a powerful tool for developing or improving your critical and reflective thinking about mathematics. By critical thinking, I mean the type of thinking that is careful and reasoned. And by reflective thinking, I mean thinking that is inquisitive, thoughtful, and deliberate. Reflective thinking also can lead one to think about how one thinks. Both critical and reflective thinking can lead one to search for and find meaning and understanding. These, I believe, can be the benefits to you of this research project.

I intend to co-author a paper, with those who collaborate with me, on the results and findings of this project. Let me know if you would like to work with me. The first meeting of the research team will be held in my office during the free period (1 p.m.) on Wednesday, 16 September.

Sincerely,

Arthur B. Powell
asked to freewrite. Freewriting topics were of their choosing; when none came to mind readily, they could write about that. Students were made aware that these writings were neither to be evaluated nor collected. It was suggested, though, that they maintain a collection of these writings for their own purposes. It was hypothesized that freewriting would promote reflection and provide a meditative period in which to collect and give oneself perspective. As we shall see, freewriting served another, unexpected purpose. No attempt was made to monitor the freewriting, but although the instructor also freewrote, students were observed to be freewriting during the allotted time.

Journal writing was the central, more substantive activity; indeed it was initially conceived as the only writing-to-learn activity upon which this case study was to be based. Students were asked to write daily, or at least for each class or assignment, on any topic or issue related to their learning of, or feelings about, the mathematics of the course or the course itself. To help remove the chore-like conception some have of writing and to relieve anxieties many associate with the quantity to be produced, students were advised that five minutes of writing was sufficient for each journal entry. After becoming accustomed to journal writing, most found themselves spending more time than this to express their thoughts. A list of topics was offered only to stimulate thought and reflection (see Figure 13.4).

Journals were collected weekly and returned with comments on the substance of what was written. The comments were intended to be non-judgmental and most often took the form of questions about or suggestions on issues, ideas, and so on to encourage further exploration. The objective was to use journal writing as a tool for learning mathematics. Therefore, it was emphasized to students that neither their grammar nor syntax were of concern, only what they had to say. Aside from moral and other intrinsic incentives, neither penalties nor rewards, in the form of grades or otherwise, were given. To have done so would have indirectly communicated to students that there was a way in which they were to process concepts and feelings. The eighteen students who completed the course submitted journals 93 percent of the time.

Freewriting and journals were two of three processes-products of this research project. The third process-product entailed student-instructor collaboration in the analysis of the student's writings and mathematics as well as in writing this case study. As co-authors, we held periodic meetings to discuss and analyze José's journal and, eventually, freewriting entries. Notes of these discussions were kept; later they were discussed, elaborated, and revised. At the end of the semester, we met several times to decide on the shape of this report and to write drafts of it.

RESULTS

Freewriting

In the original design of this study, we did not intend to analyze José's freewriting entries. At the end of the course, however, he suggested that an examination of selected entries could be informative. Indeed it was, and we found that freewriting served four purposes for José, each within the category of expressive writing. First, it focused one; it was a meditative activity, allowing one to make contact with and to take control of one's inner reality and gain a degree of confidence. Second, freewriting provided a way to clear the mind of preoccupations. These preoccupations were varied, and entries included discussions of feelings, related and unrelated to the course and mathematics; concerns about chores to be accomplished; the anxiety of making presentations in front of the class; career choices and future life; evaluations of the course's content, structure and teaching ap-
proach; and issues involving social interactions. Third, freewriting functioned as a means to reflect on mathematical processes. The following entry illustrates these three functions of freewriting (this entry and all others, unless otherwise indicated, are from the writings of the student-author):

Oh Boy! Here I am again. I think the page hates when I write on it. What else. What else. What? I made various observations of the patterns in the multiplication chart. It was helpful to hear the feedback from the group. Start on Jones' assignment tonight, study for chem exam, outline psych.

Written during the sixth week of the semester and after the same number of weeks of freewriting, this was the first entry to contain a specific reference to mathematics or a mathematical process of the mind (recognizing patterns). Later, the following entry was written. José details a specific observation and the insight it engendered.

