

Keeping the Devil in the Details: A Feasible Approach to Aggregating Trade Distortions

by

Vlad Manole^{*} and Will Martin^{}**

August 30, 2005

^{*} Washington University in St. Louis and Development Economics Research Group, The World Bank, Washington D.C. Email: vmanole@worldbank.org

^{**} Development Economics Research Group, The World Bank, Washington D.C. Email: wmartin1@worldbank.org

The views expressed in the paper are those of the authors and should not be attributed to the World Bank or its Executive Directors.

Introduction

How do we measure the welfare gains from trade? How do we measure the costs of trade protectionism? In a more and more interconnected world, the right answer to these questions can have enormous importance. The answer would be simple if a single good with a single tariff were traded. But countries have tariff schedules with thousands of tariff lines, and very high variation in tariff rates¹. Some degree of aggregation is thus necessary if we are to capture the welfare gains or losses from trade reforms. At minimum for economic modeling, the aggregation must convert individual tariff lines into aggregates that conform to the higher-level aggregation available for production/consumption data. A major challenge is to aggregate right, yet few people do it. As Martin, van der Mensbrugghe, and Manole (2003) show, using a proper aggregation method can significantly change the estimated welfare consequences of reforms. They find an increase of the estimated global benefits of EU agricultural trade reform by over 150 percent after switching from the trade-weighted average to appropriate tariff aggregates—and this is beginning from a multi-sector computable general equilibrium model with significant sectoral decomposition.

Several forms of aggregation have been used but none of them is without problems. Historically, measures such as the arithmetic or the trade-weighted average

¹ The coefficient of variation of tariff rates ranged from 0 to 0.8 using the UNCTAD TRAINS data tariff-line level data for a large sample of countries. Chile had a uniform tariff rate of 9% in 2000. In contrast, Thailand's average tariff was 22% in 1995, with a standard deviation of 17, zero minimum tariff and 100 maximum tariff.

tariffs have been employed, but they are without theoretical foundation and in addition, they may introduce significant biases in estimation. More recently, new approaches with rigorous theoretical foundations for the aggregation problem emerged. Anderson and Neary (1993) proposed a uniform tariff that yields the same welfare as the original differentiated tariff structure. In a similar fashion, Bach and Martin (2001) proposed two new tariff aggregators that keep expenditure and tariff revenue constant respectively. However, their approach did not provide a closed-form solution for the tariff revenue aggregator, and required estimation of this key aggregator by numerical methods. Further, they did not analyze the theoretical properties of these aggregators to provide insights into their potential behavior.

This paper builds on Bach and Martin (2001). It analyzes in more detail the properties of the aggregators they proposed. For their tariff revenue aggregator, we show that there may be multiple uniform tariffs that yield the same tariff revenue. To deal with this problem, we redefine the tariff revenue aggregator so that it leads to a unique solution. We then derive some of the key theoretical properties of these tariff aggregators, and under certain assumptions, develop closed form solutions for the expenditure as well as the tariff revenue aggregators. Finally, we explore the relations between the trade-weighted average tariff, the expenditure aggregator and the revenue aggregator.

To illustrate the differences between different aggregators we use a standard general equilibrium model of a small open economy. We use a trade liberalization scenario and find that the welfare results of liberalization are significantly underreported when using the trade-weighted average tariff aggregator relative to the expenditure and tariff revenue aggregators derived by us. The largest gains are found when the

appropriate aggregators are used—that is the expenditure aggregator for aggregating expenditure on imports and the tariff-revenue aggregator when representing the behavior of tariff revenues.

The paper is structured as follows: Section 1 discusses the aggregation approach. Section 2 presents the assumptions about the structure of the economy and the definitions of expenditure tariff aggregator and tariff revenue aggregator. Their properties are discussed in Section 3. In Section 4, separability between domestic and imported goods is introduced and the properties of expenditure tariff aggregator and tariff revenue aggregator are derived in this context. In Section 5 we present an application of these aggregators to a range of countries for which the necessary, detailed data are available. The trade weighted average tariff, the expenditure tariff aggregator and the tariff revenue aggregator are used in a standard general equilibrium model and the results are compared.

1. The Aggregation Approach

Several types of tariff aggregators have been proposed. The easiest form of aggregation is the simple average (Bordo *et al.* 1999). The simple average gives the same weight to all tariffs, regardless of the share in total imports of the products to which they are applied. Clearly, this approach makes poor use of information. Some products, such as oil and oil products are more important in world trade than matches. Further, it is potentially subject to manipulation. In an extreme example, it would be possible for policy makers to combine all of their “sensitive” products into just one tariff line with a very high tariff and to create thousands of tariff lines with zero tariffs for the rest of the

products so the simple average would be close to zero, thus grossly underestimating the real trade protection.

Another form of tariff aggregation is weighted average tariffs. The most common approach is to weight by the value of imports at world prices. The most widely-used tariff aggregator is the import-weighted average tariff based on import values at external prices, such as the widely-reported cif (cost, insurance, freight) basis. As Anderson and Neary (1996) have noted, trade-weighted tariffs have major deficiencies when tariffs are high. High tariffs are typically associated with very low trade values, and therefore bias downwards the trade-weighted average tariff. One potential solution to this problem is to use value shares based on tariff-inclusive prices, reducing the rate at which the value share of high-tariff goods declines. However, such an approach remains *ad hoc*, with no firm conceptual basis as an indicator of the impacts of a tariff change on economic welfare.

Weights other than imports have also been suggested. Leamer (1974) proposed using production shares as weights, an approach that underlies the aggregation involved in the Producer Support Estimates for agricultural products provided by the OECD (OECD 2005). Another possibility would be consumption shares, but these suffer from similar problems as trade weights, especially when imports are a significant part of consumption and the product has a high elasticity of demand (like luxury goods for most of the countries). Increasing tariffs for this kind of products decrease their consumption, thus lowering their shares—although consumption weights are likely to be in domestic prices, making the problem less serious than with external import value weights. A common problem for tariff aggregation using production or consumption weights is the

lack of adequate data at tariff line level.² Another possible choice of weights is the value shares in world imports. However, Leamer (1974) points out that these weights are heavily influenced by the import structure of the EU and the USA and thus not representative for many countries.

