Restraining Competition: Explicit and Implicit Ceilings in Auctions*

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Abstract

In symmetric common value auctions where bidders differ ex–post in information quality, a seller may benefit from imposing a ceiling on allowable bids. By reducing the winner's curse facing poorly informed bidders, a ceiling encourages them to bid aggressively. This may reduce information rents earned by better informed bidders, yielding the seller higher expected revenues compared to a standard auction. Such a ceiling may be explicit (a firm commitment not to accept bids above the ceiling) or implicit (a credible threat not to honor the outcome if anyone bids higher than the ceiling). Either situation can be interpreted as one where the object is offered for sale at a fixed price or the best offer; or one where the seller contractually specifies a right to buy back the object from the winner.

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*Conversations with Alessandro Citanna, Parikshit Ghosh and Faruk Gul have aided in the development of these ideas.

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1 Introduction

When faced with competing potential buyers, should a seller simply let the highest bidder win or should he specify an upper bound on allowable bids in order to maximize revenues? When seeking to place a hitherto unknown but potentially talented player with a sports club, could the player and his agent gain by specifying a cap on the acceptable salary and other terms of trade? When outsourcing a public works contract, could a government agency lower expected costs by announcing a minimum price guarantee for the suppliers? In general, could one gain by putting limits on the share of the pie that one is willing to *receive*? We consider these questions in the context of a common value auction for a single indivisible object.

In such common value environments, a bidder fears overpaying precisely when her bid is the highest and she wins the auction— the act of winning is negative information. This winner's curse problem becomes acute when, at the bidding stage, some bidders have information of inferior quality than other bidders. In such cases, a poorly informed bidder can win only when not outbid by a better informed bidder. Anticipating such an acute winner's curse, rational but poorly informed bidders bid cautiously in equilibrium. This adversely affects the incentives to bid aggressively for all types of bidders and may lower the seller's expected revenues.

We show that in such a situation the seller can benefit by imposing an *explicit ceiling* on allowable bids. By imposing a ceiling the seller restrains competitive bidding by better informed bidders. As a result, any bidder when winning at the ceiling attaches positive probability to the joint event that (i) the true–value of the good is higher than the ceiling and (ii) that he is tied with a bidder with superior information. This reduces the fear of being outbid for poorly informed bidders and encourages them to bid aggressively. Since poorly informed bidders earn no informational rents, aggressive bidding by them enables the seller to extract more rents from better informed bidders. Consequently, compared to a standard auction without a ceiling, the seller earns higher revenues on average.

A ceiling is not costless— by imposing one the seller rules out the possibility of receiving a higher price. When considering the relative merits of an auction with a ceiling and an auction without one, the seller has to trade-off the expected 'benefit' of increased bidding below the ceiling with the expected 'cost' of not allowing bids higher than the ceiling. For a fixed number of bidders, we show that imposing a ceiling is profitable for the seller if and only if the expected proportion of better informed bidders is low enough. Moreover, for any fixed proportion of better informed bidders, imposing a ceiling is better for the seller if the total number of bidders is large enough.

The intuition is as follows. When most bidders are poorly informed, the seller's expected cost from not allowing better informed bids at levels higher than the ceiling is low, while the expected benefit via more aggressive participation from poorly informed bidders is high. Furthermore, when the number of bidders is large enough, each poorly informed bidder when winning at the ceiling attaches large probability to the presence of at least one other better informed bidder. As a result, the seller is able to set a relatively high ceiling but still guarantee aggressive participation from poorly informed bidders, lowering the cost and raising the benefit from imposing a ceiling.

In IPO auctions, explicit ceilings on allowable bids are often specified.[1] This is justified on the grounds of providing a 'level playing field' or a 'bidding guide' or an 'anchor price' for small investors. Although we do not analyze the auction of a divisible good such as shares in a company, a bid ceiling performs similar roles in the context of our model.[2] In 'reverse' auctions that are held in procurement contexts, a bid ceiling takes the form of a minimum price guarantee. Given our results, such guarantees can be understood to reduce the risk of underestimating costs faced by potential suppliers.

Although auctions with explicitly announced ceilings are somewhat rare, we argue in the second part of the paper that many commonly observed selling procedures may indeed be manifestations of an *implicit ceiling* on allowable bids, at least in situations where the seller has the right to cancel the auction after the bidding concludes and resell the object later.[3] To illustrate, consider commonly observed terms of sale where a seller offers the object at a pre-specified *fixed price or the best–offer*. With such an offer it is clear that the seller is willing to entertain bids below the fixed price. It is less clear however what the seller intends to do in case a bid exceeds the fixed price. An explicit ceiling can be interpreted as such terms of sale, in situations where the seller can commit to not entertaining bids higher than the fixed price. However, even when

[1]The recent Google IPO is one example, where acceptable bids had to lie in the range $85 to $95. See http://www.wrhambrecht.com for more examples. Most IPOs in the U.S. are not held via auctions suggesting that some consideration other than maximizing issue proceeds may be guiding IPO design.

[2]Share buyback auctions are also held within a pre–specified band of prices and so our results provide some insight into this phenomenon as well.