Here we go again. Today in class I observed that when working with exponents when one moves to the right the value of the exponent increases by one. Oh! Oh! I ran out of room. Where was I? Oh! So what I said on the previous page and then the reverse is true when moving to the left. Also, the # of multiplication steps is the same as the exponent when moving to the left one takes the reciprocal of the pos. [positive] value found when moving to the right. Wow! I wonder if what I just said sounds confusing.

At times, the stream-of-consciousness nature of some freewriting requires that one read entries carefully, a few times, to extract meaning. While referring to a chart similar to the one in Figure 13.5 below, several interesting observations are mentioned in the entry above. There, the comparative statement that the number of “multiplication steps” (really factors of the base) “is the same as the exponent,” demonstrates that knowledge brought to the situation was synthesized with information observed directly in the chart. This was an act of looking and seeing beyond the mere appearance of things. It is interesting, however, that no mention was made that this insight does not hold true for expressions left of 2. Nevertheless, the above freewriting entry illustrates, in writing, José’s wrestling with a new concept, negative exponents, and achieving an understanding from among several layers of meanings of the concept. In addition, the penultimate sentence in the above entry indicates that insight was acquired into the relationship between two concepts: reciprocals and negative exponents.

The fourth purpose freewriting served for José was not initially anticipated. Often a day’s freewriting entry was reviewed immediately before writing a journal entry. That is, freewriting entries were used as notes for further elaboration in journal entries. An example of this purpose will be given after some discussion of the general results of journal writing.

Journals

As in the case of freewriting, journal entries were varied, but they were constrained in different ways. Journal entries were to be more like “learning logs” than stream-of-consciousness writings and were public documents, to be read and commented upon. Yet they retained their expressive function, almost always: Journal entries contained expressions of feelings, summaries of learning, commentaries on the course, questions, conjectures, specifications of areas of difficulty or confusion, as well as descriptions of solutions and discoveries. Perhaps because of the particular constraints on journal writing, it provided, more than freewriting, a substantive account of how and what one was learning and feeling. Journals also proved to be a powerful vehicle for dialogue between student and instructor.

Over the course of the semester, the nature of José’s journal entries and topics demonstrated growth in his understanding and enjoyment of mathematics as well as confidence in his ability to do mathematics. At first, however, entries dealt with how the class or classmates were reacting to the course, and, as such, these writings were mainly messages to the instructor, as the following excerpt illustrates:

Also, I see that the more we work with circle expressions [see Hoffman & Powell, in press] the more people feel at ease about working with them. I find or rather it is my opinion that the work groups are helping a lot of people. It gives them some sort of security to work in a group rather than on their own.

It was suggested that journal entries focus on one’s own learning, feelings, insights, discoveries, and so on. Afterward, José’s entries became
largely summaries of class discussions. During the first couple of weeks, these summaries were mostly flat, general narratives of class events.

Today we worked on the old postage stamp problem. We went over the previous information on the problem, and also we found out some new things about it. . . . Also, some conjectures were suggested.

José's writing was still outwardly focused, but over time it shifted inward and began to include reflections that claimed that patterns were being noticed and that described his feelings in relation to assignments.

As time goes by, I'm finding it easier to see patterns in the work that I'm doing. I feel confident with the work I'm doing. . . .

Also, there were statements that correlated the relative ease of solving problems with a certain degree of enjoyment.

I find, as time goes by, that it is becoming easier for me to solve math problems and that I am enjoying math somewhat more than I used to.

At this point in the semester, José's more positive affective response toward mathematics also corresponded both to increased specificity in his summary statements and to movement toward reflecting on mathematics in writing. To some extent, this movement also was stimulated by instructor nudges at the bottom of a week's journal: "It would be interesting to read about the patterns that you are seeing. If you write about these patterns, then your understanding of the material may also increase." Soon after this nudge, the following entry was written. What is particularly interesting is that it contains an elaboration of an observation first mentioned in a freewriting entry written earlier that week, illustrating that freewriting entries were used, at times, as "starting notes" for journal writings (see the entry related to Figure 13.5). This was an unexpected function of freewritings.