To understand the implications of different aggregators for welfare, it is useful to use a graphical approach showing the impacts of a change in a single tariff on the expenditure required to maintain a given utility level, and on tariff revenues. The effect of a tariff on the expenditure needed to maintain a given level of utility, is given by the concave function in Figure 1. As the tariff rises from zero, the expenditure on the good increases-- assuming it is in inelastic demand, but at a lower rate than would be implied by the Leontief demand structure that underlies the fixed-weight index. The slope of the expenditure function, e_p , declines as the quantity demanded, which is also e_p , declines.

Clearly, use of an expenditure weight obtained from a situation where a non-zero tariff is in place will underestimate the overall impact of the introduction of the tariff on the value of the expenditure function. One might attempt to solve this problem by estimating the average slope of the expenditure function over the range from $t=0$ to the value of interest. However, estimating such parameters effectively involves specifying a complete model of the economy, and solving it for the interdependent changes in quantities associated with changes in trade regimes. Once such a model is available, a better approach is probably to solve it directly for the variables of interest.

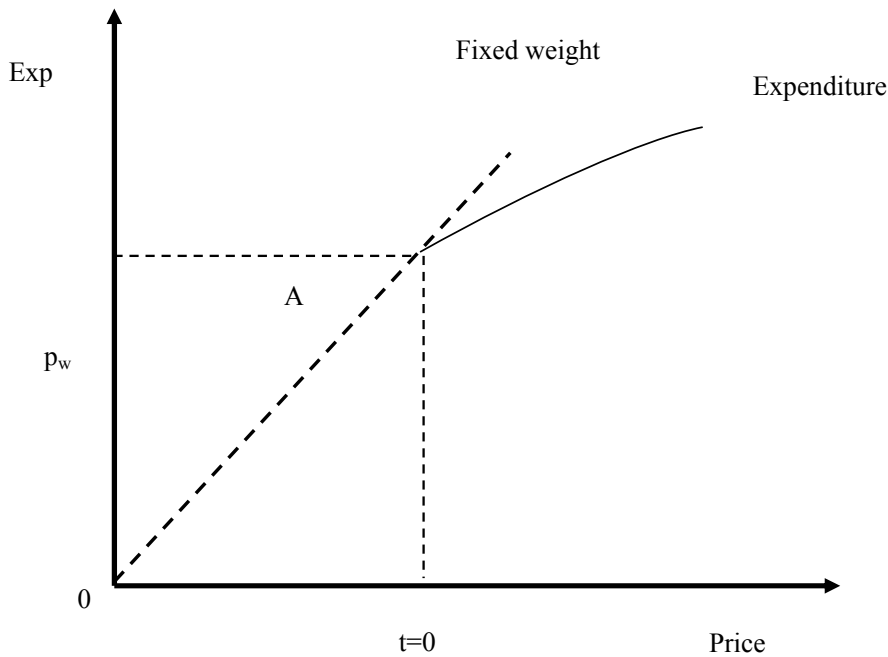
The slope of the revenue function is given by $e_p + (p-p^*)e_{pp}$, where e_{pp} is the slope of the demand curve; p is the domestic price and p^* is the international price. Clearly, this

² See CEPII procedure on www.GTAP.org.

slope is the same as that of the expenditure function when the tariff is zero, but declines as the tariff rises from zero.

A fixed weight index based on observed imports at an initial, distorted point, such as point a in the diagram will initially have the appropriate slope to represent the impacts of a price change on consumer welfare, e_p . However, as we reduce the tariff from its initial level, this weight will begin to understate the impact of this price change on expenditures. For revenues, the fixed-weight index will initially overstate the impact of the tariff change on tariff revenues, ignoring the impact of demand responses to prices. However, if tariffs are reduced sufficiently, it will come to understate the impact of a tariff reduction on revenues, because it ignores the impact of the change in price on e_p .

Figure 1. Impacts of a tariff on consumer expenditure and tariff revenues



Recently, a series of papers proposed new approaches to the aggregation problem.

In their seminal papers, Anderson and Neary (1992, 1993, 1996) proposed the Tariff Restrictiveness Index (TRI) as a uniform tariff that would be equivalent, in terms of welfare, to any pattern of distortions in a small open economy. Bach and Martin (2001) built on the TRI approach and suggested tariff aggregators that keep constant the expenditure or tariff revenue in different parts of a general equilibrium model and applied these aggregators in a computational general equilibrium framework. Several other papers have presented applications of the TRI (Bach, Martin and Stevens, 1996; O'Rourke 1997). Salvatici (2001) broadens the definition of TRI to be applied in the

Deleted: ¶

¶
 <sp><sp>¶
 <sp><sp><sp>¶
 <sp>¶
 <sp><sp>¶
 ¶
 <sp><sp><sp><sp>¶
 ¶
 <sp>¶
 <sp>¶
 <sp>¶
 ¶
 ¶
 <sp><sp>¶
 <sp><sp>¶
 ¶
 ¶

Multi-regional Applied General Equilibrium model and in the case of the liberalization of the EU agricultural policy.

Anderson and Neary (2003) propose a mercantilist index of trade policy, a new tariff aggregator that maintains the same trade volume at a given trade/ quota structure—a measure more focused on impacts on partners’ market access opportunities than the welfare-oriented TRI. Kee, Nicita and Olarreaga (2004) calculate OTRI measures of this type and use them to calculate the welfare losses associated with the existing tariff structures for 88 countries. Their results show that both simple and weighted average tariffs tend to underestimate the distortion imposed by the tariff regime by 30 percent on average. Martin, van der Mensbrugghe and Manole (2003) use the revenue tariff aggregator and expenditure tariff aggregator in a computational general equilibrium model framework and show that inappropriate aggregation may be causing very substantial underestimation of the global gains from trade reform.