[3]This may be true in informal environments such as an auction held in a college campus. However, the right to cancel is widely observed, even in formal environments. U.S. Government agencies such as the Department of the Interior and the Department of Agriculture reserve the right to cancel, respectively, offshore oil and gas lease auctions and procurement auctions even after announced reserve requirements have been met by the bidders. However, such a right is not typically exercised. See Porter (1995) and McDonald et al. (1998).

a seller cannot explicitly announce a ceiling and commit to it, we show that the threat of cancellation may allow the seller to create an implicit ceiling via such an offer. In equilibrium, bidders interpret the fixed price as a bid ceiling, expect no other bidder to bid above it, themselves do not bid above it fearing cancellation, the threat of cancellation is credible and the first auction is never cancelled.[4]

It is important to note in this respect that a threat of cancellation may in fact be contractually specified. If the seller attaches to the object a pre-specified right to buy back the object from the winner at a fixed price, then this, together with the possibility of resale, may also have the identical effect of creating an implicit ceiling at the buy–back price. In short, the threat of a buy–back discourages better informed bidders from bidding above the implicit ceiling which in turn encourages poorly informed bidders to bid up to it. Such an implicit ceiling benefits the seller whenever an explicit one does.

A mechanism where the seller offers to sell the object at a fixed posted price also reduces the fear of being outbid for poorly informed bidders, just like an auction with a ceiling. As Wang (1993), Bulow and Klemperer (2002) and Campbell and Levin (2002) show, a posted price mechanism may in fact dominate a standard auction in expected revenues. However, in our environment such a mechanism introduces the bidders to a winner's curse problem on the downside, i.e., to winning the good and paying the posted price in states where better informed bidders do not bid at the posted price. By selling the object in an auction with either an implicit or an explicit ceiling, the seller lowers both the fear of being outbid as well as this downside winner's curse problem for poorly informed bidders. As a result he does better than either an auction or a fixed price mechanism.

In private value auctions, a seller may benefit from undertaking inefficient amount of trades by imposing a reserve or floor price (i.e., a lowest allowable bid; see, e.g., Myerson (1981)). As is well-known, in common value contexts the gains from a reserve price are less pronounced, especially for a large number of bidders (see, e.g., Levin and Smith (1996)). We provide a unified analysis of reserve and ceiling prices in our model and show that the optimal reserve price is zero for a sufficiently large number of bidders. In common value environments therefore, imposing a ceiling can benefit the seller more than imposing a floor.[5]

[4]A friend, the economist A. Citanna has informed us that when he bid higher than the pre–specified price in such a mechanism, while hunting for apartments in Paris, his bid was ignored by the owner's agent in allocating the apartment and, furthermore, it was evident that he had violated the Parisian code of proper bidding behavior.

[5]We do not consider the optimal direct mechanism in this paper. Instead, we justify our focus on ceilings as

The revenue implications of bid ceilings in auctions have been studied before in Che and Gale (1998) and Gavious, Moldovanu and Sela (2002). Both papers show that imposition of bid ceilings may increase the seller's revenue, thus providing insights on the value of imposing caps on contributions by political lobbies. A common thread that connects these essays with ours is that the imposition of bid ceilings encourages aggressive bidding behavior. However, there are substantive differences. Neither of these papers study a common value environment. Further, the results of both these papers are derived for contests (i.e., all-pay auctions). In Che and Gale (1998), the results are driven by ex ante asymmetry between publicly known bidder valuations. Gavious et al. (2002) show in an independent private value environment that if bidders are ex ante symmetric, a bid ceiling may be optimal, but only if the total cost to a bidder is strictly convex in her bid.[6] Intuitively, in private value all–pay auctions, a ceiling encourages bidding by lowering a bidder's expected costs in the event that she *does not* win the auction. In contrast, in our common value winner–pays auction, a ceiling encourages bidding by lowering a bidder's expected winner's curse, in the event that she *does* win the auction.

In a preliminary note, although in a context similar to the present paper, Chakraborty (2002) provides a numerical example of a sealed-bid auction with ex ante asymmetric bidders where imposition of an explicit ceiling can improve revenue. In rather different contexts, Chen and Rosenthal (1996a, b) show that if buyers arrive randomly and sequentially, and incur inspection costs of discovering their own true valuations, a seller may benefit from imposing a price ceiling. Neither of the last two papers analyze auctions. Finally, with respect to our results on implicit ceilings and the value of the option to cancel, Horstmann and LaCasse (1997) analyze a model where an auction can be cancelled by an informed seller to convey information, thus providing a justification for secret reserve prices. In our setting however the seller is uninformed. The value of the option to cancel instead arises out of the fact that information may be revealed by the bidding process itself. Taking this into account, better informed bidders bid less aggressively and forego some information rents, thereby encouraging overall participation and raising the seller's expected revenue.

The rest of the paper proceeds as follows. In Section 2, we introduce our model, characterize the equilibrium of the standard auction and compare it with auctions that have a reserve price as well as with posted price mechanisms. In Section 3 we consider explicit ceilings, show when it dominates a standard auction and a posted price mechanism and also characterize the

a simple change in the rules of a standard auction, itself a commonly observed mechanism.

[6]In a recent paper, Sahuguet (2004) extends these results to an asymmetric environment.

optimal ceiling and floor combination. In Section 4 we consider implicit price ceilings. Section 5 concludes, while the Appendix contains the majority of the proofs.

2 The Standard Auction

A seller wants to sell an indivisible object to one of $n \geq 2$ bidders in the set $N = \{1, ..., n\}$. The value of the object to the seller equals 0 whereas the common value of the object to all the buyers is denoted by a random variable V that is distributed according to a continuous increasing distribution function F with density f that is positive on $\mathbf{V} = [0, 1]$.[7] We denote by v a realization of V while letting $\overline{v}$ be the expected value of V. Let $h(v) = \frac{f(v)}{1-F(v)}$ denote the hazard rate associated with F that we assume is monotone increasing. The payoff to a buyer from obtaining the object of value v at price p is $v - p$ and the payoff from not obtaining the object is 0. The payoff to the seller is the price that he receives in the event of making a sale, and zero otherwise.