Today in class, I observed that when working with exponents, when I move to the right the value of the exponential number increases by one. The reverse is true when moving to the left. Also, the number of multiplication steps is the same as the exponential number. When moving to the left, I take the reciprocal of the positive value I found when moving to the right. When multiplying numbers with the bases the same, but different exponential numbers, I can add these exponents. e.g. $5^2 \times 5^3 = 5^5 = 625$. When dividing numbers whose base is the same, but have different exponents, I can subtract the second exponent from the first. e.g.,

$$\frac{5^7}{5^1} = 5^{7-1} = 5^6 = 15625.$$
Journals also were used to promote critical reflection. Students were asked to review previous entries, look for instances where they used writing to think about mathematics, reflect on those instances, and comment on them in writing. During one such exercise, José’s awareness of the equivalence between “taking the reciprocal” and “raising to the negative first power,” was extended to include integers, not only fractions whose denominators are other than one.

Just as entries detailed what a student understood from a lesson or an assignment, they also revealed gaps in understanding and misconceptions. Appropriate attention to these indications could turn such entries into dynamic vehicles for challenging and augmenting a student’s mathematical awarenesses. An instance in which the journal was used to challenge a misconception occurred when we studied techniques for determining the greatest common factor (GCF) and lowest common multiple (LCM) of a group of integers. In an attempt to clarify and claim ownership of the first of these two concepts, José wrote the following:

I found that I could find the greatest common factor of two integers by first finding common factors of both integers and then by taking the largest common to both.

e.g. \((24, 30) = 1, 2, 3, 6 \) \( \text{GCF} = 6 \) or \(2^1 \times 3^1\)

Later, he attempted to internalize both concepts and accommodate them into his understanding of prime factors and prime factorization.

The way one goes about finding the LCM of a group of integers is by looking at the prime factorization of the integers in the group, then picking out the common prime factorizations thus giving one the LCM. e.g., \( \text{LCM}(28, 36) = 2^2 \times 3^1 \times 7^1 \times 5^0 \times 3^0 \times 15^0 \times 3^0 \times 5^0 \).

It appeared that José’s confusion was simply a matter of using the wrong group of three letters, LCM for GCF, not one of misconceptualization. The problem was pointed out and a question posed. Subsequently, the question, reiterated in the first sentence of the entry below, was answered and illustrated with a few examples. José described correctly how to determine the GCF of a group of positive integers and even discussed a special case.

Today, I looked at the prime factorization of a group of numbers to see if I could determine their GCF and LCM just from their prime factorization. I found that both of these answers can in fact be determined by the prime factorizations. The way one goes about determining the GCF of a group of integers is to first see what prime factors the group has in common. The common prime factors of the group is the GCF. Note, if there are no common prime factors among the group their GCF is \((1)\) one.

In this part of the entry, although José computed the LCM correctly, his available language did not permit him to describe accurately his perception and action. On the other hand, part of his confusion related to what the adjective “common” qualifies and what can be seen in the prime factorizations of a group of integers. To find the GCF, the word “common” is related to what one sees directly in the prime factorizations. Given their prime factorizations, however, the LCM of a group of integers is not related to “common” visible elements. That is, the multiples of the integers are not displayed; one cannot see the LCM of a group of integers by simply examining their prime factorizations.

José had to search for language that corresponded to his perceptions and actions. When asked to reflect critically, to review and comment, on a group of journal entries that contained the one above, he eventually did find language to describe correctly the process he actually engaged in to find the LCM. The following is excerpted from that journal entry:

To find the LCM, least common multiple, of a group of numbers one must take the distinct prime factors of the group that one expressed to the highest power.

e.g., \( \text{LCM}(2^2 \times 3^1, 2^1 \times 5^1 \times 3^1, 7^1 \times 11^1 \times 13^1) = 2^2 \times 3^1 \times 5^1 \times 7^1 \times 11^1 \times 13^1 \times 19^1 \).