2.A Model

An important lesson from index theory generally, and particularly from the papers by Anderson and Neary on tariff index theory, is that a reliable index must be based on a model that relates the index to an economic variable of interest. In this paper, we focus primarily on economic welfare in small open economies. We assume that the structure of such a competitive, small open economy can be captured by the following equations (Bach and Martin (2001)):

The income-expenditure condition,

$$(1) \quad e(\mathbf{p}, u) - r(\mathbf{p}, \mathbf{v}) - (\mathbf{e}_p - \mathbf{r}_p)(\mathbf{p} - \mathbf{p}^w) - f = 0$$

and the vector of behavioral equations³,

$$(2) \quad e_p(\mathbf{p}, u) - r_p(\mathbf{p}, \mathbf{v}) = \mathbf{m}$$

where $e(\mathbf{p}, u)$ is the expenditure function of the representative household, \mathbf{p} is a given vector of domestic sectoral price aggregates, u is the domestic utility, and $r(\mathbf{p}, \mathbf{v})$ is the domestic revenue, and \mathbf{v} is a vector of productive resources; \mathbf{m} is the vector of imports, and f is the net financial inflow from abroad. We can define \mathbf{B} as the balance-of-trade function, which captures the financial inflow necessary to keep the level of utility u^0 constant when prices \mathbf{p} change (Anderson and Neary 1996).

Based on equation (1) and considering prices \mathbf{p} and the level of utility u^0 as exogenous, \mathbf{B} can be written as:

$$(3) \quad \mathbf{B}(\mathbf{p}, u^0) = e(\mathbf{p}, u^0) - r(\mathbf{p}, \mathbf{v}) - (\mathbf{e}_p - \mathbf{r}_p)(\mathbf{p} - \mathbf{p}^w) - f$$

The majority of papers that use the balance-of-trade approach are based on a single-level model and specify aggregators that capture all the relevant variations across tariffs. We use a two-stage modeling approach that is compatible with multi-sectoral economic models. In the first stage we compute indices that capture the information about tariffs within groups. In the second stage, we go one level up and aggregate these indices into an economy wide aggregator.

Deaton and Muellbauer (1980) provide the theoretical underpinnings for two-stage modeling approach from the consumption side. If weak separability exists, then the consumer's maximization problem can be decomposed into the maximization of sub-

³ We use bold letters for vectors.

utility functions over categories of products, and at a higher level, maximization of total utility over the sub-utility functions. In addition, homotheticity of preferences at the lower level allows the use of price and quantity indices at higher levels of aggregation, and hence the simplifications resulting from use of a two-stage budgeting approach.

In a similar fashion, Chambers (1988) and Lloyd (1994) show that weak separability of the production function, and homotheticity of the sub-aggregator functions allow aggregation of multi-stage production technologies.

In the rest of the paper we base our analysis on the assumption that the conditions needed for the formation of sub-aggregate price and quantity indexes have been met. This is not an unrealistic assumption since these conditions are routinely used in all modeling approaches involving aggregation (Hertel and Tsigas, 1997), irrespective of whether the aggregators used are grounded in economic theory. We construct two different types of aggregators – one that is optimal for decisions regarding expenditure levels and the demand for consumption goods, and another one that is optimal for aggregating tariffs and estimating tariff revenues. We further assume, following Armington (1969), that domestic products are differentiated from imported products. If the prices of domestically-produced goods are determined by returns on export markets, then $r(\mathbf{p}, \mathbf{v})$ will be invariant to changes in tariffs, and import demand will equal $\mathbf{e}_p(\mathbf{p} - \mathbf{p}^w)$, allowing further simplification of the model. In the following section, we develop aggregators for the two components of the model—the expenditure and tariff revenue functions—needed to capture the welfare impacts of tariffs in a small, open economy.

2.1 The expenditure function aggregator

Based on the previous assumptions, there is an expenditure function for each of the sub-utility functions used in the analysis. Let e_j be the expenditure function for commodity group j :

$$(4) \quad e_j(\mathbf{p}_j, u_j^0)$$

where \mathbf{p}_j is the domestic price of goods and u_j^0 is the utility level. Bach and Martin (2001) define the tariff aggregator for expenditure on the commodity group j as the uniform tariff, τ_j^e , which requires the same level of expenditure on imported commodities in the group as the observed vector of tariffs to maintain utility level u_j^0 .

The goods in the commodity group j can be divided in domestic (non-tradable) goods - with price \mathbf{p}_j^d , and imported goods - with domestic price \mathbf{p}_j^t , so the complete domestic vector price may be written as $\mathbf{p}_j = (\mathbf{p}_j^d, \mathbf{p}_j^t)$.

Furthermore, \mathbf{p}_j^t can be written as:

$$\mathbf{p}_j^t = \begin{bmatrix} 1 + \tau_{j1} & & 0 \\ & \ddots & \\ 0 & & 1 + \tau_{jn} \end{bmatrix} \mathbf{p}_j^w$$

where \mathbf{p}_j^t is the domestic price of imported good, τ_{ji} is the *ad-valorem* tariff for the good i in the commodity group j and n is the number goods in the same commodity group, and

\mathbf{p}_j^w are world prices for the imported goods. Note that the disaggregated tariffs enter the definition via domestic prices of imported goods - \mathbf{p}_j^t .

We can define the tariff aggregator for expenditure on commodity group j as the uniform tariff τ_j^e :

$$(5) \quad \tau_j^e = [\tau_j^e \mid e_j(\mathbf{p}_j^d, \mathbf{p}_j^w(1+\tau_j^e), u_j^0) = e_j(\mathbf{p}_j^d, \mathbf{p}_j^t, u_j^0)]$$

If the commodity group does not include domestic goods, then we can use the homogeneity of degree one of the expenditure function in prices to extract $(1+\tau_j^e)$ in the LHS of expression (5), and, in consequence, $\tau_j^e = e_j(\mathbf{p}_j^d, \mathbf{p}_j^t, u_j^0) / e_j(\mathbf{p}_j^d, \mathbf{p}_j^w, u_j^0) - 1$.