The realized value of V is not common knowledge, although each bidder may have private information about it. We impose an extremely simple and tractable information structure on the bidders. Each bidder $i \in N$ knows the realization v of V with probability $\alpha \in (0, 1)$; otherwise, he receives no additional information and only knows the prior. The probability that any one bidder is informed is independent across bidders and independent of V. Each bidder knows whether or not he is informed but does not know how many other bidders are informed. For $0 \leq k \leq K \leq n$, let $\pi(k, K) = \binom{K}{k}\alpha^k(1-\alpha)^{K-k}$ be the probability that k out of K bidders are informed. The seller is uninformed.

While the information structure described above is special, it captures in a simple way the idea that bidders who are ex–ante identical may be ordered (in the sense of Blackwell), in the quality of their information at the interim stage. We hope to convince the reader in what follows that the intuition underlying our results on the desirability of a bid ceiling needs only this ordering property of the quality of information.[8]

We suppose that the seller sells the object in a continuous ascending clock English (or

[7] See Section 3 for a discussion of how our results are affected when V has unbounded support on $\mathbb{R}_+$.

[8] The case where each bidder either obtains the *same* (across bidders) noisy signal of true value, and is uninformed otherwise (or obtains a coarser signal of inferior quality) is immediately covered by our results. Chakraborty (2002) provides a numerical example of an asymmetric second price auction with three bidders, where bidders may either know the true value or obtain conditionally independent noisy signals, and where a ceiling may be optimal.

Japanese) auction.[9] We allow the seller to set a reserve or floor price r (i.e., a minimum allowable bid) or a ceiling price c (i.e., a maximum allowable bid) with $0 \leq r \leq c \leq 1$.[10] Given a reserve and ceiling pair $\{r, c\}$, the rules of the auction are as follows. A clock (signifying the price) rises continuously starting at r and continuing till c. At each point, each bidder has to decide whether to drop out or not, upon observing the number of other bidders remaining in the auction and when other bidders have dropped out. Drop-outs are final. Let $A(p)$ be the set of bidders who are active when the clock is at $p \in [r, c]$. For any set X, denote by $|X|$ its cardinality. Since drop–outs are final, for $p > p'$, $A(p) \subset A(p')$, i.e., $|A(p)|$ is a non–increasing function of p. For $p > r$, let $Q(p) = \cap_{p':p'<p} A(p') - A(p)$ be the set of bidders who quit at p and let $Q(r) = N - A(r)$. If $|A(r)| = 0$, then no bidder is ever active and the good is not sold. On the other hand, if $|A(p)| \geq 2$ for all $p \in [r, c]$, then the auction stops at $p = c$ and the winner is decided by uniform randomization among the bidders in $A(c)$. In such a case the winner pays a price equal to c for the object. In all other cases, the auction stops at a price $P = \inf\{p \in [r, c] | \, |A(p)| \leq 1\}$, equal to the price the winner pays. If $|A(P)| > 0$ then the winner is decided by uniform randomization among bidders in $A(P)$; while if $|A(P)| = 0$ then it must be that $|Q(P)| > 0$ and the winner is decided by uniform randomization among the bidders in $Q(P)$.

This completes the description of the auction with a reserve and ceiling pair $\{r, c\}$.[11] The standard auction is a special case of the auction above where $r = 0$ and $c = 1$. A posted price mechanism is another special case where $r = c$.

In any auction defined by a pair $\{r, c\}$, a strategy for any bidder is to choose a probability of quitting at any point $p \in [r, c]$, upon observing the past history of quits and given the bidder's private information. Due to the symmetry of the model, it is natural to look for a perfect Bayesian equilibrium in symmetric bidding strategies. Our objective to characterize the reserve

[9] See Milgrom and Weber (1982). We focus on such an auction format only because it makes equilibrium strategies particularly easy to describe. All qualitative results extend to the case where the seller instead holds a sealed-bid second price auction, for example.

[10] Throughout we will assume that the seller can commit to any ceiling or floor price that he may impose and in fact to honoring the rules of the auction. In Section 4, we turn to the question of a lack of commitment and implicit ceilings.

[11] While this description of the auction is adequate for presenting our main results, it cannot capture the possibility of simultaneous randomized drop–outs by a subset of at most $n-2$ bidders, a possibility that does not arise in any of the equilibria characterized in Propositions 1—7. A richer structure that captures this possibility is described in footnote 18.

and ceiling combination $\{r, c\}$ that maximizes the seller's ex–ante expected revenues, given that bidders play a symmetric equilibrium.

We begin by deriving the symmetric equilibrium for the standard auction. In order to do so, it is convenient to define the function $H : \mathbb{R}_+ \to \mathbb{R}_+$ as follows:

$$H(x) = (1 - F(x)) [E(V|V > x) - x] = \int_x^\infty [1 - F(u)]du \tag{1}$$

As will become clear below, in the standard auction $H(x)$ is the informational rent an informed bidder earns from winning at a price x. Observe that $H(x)$ is a positive, decreasing and convex function with $H(0) = \overline{v}$ and $H(x) = 0$ for $x \geq 1$.