There are three fascinating aspects to this journal-entry excerpt. First, integers are represented not in standard form such as 750, but in their prime factored form, \(2^1 \times 5^2 \times 3^1\); that is, a level of ease in handling a more abstract representation of integers is evident. Second, José’s description of how to determine the LCM is general and concise. He achieved this level of generality and conciseness by reflecting and critically reflecting through writing and revising his written conceptualizations. Third, in the description above, if the word “take” were replaced by “multiply” and the phrase “that one” replaced by “each,” which is how they are interpreted in practice, then José’s description would appear to have come from an edition of James and James (1963, p. 262)!
Student–Instructor Collaboration

In conventional research models, including classroom-based ones, there is a researcher and objects of research. Even in more progressive models, such as classroom-based research models, the instructor is the researcher and students are the objects. We rejected such models in favor of one that can be called participatory. In the methodological design of this case study, in terms of both processes and products, student and instructor co-labored to analyze the student’s writings and to write this report. In the following excerpt, José reflects on several features and benefits of his involvement in the collaborative project:

I became interested in the study due to my poor math skills. I felt that if I took a more active role in the learning of mathematics I might be able to do better in the course. Throughout the semester I kept a journal detailing my observations of the class, course, and my learning of mathematics. We met after classes and whenever our schedules allowed us to discuss what I felt that I had gained as a result of writing in a mathematics course. I was then asked to comment on the writing experience and the journals that I had kept, to see exactly how it was that I had gained a better understanding of the mathematics I was learning. I found many instances where certain ideas or concepts became clearer to me as a result of writing about them. We then proceeded to put together our paper with the focus being on my learning process and understanding of mathematical concepts through writing.

As a result of the writing that I had done during the course of the semester, I felt more confident in my problem-solving abilities and understood the material better. I was not only more efficient and understood better, but I found that by writing about mathematics I had eliminated some of the anxieties I had once had about mathematics. As a result, I had one of the two highest final exam scores and semester grades in the class. Previously, I disliked mathematics and performed poorly.

José’s decision to participate in the project was an act directed toward improving his mathematics. Indeed, his insight into and facility in mathematics improved. These improvements occurred as José began to overcome his anxieties of, and estrangement from, mathematics. Explicit attention to reflection and critical reflection were the vehicles for these transformations to occur.

CONCLUSIONS

In this participatory study, we have analyzed select freewriting and journal entries of one first-semester freshman, the student-author, who was enrolled in a developmental computation course. The analysis of these reflections and critical reflections was undertaken to determine the degree of reasonableness of the claim that writing can facilitate the learning of mathematics. As we have seen in this study, writing is a heuristic tool with which one can negotiate meaning; in negotiating meaning, one is generating knowledge and learning. Thus, the claim is more than reasonable. In addition to this, we have derived other conclusions. Though these are stated in general terms below, we will discuss limitations on their interpretations. Finally, we will mention a few questions suggested by this study.

Our conclusions are grouped into three areas: student–instructor communications, students’ affectivity, and students’ learning. With respect to student–instructor communications, students are encouraged to write expressively and tend do so freely. Journals are a powerful medium for dialogue between instructor and students. Moreover, this means of personal dialogue can serve to reassure students that their concerns are taken into account. Instructors have an opportunity to provide feedback to students’ statements, interpretations, questions, discoveries, and misconceptions. Rich opportunities for encouraging students to reconsider their conceptualizations and to extend and deepen them often present themselves. In addition, the revelatory nature of students’ expressive writings provides instructors with feedback on important dimensions of their pedagogy.

Reflecting critically in writing about the mathematics they are learning gives students greater potential to control their learning and to develop criteria for monitoring their progress. The development of control and monitoring capabilities engenders in students feelings of accomplishment. These feelings, in turn, have a positive effect on their affective response toward the mathematics they were learning. Finally, from acquiring greater control over their learning, developing criteria for personal standards of progress, and conceptually understanding the mathematics in which they are engaged, students derive a great deal of satisfaction with themselves as learners capable of doing and understanding mathematics.

Reflecting critically on one’s mathematical experiences in writing presupposes an active, not a passive, learner. This action, coupled with the revelatory nature of reflective writing, suggests that writing can have a significant impact on learners’ cognition and meta-cognition. Writing, because the writer and others can see it, allows one to explore relationships, make meaning, and manipulate thoughts; to extend, expand, or drop ideas; and to review, comment upon, and monitor reflections. Expressive writing supports these cognitive and meta-cognitive acts. After one establishes a degree of confidence in one’s ideas, it seems almost natural to move from expressive to transactional prose. This is what occurred to José as he wrestled with his ideas on how to determine the least common multiple of a group of integers. He constructed and reconstructed meaning. He wrote and revised his reflections. The process was mediated by external com-
ments. As José began to express his ideas with greater clarity and confidence and selected language that more accurately described his perceptions and actions, his writing shifted from expressive to transactional.