2.2 Tariff revenue aggregator

Bach and Martin, (2001) propose a tariff revenue aggregator defined in a similar fashion to the expenditure aggregator. A tariff revenue aggregator on the commodity group j may be defined as the uniform tariff that will yield the same tariff revenue as the observed vector of disaggregated tariffs for that particular group of commodities, conditional on the utility level underlying the expenditure function and the resource endowments underlying the domestic revenue function:

$$(6) \quad \tau_j^R = [\tau_j^R \mid tr_j(\mathbf{p}_j^w(1+\tau_j^R), \mathbf{p}_j^w, u_j^0, v_j^0) = tr_j(\mathbf{p}_j^t, \mathbf{p}_j^w, u_j^0, v_j^0)]$$

Expression (6) is based on the implicit assumption that the equation has a solution and that this solution is unique. In Proposition 2 we show that, the assumption of a unique solution need not hold, suggesting a need for caution with the original Bach-Martin approach.

3. Properties of the aggregators

The definitions given in equations (5) and (6) do not guarantee existence, uniqueness or economic meaning for the proposed tariff aggregators. This raises particular concerns if these aggregators are to be generated on a routine basis using trade analysis software. In the rest of the paper we consider a Constant-Elasticity of Substitution (CES) functional form for the expenditure function and the import demand functions. A reason for this assumption is the significant use of CES type functions in general equilibrium modeling. In addition, using an explicit functional form allows us to derive the properties of tariff aggregators. To simplify the notation, we drop the commodity group index for the remainder of the paper.

Based on the previous assumptions, the expenditure function and the tariff revenue function have the following forms:

$$e = \left(\beta^d p^d + \sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} u^0$$

$$tr = \sum_i \beta_i \left(\frac{p}{p_i} \right)^\sigma (p_i - p_i^w) u^0$$

where $p = \left(\beta^d p^d + \sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is an index price, p^d is the price of the domestic

good and p_i and p_i^w are the domestic and world prices of the import good i , the parameters $\beta^d, \beta_1, \beta_2, \dots$ are the expenditure shares (domestic and import). By

appropriate selection of the units of measurement, all domestic prices are equal to 1 in the base equilibrium and, in consequence, $p_i^w = \frac{1}{1 + \tau_i}$.

As Bach and Martin (2001) have shown, there is a closed-form solution for the initial value of the expenditure aggregator. The expenditure index, τ^e , can be estimated by setting the value of the expenditure function (5) equal to the expenditure function with a uniform tariff and solving for that uniform tariff. With all domestic prices are equal to 1 in the base equilibrium, the uniform tariff equivalent in the expenditure case has the closed-form solution:

$$(7) \quad \tau^e = \left(\frac{1 - \beta^d}{\sum_{i=1}^n \beta_i (1 + \tau_i)^{-(1-\sigma)}} \right)^{\frac{1}{1-\sigma}} - 1$$

The closed-form equation for the expenditure index in the CES case guarantees the existence and the uniqueness of the tariff aggregators. The only question remaining is whether the solution has economic meaning. We prove that the expenditure aggregator is always positive, and therefore is economically relevant. As for the tariff revenue aggregator, we show that there are likely to be two solutions, and define the conditions needed to ensure that the estimated index is the economically relevant one.

Proposition 1. In the context of Formula 7, for $\sigma > 1$, the closed form solution for the expenditure aggregator, τ^e is positive.

Proof: In the context of the (7) and with the domestic prices set to 1 in the base equilibrium, world prices are $p_i^w = \frac{1}{1 + \tau_i}$ and we see that

$$\sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} = \sum_{i=1}^n \beta_i (1 + \tau_i)^{\sigma-1} > \sum_{i=1}^n \beta_i \quad \text{because } \sigma > 1 \text{ (we assume that there is at least}$$

one positive tariff). As the denominator is $1 - \beta^d = \sum_{i=1}^n \beta_i$, the fraction is less than 1. With

a negative power, the expression is greater than one, so the tariff revenue index is positive.

The situation is more complex with respect to τ^R . The tariff revenue index, τ^R can be obtained by setting the tariff revenue function (6) equal to the corresponding expression with a uniform tariff, and solving for τ^R . This is similar to solving:

$$(8) \quad c = h(\tau^R)$$

where $h(\cdot; \mathbf{p}_j^w, u_j^0, v_j^0)$ is a real function of τ^R and c depends on disaggregated tariffs. As τ^R cannot be factored out of (8), the tariff revenue aggregator must be obtained numerically. This is a problem for widespread use of the aggregator. Further, the solution may not be unique.

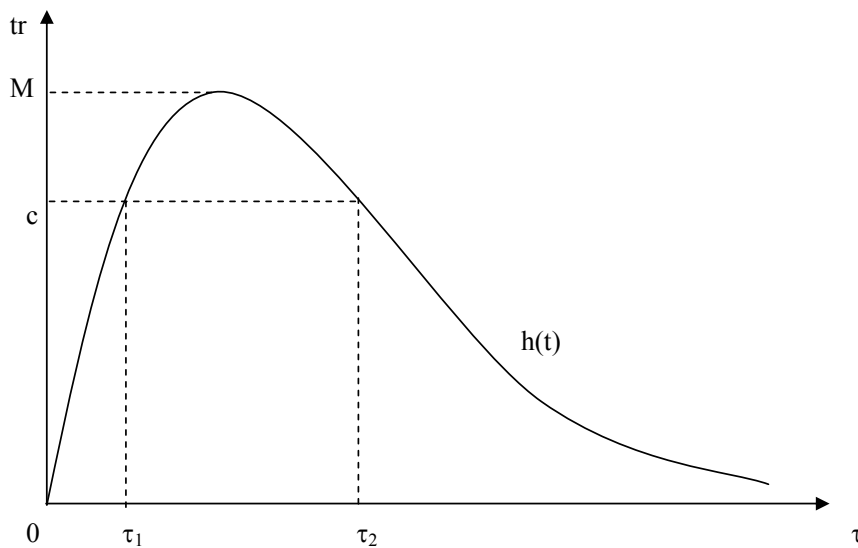
Proposition 2. In the context of Equation (8), for certain values of the parameter c , there are at least two solutions to the equation.