Proposition 1 *Symmetric equilibrium strategies in the standard auction are as follows. Any informed bidder who knows $V = v$ drops-out only at $p > v$, staying in otherwise, regardless of the history of quits. Any uninformed bidder quits as soon as anyone else quits or at $p > \overline{v}$. The ex–ante expected revenue for the seller equals*

$$R_s = \overline{v} - n\alpha(1 - \alpha)^{n-1} H(\overline{v}) \tag{2}$$

Proof. Consider bidder i when he is informed that $V = v$. At any price $p < v$, regardless of the history of drop–outs (and, in fact, the strategies of the other bidders), bidder i makes zero expected profits from dropping out and non–negative expected profits from continuing so that it is in his interest to continue. At a price $p = v$, such a bidder makes zero profits from both dropping out and continuing and so it is in his interest to continue. Finally, at $p > v$ he makes weakly lower profits from continuing compared to from dropping out. Thus, it is a best–response for informed bidders to behave as specified.

Next consider uninformed bidders. Fixing the behavior of informed bidders, if uninformed bidders observe a drop-out at any p for which, given the observed history, no uninformed bidder drops out with positive probability, all uninformed bidders should infer that the value of the good is equal to p. In such a case it is a best–response for each uninformed bidder to drop–out, as specified. Furthermore, in any symmetric equilibrium, any uninformed bidder cannot drop–out with positive probability at any price $p < \overline{v}$, conditional on a history that is consistent with all other bidders being uninformed. For if not, then by continuing with probability 1 at p and dropping out immediately after, any such bidder wins the object with a strictly higher probability at the same price p whenever all other bidders are uninformed and drop out at p, leading to strictly higher expected profits in that state; and the same expected profits in all other states where at least one bidder is informed. Similarly, in any symmetric equilibrium, uninformed bidders cannot continue with positive probability at any price $p > \overline{v}$, conditional on a history that is consistent with all other bidders being uninformed, as it is strictly better to drop out at p. Finally, at $p = \overline{v}$, conditional on a history consistent with all other bidders being uninformed, any uninformed bidder is indifferent between dropping out or continuing. It follows that it is a best–response for all uninformed bidders to behave as specified, conditional on observing no drop–outs.

Given the specified behavior, a bidder earns strictly positive profits when all three of the following conditions are satisfied: (i) he is informed, (ii) every other bidder is uninformed and (iii) the realized value of the good is higher than its expected value $\overline{v}$. Thus the sum of the ex–ante expected profits of all of the n bidders equals $n\alpha(1-\alpha)^{n-1}H(\overline{v})$. Since the object is always sold in equilibrium, the sum of the expected revenues for the seller and the expected profits of all bidders must equal $\overline{v}$, yielding expression (2) for seller revenues.■

The equilibrium characterized by Proposition 1 is identical to that characterized by Milgrom and Weber (1982), in a more general model. In the context of our information structure, it is essentially a dominant strategy for informed bidders to continue till the true value. Given this, uninformed bidders must quit as soon as they observe a quit and so earn zero profits when competing with at least one informed bidder. Furthermore, even when they observe no prior drop–outs, uninformed bidders quit at $\overline{v}$, the maximum price consistent with earning non–negative profits when all bidders are uninformed. Thus, uninformed bidders make zero profits regardless of how many other bidders are informed. An informed bidder makes positive profits when he is the only informed bidder and the value of the object is higher than its ex–ante expected value.[12]

With the standard auction in place as our benchmark mechanism we turn next to the question of imposing a reserve price $r > 0$ but no ceiling (i.e., $c = 1$). In the context of a more general model Levin and Smith (1996) show that the benefits from a reserve price asymptotically approach 0 as the number of bidders increases. We confirm this finding in our context with our next result which shows that there is no benefit from imposing a reserve price for finite n.

[12]The arguments made in the proof of Proposition 1 suggest that the equilibrium characterized is the unique symmetric equilibrium, modulo some freedom in specifying behavior off the path of play and also knife–edge cases where an informed bidder (who has observed $V = v$) differs in his behavior at $p = v$, and\or uninformed bidders differ at $p = \overline{v}$ conditional on not having observed drop–outs. Such differences will not affect the seller's ex–ante expected revenues.

In order to do this, for each α we let n_α^r be the smallest value of n satisfying:

$$\frac{\overline{v}f(\overline{v})}{1-F(\overline{v})} \geq \frac{\pi(1,n)}{1-\pi(0,n)} \tag{3}$$

Proposition 2 *For each α, if $n \geq n_\alpha^r$, then the expected revenues of the seller from the standard auction is higher than the expected revenue from any auction with a positive reserve price.*

Proof. In the Appendix.□

For any positive reserve price r, informed bidder behavior will remain unchanged from the standard auction when $v \geq r$ (with no participation when $v < r$). However, for any positive reserve price, uninformed bidders will not participate with strictly positive probability, and will not participate at all for a sufficiently high reserve, e.g., when $r > \overline{v}$. In the Appendix we show that it does not pay the seller set such a high reserve price that excludes uninformed bidders completely and lowers the probability of making a sale, provided there are sufficiently many bidders.[13]

For a sufficiently low reserve price, uninformed bidders will participate with positive probability. However, they will bid less aggressively than in the standard auction, typically quitting below $\overline{v}$. As a result, informed bidders will make higher profits in comparison to the standard auction. Since uninformed bidders make zero profits and the object is not sold with positive probability, the seller is strictly worse off from such a low reserve price relative to the standard auction, regardless of n.