Through this case study, we have also shown that writing helps students to acquire a rich vocabulary and use it in the context of their understanding of mathematics. Mayher, Lester, & Pradl (1983) make this point with regards to learning in general:

Writing's capacity to place the learner at the center of her own learning can and should make writing an important facilitator of learning anything that involves language. Writing that involves language choice requires each writer to find her own words to express whatever is being learned. Such a process may initially serve to reveal more gaps than mastery of a particular subject, but even that can be of immense diagnostic value for teacher and learner alike. And as the process is repeated, real and lasting mastery of the subject and its technical vocabulary is achieved. (p. 79)

By providing students with opportunities to work with mathematical concepts and terms in their own language and on their own terms, writing also helps students build their confidence in the context of mathematics and become engaged in the material more thoroughly.

Writing to learn mathematics is transformative not only for learners but for instructors as well. Useful writing-to-learn activities are those that engage learners in exploring the contents of their minds; that is, they should maximize the extent to which learners choose language to describe their thoughts, actions, and perceptions. As Mayher, Lester, and Pradl (1983) have claimed, “Writing that involves minimal language choices, such as filling-in-blanks exercises or answering questions with someone else’s language—the textbook’s or the teacher’s—are of limited value in promoting either writing or learning” (p. 78). The more learners are involved in choosing language, the more they are engaged in constructing and reconstructing meaning and making sense of mathematics for themselves. For learners to develop their reflective and critical reflective abilities, learning environments must promote, as Freire has argued, “acts of cognition, not transferrals of information” (1970, p. 67).

What can a case study tell us about the usefulness of writing to learn mathematics generally? It seems likely that most of the conclusions presented above hold true in general. Yet it is also true that one cannot generalize from the case of one to that of many. Broader studies are required to determine precisely for how many other learners our conclusions are true. Conversely, individual learners cannot be known solely on the bases of generalizations derived from the study of groups. A number of our conclu-

sions, however, do point to questions and patterns to look for in the writings of other learners.

This study raises a number of broad questions: How can writing activities best be structured to promote mathematics learning? Do such activities differ in relation to different levels of mathematics? Can writing activities be used to promote collaborative learning? What types of instructor responses prompt students to write more meaningfully and with greater clarity? In what ways can writing be used as an independent, reflective, and meaning-seeking learning tool? We invite others to examine these and other questions that involve students in writing as a vehicle to learn mathematics.

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NOTES

1. The notion that feelings are critical to learning and understanding mathematics is not usually made explicit by mathematicians or acknowledged by mathematicians. Henderson (1987), a mathematician at Cornell University, has expressed introspectively that feelings are an essential component of understanding mathematics: “When I understand something my perception of my universe of experience has been broadened—deepened. . . . To be complete this understanding (increased perception, changed meaning) must include the components of knowing, feeling, and acting” (p. 1; author’s emphasis). Also see Mason, Burton, & Stacey (1985) for practical suggestions on how to attend to affectivity to do mathematics successfully.

2. Transactional writing uses language “to get things done: to inform people (telling them what they need or want to know or what we think they ought to know), to advise or persuade or instruct people.” It is used whenever an “accurate and specific reference to what is known about reality” is needed. Expressive writing is “‘thinking aloud on paper.’ It has the function of revealing the speaker, verbalizing his consciousness . . . submits itself to the free flow of ideas and feelings. . . .” (Britton et al., 1975, pp. 88–90).

3. Some educators, such as King (1982), classify and describe writing activities as being either expressive or transactional. However, this results in a problematic classification scheme principally for two reasons. First, as categories of writing,
expressive and transactional refer to the function of a piece of writing for the writer, not to the characteristics of a writing task or to an instructor's expectations (Britton et al., 1975, pp. 88–91). Though the type of writing to which an instructor gives greater value can be distinguished as expressive and transactional. Second, as described by King, all activities, including those classified as "expressive," are actually product-oriented and, from a developmental perspective, are static. The point is that any of King's activities could be used properly to move from "close-to-the-self" or expressive writing to product-oriented impersonal or transactional writing.

REFERENCES