Proof: We sketch the ideas first, following with a more detailed exposition.

Analyzing the equation (8), we notice that the function $h(t)$ is a continuous and positive function with $h(0)=0$, $\lim_{t \rightarrow \infty} h(t) = 0$ and has a maximum on its domain and it reaches this maximum at M . For any tariffs, imports and domestic consumption such that $c < M$ we may apply the intermediate value property and find at least two solutions.

For a better understanding of the problem, let us consider Figure 2:

Figure 2. Uniform tariff solutions for constant tariff revenue



Deleted: ¶
¶
¶
¶
¶

Figure 2 is the (in)famous Laffer Curve for tariff revenues. As tariff rates rise, tariff revenues increase up to some point – M . A further increase in tariffs would cause a decline in revenues, because the reductions in import volumes associated with increased tariffs outweigh the revenue gains. If $c < M$, there are two values τ_1 and τ_2 such that $c = h(\tau_1) = h(\tau_2)$. However, only one of these tariff rates is in the economically relevant

range. No well-informed government would set tariff rates beyond the revenue-maximizing level. Assuming that the objective is to keep tariff revenue constant and assuming economic rationality, we use the lowest uniform tariff that keeps tariff revenue constant. This aggregator has the property that tariff revenues are increasing in this range.

For a detailed proof, we write an explicit form for equation (8):

$$(9) \quad \frac{\sum_{i=1}^n \beta_i p_i^W \tau_i}{\sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma}} = \tau^R (1 + \tau^R)^{-\sigma} \left[\beta^d + (1 + \tau^R)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$$

We define the function $h: R^+ \rightarrow R^+$, $h(t) = t(1+t)^{-\sigma} \left[\beta^d + (1+t)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$.

The function h can be written as $h(t) = k(t) * m(t)$, with $k(t) = t(1+t)^{-\sigma}$ and

$m(t) = \left[\beta^d + (1+t)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$. The function $k(t)$ has the following properties:

1. $k(0) = 0$
2. $\lim_{t \rightarrow \infty} \frac{t}{(1+t)^\sigma} = \lim_{t \rightarrow \infty} \frac{1}{\sigma(1+t)^{\sigma-1}} = 0$, where we applied l'Hospital.
3. $k'(t) = \frac{1 - (\sigma-1)t}{(1+t)^{\sigma+1}}$, the derivative being positive for $t < \frac{1}{\sigma-1}$, zero for $t = \frac{1}{\sigma-1}$ and negative for $t > \frac{1}{\sigma-1}$.

The function $k(t)$ starts from zero, increases until it reaches the maximum in $t = \frac{1}{\sigma-1}$

after which it decreases, converging asymptotically to zero.

The function $m(t)$ has the following properties:

$$1. \quad m(0) = \left[\beta^d + \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}} \quad \text{with } m(0) > 0. \quad \text{Similar with the proof of}$$

Proposition 2, $\beta^d + \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} > 1$ so $m(0) < 1$.

$$2. \quad \lim_{t \rightarrow \infty} m(t) = (\beta^d)^{\frac{\sigma}{1-\sigma}}. \quad \text{As } \beta^d < 1, \quad \lim_{t \rightarrow \infty} m(t) > 1.$$

$$3. \quad m'(t) = \frac{\sigma}{1-\sigma} \left[\beta^d + (1+t)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}-1} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} (1-\sigma)(1+t)^{-\sigma} > 0,$$

so $m(t)$ is an increasing function.

Note that the function $h(t)$ is a continuous and positive function with $h(0)=0$, $\lim_{t \rightarrow \infty} h(t) = 0$, has a maximum on its domain and it reaches this maximum M . For any

tariffs, imports and domestic consumption such that $c = \frac{\sum_{i=1}^n \beta_i p_i^W \tau_i}{\sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma}} < M$ we may

apply the intermediate value property and find at least two solutions.

Observation. The definition of the tariff revenue aggregator may be amended as follows: A tariff revenue aggregator on the commodity group j may be defined as **the low value** uniform tariff that will yield the same tariff revenue as the observed vector of disaggregated tariffs for that particular group of commodities

4. Separability between domestic and imported goods

If we make the additional assumption of separability between domestic and imported products - made routinely in CGE models - it becomes possible to define the expenditure aggregator only over foreign products. This specification requires the usual assumptions for such two-stage budgeting, such as weak separability in demand and homotheticity of the sub-utility functions at the lower level of nesting, but these assumptions are virtually universal in trade applications.

4.1 The Expenditure Tariff Aggregator

Based on the above assumptions, the expenditure aggregator can be defined over a group of imported commodities (no domestic product). Using the homogeneity of the expenditure function to obtain a closed-form solution for τ_j^e from equation (5), the tariff expenditure aggregator is derived as:

$$(10) \quad \tau^e = \left(\sum_{i=1}^n \beta_i (1+t_i)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} - 1$$

Because of the prevalence of the assumption of separability between domestic and imported goods in modern Computational General Equilibrium models, equation (10) is the logical basis for an expenditure-based tariff aggregator.

4.2 The Tariff Revenue Aggregator

The tariff revenue aggregator is designed to return a tariff revenue aggregator that will represent the value of tariff revenues generated by a vector of non-uniform tariffs.

While the trade-weighted average has this property in the initial equilibrium, the weighted average corresponding to a new tariff vector will not generate the actual value of the tariff revenues associated with this average tariff vector.

The tariff revenue index, τ^R , can be obtained by setting the second term in (6) equal to the corresponding expression with a uniform tariff, and solving for τ^R . As τ^R cannot be factored out of (6), it must therefore be estimated numerically.