Before we end this section we turn briefly to posted price mechanisms where the seller offers up the object at a fixed price. Wang (1993), Bulow and Klemperer (2002) as well as Campbell and Levin (2002) demonstrate that posted price mechanisms can be attractive to a seller in comparison to the standard auction because the rent transferred to bidders when the true value of the object is higher than the posted price mitigates the winner's curse and encourages aggressive bidding. In our model, a posted price mechanism is not without its costs however as an uninformed bidder may end up winning the object with a greater chance when informed bidders know the value of the object is below the posted price and so do not bid. As we show next, the 'upside' benefit of reducing the winner's curse that a posted price has may not be enough to swamp its 'downside' risk, so that the standard auction will yield the seller higher expected revenues compared to any posted price mechanism, at least for n large enough.

[13]Note from (3) that for the uniform distribution such a high reserve price is not optimal for all $n \geq 2$; a condition that also holds for all distributions that dominate the uniform distribution in the hazard rate, for example.

Proposition 3 *For each* α*, there exists* n_α^p *such that if* $n \geq n_\alpha^p$*, the expected revenues of the seller from the standard auction is higher than the expected revenue from any posted price mechanism.*

Proof. In the Appendix.□

The intuition for this result is similar to that for the last one. For any fixed price p, informed bidders participate if and only if $v > p$. For a high enough p (e.g., $p \geq \overline{v}$), uninformed bidders will not participate. In such a case, the seller can obviously do better by instead holding an auction with a reserve price equal to p. From Proposition 2 however, such an auction is dominated by the standard auction when n is large enough.

For a sufficiently low posted price, uninformed bidders will participate with positive probability. However, conditional on winning at the low posted price $p < \overline{v}$, informed bidders will make higher profits compared to winning at a price $\overline{v}$ in the standard auction. Furthermore, an informed bidder will make positive profits from winning not only when he is the only informed bidder (as in the standard auction) but also when there is more than one competing informed bidder present. Since the probability of being the only informed bidder becomes vanishingly small when there are many bidders, any posted price mechanism is dominated by the standard auction for n large enough.

3 Explicit Ceilings

The discussion of posted price mechanisms in the last section illustrates why imposing a ceiling in an auction may be better for seller revenues than both a posted price as well as a standard auction. Like a posted price mechanism (but unlike a standard auction), a ceiling on allowable bids reduces the fear of uninformed bidders from being outbid precisely when the object is worth bidding for. Unlike a posted price mechanism (but like the standard auction), by holding an auction at prices below the ceiling the seller reduces the downside winner's curse problem for uninformed bidders as they typically pay a price lower than the ceiling when the value of the good is low. In Section 3.1 we consider a simple example of a ceiling and show that it dominates the standard auction when the number of bidders is large enough, while in Section 3.2, we turn to the question of the optimal ceiling.

3.1 A Simple Ceiling

Let $c^* \in (\overline{v}, 1)$ be a candidate ceiling price that solves the following equation:

$$\pi(0, n-1)[\overline{v} - c^*] + (1 - \pi(0, n-1))H(c^*) = 0 \tag{4}$$

It is straightforward to verify the existence of a unique c^* that solves (4). In Proposition 4 we characterize the symmetric equilibrium of the auction with a ceiling c^* (and $r = 0$, henceforth referred to as the c^*–auction). In symmetric equilibrium, informed bidders stay in till the true value or the ceiling c^*, whichever is lower, while uninformed bidders bid till the ceiling, dropping out before as soon as any one else drops out. Expression (4) states that an uninformed bidder earns zero expected profits conditional on bidding at the ceiling, given that no quits have occurred before— the first term is his probability weighted expected payoff from winning in the event that all other bidders are uninformed and have bid till the ceiling, while the second term is his probability weighted expected payoff from winning in the event that at least one other bidder is informed and $V > c^*$.[14] Of course, the bidder earns zero profits when he does not win. Finally, if the true value of the good $V < c^*$ and at least one informed bidder is present, the latter drops out at V triggering drop outs by all uninformed bidders immediately after, so that uninformed bidders earn zero expected profits overall. We will show later that the ceiling c^* is the highest possible ceiling consistent with uninformed bidders bidding up to the ceiling with probability 1. For the moment observe that c^* is increasing in n— when n becomes large an uninformed bidder attaches a large probability to at least one other bidder being informed and so is willing to bid till a high ceiling.

With the ceiling c^* a bidder earns profits only when both the following conditions are satisfied: (i) she is informed, and (ii) the value of the good is greater than c^*. We therefore obtain expression (5) below for the seller's ex–ante expected revenues. Note than in contrast to the standard auction (but like a posted price mechanism), an informed bidder may earn strictly positive profits even when other bidders are informed.

Proposition 4 *Symmetric equilibrium strategies in the c^*–auction are as follows. Any informed bidder who knows $V = v$, quits only when $p > v$ if $v < c^*$ and staying in till c^* otherwise,*

[14]Note that in both cases, the bidder will be tied with n other bidders when winning, and so will win with probability $\frac{1}{n}$, which thus cancels out from the zero–profit condition (4). Furthermore, the probabilities in expression (4) should both be conditional probabilities. But since this only has the effect of dividing the expression by a constant, it is harmless to drop it.

regardless of the history of quits; while any uninformed bidder quits whenever anyone else quits but otherwise does not quit at any $p \in [0, c^*]$*. The ex–ante expected revenue for the seller equals*

$$R(c^*) = \overline{v} - \alpha H(c^*) \tag{5}$$

Proof. Fix the ceiling c^* defined by (4). It is straightforward to show, by using arguments identical to those used previously, that it is essentially a weakly dominant strategy for an informed bidder to stay in the auction till the true value, or the ceiling, whichever is lower.

Consider next uninformed bidders. Due to arguments identical to those contained in the proof of Proposition 1 it follows that uninformed bidders will not drop out at any price strictly less than $\overline{v}$ conditional on having observed no drop–outs, in a symmetric equilibrium.

Let $\sigma_{\overline{v}} \in [0, 1]$ be the probability with which uninformed bidders continue at $p = \overline{v}$, given no prior drop–outs. Observe first that by dropping out at $\overline{v}$, an uninformed bidder earns zero expected payoffs, and this is true regardless of how other uninformed bidders behave. Observe next that $\sigma_{\overline{v}} > 0$ in a symmetric equilibrium. For if not, then by continuing at $\overline{v}$ till c^*, dropping out immediately after a drop–out, an uninformed bidder competes only against informed bidders and so makes strictly positive expected payoffs when he wins at the ceiling and zero payoffs otherwise.

We show next that, in fact, $\sigma_{\overline{v}} = 1$. Suppose not, so that an uninformed bidder is indifferent between continuing and dropping out at $\overline{v}$. Let $k' = 0, ..., n$ be the number of bidders who continue at $\overline{v}$. Then, conditional on observing $k' \geq 2$ bidders continuing at $\overline{v}$, any uninformed bidder who continues at $\overline{v}$ must continue with probability 1 at each $p \in (\overline{v}, c^*]$, given no further drop–outs. For suppose to the contrary that for some such k' uninformed bidders continue with probability $\sigma_p < 1$ at some $p > \overline{v}$. The expected payoffs from dropping out at p conditional on having reached it is equal to $(\overline{v} - p) < 0$, times the probability of winning the object upon dropping out at p, an event that occurs with positive probability only when all other bidders are uninformed. Observe that since $\sigma_p < 1$, this is equal to the expected payoffs overall from following this strategy of continuing at p with probability σ_p. However, for any such k' and p, by dropping out earlier, at some $p' = \overline{v} + \varepsilon < p$, any such uninformed bidder can earn strictly higher expected payoffs, for ε small enough, a contradiction establishing that uninformed bidders must continue till c^*, for each $k' \geq 2$, after continuing at $\overline{v}$, given no further drop–outs. It follows from this that, for each k', an uninformed bidder must make zero expected payoffs given that he continues at $\overline{v}$ and observes that k' bidders are continuing. For if from continuing at $\overline{v}$ and observing k', an uninformed bidder earns negative expected payoffs from continuing till c^*, for some k', then it is better for him to drop–out immediately and earn zero expected payoffs. Given this, if an uninformed bidders earns strictly positive payoffs after continuing at $\overline{v}$ and observing k', for some k', it is strictly better for him to continue at $\overline{v}$ rather than drop–out with positive probability, a contradiction with the supposition that he is indifferent between dropping out and continuing at $\overline{v}$. However, the expected payoffs from continuing at $\overline{v}$, observing k' and then continuing till c^*, in the case that $k' = n$, is equal to (ignoring multiplicative constants),

$$\pi(0, n-1)\xi_{\overline{v}}(n-1, n-1)[\overline{v} - c^*] + \sum_{k=1}^{n-1} \pi(k, n-1)\xi_{\overline{v}}(n-1-k, n-1-k)H(c^*) \qquad (6)$$

where $\xi_{\overline{v}}(k, K) = \binom{K}{k}\sigma_{\overline{v}}^k(1-\sigma_{\overline{v}})^{K-k}$ is the probability that k out of K uninformed bidders continue at $\overline{v}$. Since $\sigma_{\overline{v}} \in (0, 1)$, from the definition of c^* in (4) it is easily seen that the expression in (6) is strictly positive, a contradiction with the fact established above that the expected payoffs from continuing at $\overline{v}$ and observing $k' = n$ is zero. Thus, $\sigma_{\overline{v}} = 1$.

Given that uninformed bidders continue with probability 1 at $p = \overline{v}$, it follows that they must continue with probability 1 at any $p \in (\overline{v}, c^*]$, given no drop–outs— for if not, they make strictly negative profits from dropping out at any price $p > \overline{v}$ and zero profits from dropping out at $\overline{v}$. Finally, given these strategies of informed and uninformed bidders, it is a best–response for uninformed bidders to drop–out upon observing a drop–out at any $p < c^*$ since such a drop–out can come only from an informed bidder.

In equilibrium, from (4) it follows that uninformed bidders earn zero expected payoff, while informed bidders obtain a positive payoff equal to $H(c^*)$ only when the true value of the object is at least c^* and he wins with probability $1/n$. Since the good is always sold and there are n ex–ante identical bidders, the ex–ante expected revenue for the seller equals

$$R(c^*) = \overline{v} - \alpha H(c^*)$$

This concludes the proof.■

How does the seller do in the c^*–auction compared to the standard auction? In Proposition 5 we show that if either α is small enough or n is large enough, the c^*–auction dominates the standard auction in expected revenues.