With a single-level CES aggregator and with the domestic prices all set to unity, the equation that defines the tariff revenue aggregator is:

$$(11) \quad \sum_{i=1}^n \beta_i p_i^w \tau_i = \tau^R (1 + \tau^R)^{-\sigma} \left[\beta_d + (1 + \tau^R)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}} \sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma}$$

Using the assumption of separability between domestic and imported goods, a tariff aggregator ranging over the imported goods alone can be defined. After a few manipulations, we obtain a closed-form solution for τ^R :

$$(12) \quad \tau^R = \frac{\sum_{i=1}^n t_i (1 + t_i) \beta_i}{\left(\sum_{j=1}^n \beta_j (1 + t_j)^{\sigma-1} \right)^{\frac{1}{1-\sigma}}}$$

where the β_j 's are value shares of imports at domestic prices ($\beta_j = \frac{M_j (1 + t_j)}{\sum_k M_k (1 + t_k)}$), where

M_k is the import of product k and t_k is the tariff for the product k), p_j^w is the world price for product j , where ($p_j^w = \frac{1}{1 + t_j}$) because the domestic price is unity, and σ is the

elasticity of substitution.

Next, we use the following lemma:

Lemma. Consider x_1, x_2, \dots, x_n n positive real numbers not all equal and w_1, w_2, \dots, w_n n

positive real numbers such that $\sum_{i=1}^n w_i = 1$. For r a positive real number, we define the r -

weighted mean of the x_1, x_2, \dots, x_n numbers as $A_r = \left(\sum_{i=1}^n w_i x_i^r \right)^{\frac{1}{r}}$. For the given weights

w_1, w_2, \dots, w_n the r -weighted mean of the x_1, x_2, \dots, x_n numbers has the following properties:

1. If $r > s$ then $A_r > A_s$.
2. $\lim_{r \rightarrow 0} A_r = x_1^{w_1} x_2^{w_2} \dots x_n^{w_n}$.

In this framework, we present the following properties for the expenditure and revenue tariff aggregator indices.

Proposition 3. For $\sigma > 0$ the expenditure tariff aggregator is always larger or equal than the weighted average tariff.

Proof. We use the notation t_{wa} for the weighted average tariff ($t_{wa} = \sum_{i=1}^n w_i t_i$), where

$w_i = \frac{M_i}{\sum_{k=1}^n M_k}$, $i = 1, \dots, n$ are the weights). From Lemma 1, we define

$A_{1-\sigma} = \left(\sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ as the $1-\sigma$ weighted mean of world prices $p_1^w, p_2^w, \dots, p_n^w$ with

the weights $\beta_1, \beta_2, \dots, \beta_n$. If there is a uniform tariff, $t_i = t$, $i = 1, \dots, n$ then the expenditure aggregator is equal with trade weighted average. If the tariff is not uniform, we may use Lemma and as $1 - \sigma < 1$, then $A_{1-\sigma} < A_1$, that is:

$$(13) \quad \left(\sum_{j=1}^n \beta_j (p_j^*)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} < \sum_{i=1}^n \beta_i p_i^*$$

We can write:

$$\sum_{i=1}^n \beta_i p_i^w = \sum_{i=1}^n \frac{M_i (1+t_i)}{\sum_{k=1}^n M_k (1+t_k)} \frac{1}{1+t_i} = \sum_{i=1}^n \frac{M_i}{\sum_{k=1}^n M_k (1+t_k)} = \frac{\sum_{i=1}^n M_i}{\sum_{k=1}^n M_k + \sum_{k=1}^n M_k t_k} = \frac{1}{1+t_{wa}}$$

and (13) becomes:

$$\left(\sum_{j=1}^n \beta_j (p_j^w)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} < \frac{1}{1+t_{wa}}$$

and further:

$$(14) \quad \tau^e = \left(\sum_{j=1}^n \beta_j (1+t_j)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} - 1 > t_{wa}$$

Proposition 4. For $\sigma > 0$ the revenue tariff aggregator is always larger or equal than the weighted average tariff and lower or equal than the expenditure tariff aggregator.

Proof. Using (10) and (12), we can write the revenue tariff aggregator as a function of the expenditure tariff aggregator:

$$(15) \quad \tau^R = (\tau^e + 1) \sum_{i=1}^n \beta_i p_i^w t_i$$

We show that right hand side sum can be expressed as a function of the weighted tariff average:

$$\sum_{i=1}^n \beta_i p_i^w t_i = \sum_{i=1}^n \frac{M_i(1+t_i)}{\sum_{k=1}^n M_k(1+t_k)} \frac{t_i}{1+t_i} = \sum_{i=1}^n \frac{M_i t_i}{\sum_{k=1}^n M_k(1+t_k)} = \frac{\sum_{i=1}^n M_i t_i}{\sum_{k=1}^n M_k + \sum_{k=1}^n M_k t_k} = \frac{t_{wa}}{1+t_{wa}}$$

We can write (15) as:

$$(16) \quad \tau^R = (\tau^e + 1) \frac{t_{wa}}{1+t_{wa}}$$

From Proposition 3:

$$(17) \quad \tau^e + 1 \geq 1 + t_{wa} \Rightarrow \tau^R = (\tau^e + 1) \frac{t_{wa}}{1+t_{wa}} \geq t_{wa}$$

Let us consider the function:

$$f: R_+ \rightarrow R \quad f(x) = \frac{x}{1+x}$$

Note that $f'(x) = \frac{1}{(1+x)^2} > 0$ so f is an increasing function. From Proposition 3:

$$\tau^e \geq t_{wa} \Rightarrow \frac{\tau^e}{1+\tau^e} \geq \frac{t_{wa}}{1+t_{wa}} \Rightarrow \tau^e \geq (\tau^e + 1) \frac{t_{wa}}{1+t_{wa}} = \tau^R$$

We proved that:

$$(18) \quad \tau^e \geq \tau^R \geq t_{wa}$$

5. Application

To illustrate the differences between different aggregators, we use a standard two-sector general equilibrium model of small open economies. This is a dual version of the model presented in Devarajan *et al.* (1997), and de Melo and Tarr (1992).⁴ The model is known as the “1-2-3 model”, reflecting the fact that it has one country, two sectors and three goods. The two sectors are producing an export good E , that is not consumed in the country, and a domestic good D (E and D are different), which is consumed only domestically. The third good in the model is the imported good M . The model uses the Armington assumption (Armington, 1969) that domestically produced goods and imported goods are imperfect substitutes. The aggregate production \bar{X} is fixed – an implicit assumption of full employment of all inputs. In the model the country is small, so the world prices ($\bar{\pi}^m$ for imports and $\bar{\pi}^e$ for exports) are constant. The equations of the model are presented in Table 1.

⁴ The model follows Devarajan *et al.* (1997) and De Melo and Tarr (1992). Different versions and applications for the model are presented Devarajan, Lewis and Robinson, (1990, 1993), Go and Sinko (1993), De Melo and Robinson (1989), Benjamin and Devarajan (1985), Benjamin, Devarajan and Weiner (1989). Devarajan and Go (1993) present a dynamic version of the model.

Table 1. The 1-2-3 in Dual Form

| | | |
|-------|--|------------------------------------|
| (A1) | $P = F(P_d, P_m)$ | Consumer price (CES - aggregation) |
| (A2) | $P_o = G(P_d, P_x)$ | Producer price (CET - aggregation) |
| (A3) | $P_m = \bar{\pi}^m (1+T)$ | Import price |
| (A4) | $P_x = \bar{\pi}^x$ | Export price |
| (A5) | $Q_m = f_2(P_m, P_d)$ | Import demand equation |
| (A6) | $Q_x = g_2(P_x, P_d)$ | Export supply equation |
| (A7) | $Q_d^d = f_1(P_m, P_d)$ | Domestic demand equation |
| (A8) | $Q_d^s = g_1(P_x, P_d)$ | Domestic supply equation |
| (A9) | $Rev = P_o \cdot Q$ | Revenue function |
| (A10) | $Trev = T \cdot Q_m \cdot \bar{\pi}^m$ | Tariff revenue |
| (A11) | $u = (Rev + Trev + f)/P$ | Utility function |
| (A12) | $e = P \cdot u$ | Expenditure function |
| (A13) | $D^d - D^s = 0$ | Domestic demand-supply equilibrium |

Where

M, E = imports, exports
 D^d, D^s = demand and supply of the domestic good D
 \underline{Q} = composite consumer good
 \bar{X} = composite production
 $\bar{\pi}^m$ = world price of imports
 $\bar{\pi}^e$ = world price of exports
 P^m = domestic price of imports M
 P^e = domestic price of exports E
 P^d = domestic price of domestic sales D
 P^q = domestic price of composite consumer good Q
 P^x = domestic price of composite output X
 f = net foreign capital inflow

In equation 1, $F(.,.)$ is a constant elasticity of substitution (CES) function for the consumer price aggregator. In equation A2, $G(.,.)$ is a constant elasticity of transformation (CET) function for the export price aggregator. The exchange rate is equal to unity, so equations A3 and A4 link world prices to domestic prices. The import tariff aggregator enters in equation A3.

From the first-order conditions we derive the demand for imports – equation *A5*, supply for export – equation *A6*, demand and supply for domestic good – equations *A7* and *A8*. Equations *A9*, *A10*, *A11* and *A12* characterize revenue, tariff revenue, and utility and expenditure functions. The equilibrium condition for the domestic good is presented in the last equation.

One of the advantages of the “1-2-3 model” is the fact that the necessary data to run the model are readily available. We obtained macro-economic data from the World Development Indicators 2004 - World Bank. The trade data was obtained from UNCTAD’s TRAINS database and from the UN Statistical Division’s COMTRADE database. We used WITS⁵ to extract the trade data.

The goal of this exercise is to compute and to compare the welfare gains from trade liberalization considering three different scenarios: (1) tariffs are aggregated using the import-weighted average tariff; (2) tariffs are aggregated using the expenditure tariff aggregator; (3) tariffs are aggregated using the expenditure tariff aggregator but the balance of trade is corrected by a tariff revenue aggregator factor as suggested by Bach and Martin (1996). We applied the analysis to the following countries: Brazil, India, Indonesia, Norway, Pakistan, South Africa, Thailand and Venezuela. Table 2 presents the results of the exercise. The columns 2-4 present three types of tariff aggregators: the trade-weighted average tariff, the tariff revenue aggregator and the expenditure aggregator. The welfare effects (measured as percentage of GDP) generated by trade

Deleted: Mozambique,

⁵ WITS stands for *World Integrated Trade Solution* and it is an World Bank program that facilitate access to the most important trade databases and has analytical capabilities. At this moment, WITS may be used to access import-export data from COMTRADE, protection (tariffs and non-tariff measures) and trade data from TRAINS, tariff schedules (bounded and applied) and trade data from WTO. WITS may be also used to compute tariff aggregators like TRI, expenditure aggregators and tariff revenue aggregators – a practical implementation of the aggregators discussed in this paper.

liberalization corresponding to the three scenarios described above are presented in columns 5-7.

The empirical relation between the tariff aggregators is the one demonstrated in the theoretical section of this paper – that is, the trade weighted average is lower than tariff revenue aggregator and the latter is lower than the expenditure aggregator. For almost all the countries in our sample, there is a difference of up to 1% between the trade-weighted average and the tariff-revenue aggregator, with the exception of Pakistan, where there is a 3% difference. In contrast, the difference between the tariff-revenue and the expenditure aggregators is much bigger, from 2% in the case of Brazil, up to 19% in the case of Pakistan. One would expect that trade liberalization should produce significant welfare effects in Pakistan since its degree of trade protection is high. However, the welfare effects vary considerably, depending on the type of aggregator used. It varies from 0.13% of GDP using the first scenario (trade-weighted average aggregator) to 0.56% of GDP in the second scenario (expenditure aggregator) to 3.88% of GDP using the third scenario (expenditure tariff aggregator corrected by a tariff revenue aggregator factor). In the case of Thailand, the welfare gains from trade liberalization vary from 0.06 % of GDP if using the first scenario to 0.12% of FDP using the second scenario to 2.05 % of GDP when using the third scenario.