Proposition 5 *1. For each $n \geq 2$, there exists $\alpha_n^* \in (0, 1)$ such that c^*–auction earns the seller higher revenues than the standard auction iff $\alpha < \alpha_n^*$.*

2. For each $\alpha \in (0, 1)$, there exists n_α^* such that for all $n \geq n_\alpha^*$, the c^*–auction earns the seller higher revenues than the standard auction.

Proof. See the Appendix.□

In the standard auction, the seller leaves rents for bidders when only one of the bidders is informed and the true value of the object is at least the uninformed bidders' maximum possible bid $\overline{v}$. With a ceiling c^* however, since uninformed bidders bid till c^*, the seller gains in states where there is only one informed bidder and the value of the good $V > \overline{v}$, as he receives a price higher than $\overline{v}$ in these states. The seller also gains in states where all bidders present are uninformed, when he receives a price c^* that is higher than $\overline{v}$. However, a ceiling has some costs as the seller loses in states where multiple informed bidders are present and $V > c^*$. When $\alpha < \alpha_n^*$ the seller's expected loss in the last case is swamped by his gains from increased bidding by uninformed bidders. In such cases, the seller earns higher revenues by imposing a ceiling.

To gain intuition for the second part of the result note first that for a large number of bidders there are almost always two informed bidders present. Thus, the seller almost always loses from a ceiling whenever $V > c^*$. However, in order to guarantee participation from uninformed bidders till the ceiling, the expected loss to an uninformed bidder from winning at the ceiling when all other bidders are uninformed must balance out his expected gains from winning at the ceiling against at least one informed bidder. With a large number of bidders, each uninformed bidder knows that at least a few of the other bidders are likely to be informed and so is willing to bid till a high ceiling. In other words, as n grows, c^* grows as well so that the probability that $V > c^*$ times the expected loss to the seller from a ceiling in that event becomes small and, by the zero–profit condition for uninformed bidders, approximately cancels out the seller's gain from a ceiling in the event that all bidders are uninformed. Since the seller also gains from a ceiling when there is exactly one informed bidder present, he is better off overall from imposing a ceiling. From Propositions 2 and 3 it then follows that the c^*–auction also dominates any auction with a reserve price as well as any posted price mechanism for n large enough.[15]

A ceiling is beneficial in our model exactly because it encourages uninformed bidders to bid beyond $\overline{v}$, the ex–ante expected value of the object, in return for the positive probability of winning at the ceiling against an informed bidder and making positive profits when $V > c^*$. In a sense then, a ceiling acts as a bidding guide or anchor price for uninformed bidders. By imposing a ceiling the seller gives up on high bids from better informed bidders, but the added participation that a ceiling generates from poorly informed bidders more than compensates the seller when there are many bidders or when most bidders are likely to be poorly informed.

[15] It does not however imply that the c^*–auction dominates an auction with $0 < r < c < 1$. See the next section for a characterization of the optimal ceiling and floor combination.

To end this section, we present a numerical example with the uniform distribution that illustrates the results obtained so far. Before doing so a few more remarks are in order. The desirability of a ceiling is not limited to the case where V has compact support. It is easy to check that the equilibria characterized so far in Propositions 1 through 4 continue to remain equilibria when V has unbounded support on $[0, \infty)$, implying that all associated revenue expressions hold unchanged. With respect to Proposition 5, the first part of the result remains unchanged for such distributions; and so does the second, provided the hazard rate $h(v)$ is unbounded above. If however, $h(v)$ is bounded above then the second part of the result is weakened to yield the existence of an upper bound on α below which a ceiling dominates for large n.[16]

Example Suppose that V follows the uniform distribution on $[0, 1]$ so that $\overline{v} = \frac{1}{2}$ and

$$H(x) = \frac{1}{2}(1 - x)^2$$

The expected revenue in a standard auction equals

$$R_s = \frac{1}{2} - n\alpha(1 - \alpha)^{n-1}\frac{1}{8}$$

Note that for the uniform distribution, condition (3) holds for all $n \geq 2$, so that from Proposition 2, a standard auction dominates any auction with a positive reserve price. Furthermore, note from the proof of Proposition 3, that any posted price $p \geq \overline{v}$ is dominated by an auction with the same reserve price which, by the previous remark, is dominated by the standard auction for all n. Finally, it is easily seen that a posted price $p < \overline{v} < c^*$ is dominated by the c^*–auction for all n. To see this, observe that for such a posted price an informed bidder makes profits equal to $H(p)$ conditional on winning, and wins with probability at least $1/n$ (when all other bidders bid with him) for a total ex–ante expected profits for all bidders that is at least $\alpha H(p)$ (since uninformed bidders also make non–negative profits). But since $p < c^*$, $\alpha H(p) > \alpha H(c^*)$, the total ex–ante expected profits for all bidders in the c^*–auction.[17] Thus, either the standard auction or the c^*–auction dominate all other mechanisms that we have considered so far. Consequently, we will focus on these two mechanisms for the rest of this example.