The results presented strongly suggest that aggregating correctly is very important and that the welfare gains from trade liberalization are severely underestimated when using the traditional tariff aggregator. This can have very important policy implications, especially for developing countries, which usually have high degree of protection.

Table 2. Welfare effects after trade liberalization (% of GDP)

| Countries | Tariff aggregators (%) | | | Welfare Effects after Trade Liberalization % of GDP | | |
|--------------|------------------------|---------------------------|------------------------|---|------------------------|--|
| | Trade Weighted Average | Tariff Revenue Aggregator | Expenditure Aggregator | Trade Weighted Average | Expenditure Aggregator | Expenditure Aggregator Corrected results |
| Brazil | 10.0 | 10.1 | 11.7 | 0.04 | 0.05 | 0.20 |
| India | 26.6 | 28.3 | 34.6 | 0.23 | 0.35 | 0.79 |
| Indonesia | 6.0 | 6.3 | 11.8 | 0.02 | 0.07 | 0.73 |
| Norway | 1.2 | 1.3 | 8.5 | 0.001 | 0.04 | 1.15 |
| Pakistan | 16.1 | 19.1 | 37.6 | 0.13 | 0.56 | 3.06 |
| South Africa | 5.0 | 5.1 | 7.7 | 0.02 | 0.04 | 0.52 |
| Thailand | 9.4 | 9.8 | 14.1 | 0.06 | 0.12 | 1.86 |
| Venezuela | 13.7 | 13.9 | 15.8 | 0.09 | 0.11 | 0.31 |

Deleted: Mozambique [1]

6. Conclusions

We analyze the tariff aggregators proposed by Bach and Martin (1996) and show that for the tariff revenue aggregator there are multiple uniform tariffs that yield the same revenue over a large set of parameters. We consequently redefine the tariff revenue aggregator so that it leads the existence and the uniqueness of the tariff aggregators. We prove that the expenditure aggregator is always positive, therefore is economically relevant. We derive the theoretical properties of the expenditure and tariff revenue aggregators, and under certain assumptions, we develop closed form solutions for both the expenditure as well as the tariff revenue aggregators. Finally, for a positive elasticity of substitution between products, we show that the trade-weighted average tariff is lower

than tariff revenue aggregator and the latter is lower than the expenditure aggregator. Illustrative examples provided in the final section of the paper show that use of appropriate aggregators can greatly change the results obtained from the analysis.

Reference

- Anderson, J. E. 1995. "Tariff - Index Theory," *Review of International Economics* 3(2): 156 – 173.
- Anderson, J. E., and J. P. Neary. 1992. "Tariff reform with quotas, partial retention and tariffs", *Econometrica* 60(1):57-76.
- Anderson, J. E., and J. P. Neary. 1993. "A New Approach to Evaluating Trade Policy," Mimeo, Boston College.
- Anderson, J. E., and J. P. Neary. 1994. "Measuring the Restrictiveness of Trade Policy," *The World Bank Economic Review*, 8 – 2: 151 – 170.
- Anderson, J. E., and J. P. Neary. 1996. "A New Approach to Evaluating Trade Policy," *Review of Economic Studies*, 63: 107-25.
- Anderson, J. E., and J. P. Neary. 2003. "The Mercantilist Index of Trade Policy," *International Economic Review*, 44 (2): 627 -49.
- Bach, C. F., and W. Martin. 2001. "Would the Right Tariff Aggregator for Policy Analysis Please Stand Up?," *Journal of Policy Modeling*, 23: 621-35.
- Bach, C. F., W. Martin, and J. A. Stevens. 1996. "China and the WTO: Tariff Offers, Exemptions, and Welfare Implications," *Weltwirtschaftliches Archiv – Review of World Economics*, 132 (3): 409 – 431.
- Bordo, M. D., B. Eichengreen, and D. Irwin. 1999. "Is Globalization Today Really Different than Globalization a Hundred Years Ago?," NBER Working Paper No. 7195.

- Chambers, R.G. 1988. *Applied Production Analysis. A Dual Approach*. Cambridge: Cambridge, University Press.
- Deaton, A., and A. Muellbauer. 1980. *Economics and Consumer Behavior*. Cambridge: Cambridge University Press.
- Hertel, T. W., and M. E. Tsigas. 1997. "Structure of GTAP" in T. W. Hertel (ed.) *Global Trade Analysis: Modeling and Applications*. Cambridge: Cambridge University Press.
- Kee, H.L., A. Nicita and M. Olarreaga, 2004. "Import Demand Elasticities and Trade Distortions," World Bank Paper No. 3452.
- Leamer, E. 1974. "Nominal Tariff Averages with Estimated Weights," *Southern Economic Journal*, 41:34-46.
- Lloyd, P.J. 1994. "Aggregation by Industry in High-Dimensional Models," *Review of International Economics*, 2(2): 97-111.
- Martin, W., D. van der Mensbrugghe, and V. Manole. 2003. "Is the Devil in the Details?: Assessing the Welfare Implications of Agricultural and Non Agricultural Trade Reforms," presented at The 6th Annual Conference on Global Economic Analysis, June 12 - 14, Scheveningen, The Hague, The Netherlands.
- OECD (2005), *Agricultural Policies in OECD Countries: Monitoring and Evaluation*, Organisation for Economic Co-Operation and Development, Paris.
- O'Rourke, K. 1997. "Measuring Protection: A Cautionary Tale," *Journal of Development Economics*, 53:169-83.

Salvatici, L. 2001. "Trade Distortion Indexes and Multiregional AGE Models: The Case of the Common Agricultural Policy," Working Paper No. 45, Universita' Degli Studi di Roma "La Sapienza", Dipartimento di Economia Pubblica, Roma,

| | | | | | | |
|------------|-----|-----|------|------|------|------|
| Mozambique | 9.1 | 9.3 | 10.8 | 0.06 | 0.09 | 0.13 |
|------------|-----|-----|------|------|------|------|