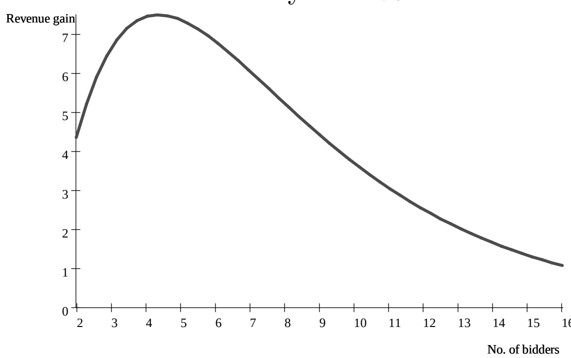
For the uniform distribution, exploiting (4), it can be seen that

$$c^* = \frac{1}{1 + \sqrt{(1 - \alpha)^{n-1}}}$$

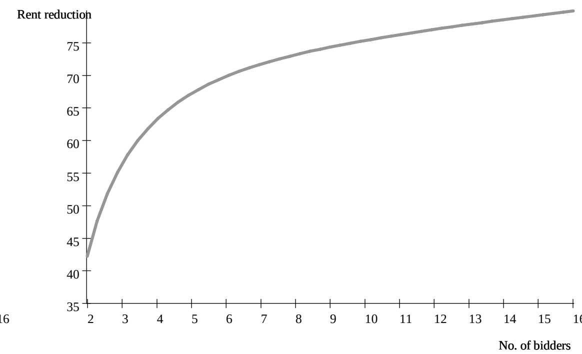
[16] See the proof of Proposition 5 (footnote 21 in particular) for more on this.

[17] While we do not state this as a proposition, this last argument clearly does not depend on the distributional assumptions made in this example.

For $n = 2$, the cut–off value $\alpha_2^* = 0.828$ below which the c^*–auction dominates the standard auction. Furthermore, it not difficult to verify that for $n \geq 4$, the c^*–auction dominates the standard auction for all α. The figures below depict the percentage gain in revenue and the percentage reduction in rents from the c^*–auction over the standard auction, both as a function of n when $\alpha = 0.25$. It is easy to verify that the percentage revenue–gain is the highest at $n = 4$, when $c^* = 0.60624$, $R(c^*) = 0.480\,62$ and $R_s = 0.447\,27$ for an expected gain in revenue of 7.46% and a reduction in rents by 63.25%.



Gain in revenues



Reduction in rents

3.2 Optimal Ceilings and Floors

While the results above show that imposing a ceiling may be desirable when n is large and α is low, we have so far not characterized the optimal ceiling and floor combination. In this section, we attempt a partial answer to this question. In order to do so we will focus on the simplest case of 2 bidders and characterize the unique symmetric equilibrium with 2 bidders for every reserve and floor combination in the set

$$Z = \{r, c \mid 0 \leq r \leq c \leq 1\}$$

Focusing attention on the two bidder case is restrictive. For $n > 2$, a full characterization of the equilibrium set is tedious and adds no insight beyond what is obtained from the two bidder case. Furthermore, as noted before, the description of the auction provided in Section 2 has to be slightly enriched in order to adequately handle the possibility of randomized drop–outs by a subset of at most $n - 2$ bidders.[18] Since our main focus is demonstrating that a ceiling may sometimes be desirable, we avoid such tedious complications in this paper. Furthermore, in order

[18]This can be done as follows. Any time there is a drop–out at some price p, the clock stops and a discrete counter (e.g., a bell) starts, allowing bidders to successively drop–out at the same price p. The clock starts again when the number of active bidders in two successive rings of the bell are identical. Otherwise, the bell continues to

to rule out the optimality of setting a high reserve price that completely excludes uninformed bidders from the auction, we will invoke the results obtained in Proposition 2 and assume that condition (3) holds for the case $n = 2$.

Given our assumptions, it will turn out that the optimal mechanism involves setting a reserve price equal to 0, regardless of the ceiling. With respect to a ceiling, either the optimal ceiling is equal to 1 (corresponding to the standard auction) or it is equal to c^* (i.e., the simple ceiling of the previous section). From Proposition 5 it then follows that if α is greater than the cut–off value α_2^* (respectively, less than) then the standard auction (resp., c^*–auction) is optimal in the class of all possible mechanisms that we consider. Interestingly, in either case the revenue maximizing mechanism is efficient, i.e., the object is sold with probability 1.

Proposition 6 *Suppose* $n = 2$ *and* $\frac{\overline{v}f(\overline{v})}{1-F(\overline{v})} \geq \frac{2\alpha(1-\alpha)}{1-(1-\alpha)^2}$. *A symmetric equilibrium exists for each* $\{r, c\} \in Z$ *and the seller's expected revenue is maximized by holding either a standard auction or one with the simple ceiling* c^*.

Proof. See the Appendix.□

The intuition why setting a positive result price is not optimal for the seller is the same as that for Proposition 2— a positive reserve price discourages participation from uninformed bidders, raising the informational rents earned by informed bidders and lowering the probability of a sale. The intuition underlying the optimal ceiling price is also similar. With a ceiling less than c^*, uninformed bidders earn positive profits and the seller can raise the ceiling without lowering participation from them. Such a ceiling is thus clearly not optimal. With a ceiling greater than c^* however, uninformed bidders do not bid till the ceiling with positive probability. As we show in the Appendix, this implies that the seller is better off either by lowering the ceiling to c^* and increasing participation from uninformed bidders, or by not having a ceiling at all.

4 Implicit Ceilings

So far we have considered a situation where the seller is able to explicitly impose a ceiling on allowable bids and commit to honoring the outcome of the resulting auction. In many situations such an ex–ante commitment may not be feasible, potentially preventing the seller

ring till there is at most one bidder left. The winner is the last bidder active. It is easy to see that all equilibrium outcomes characterized in Propositions 1–4 continue to remain so under this new enriched description.

from exploiting the benefits of a ceiling. In this section we show that in such cases, if the seller is able to cancel the outcome of the initial auction and resell the object subsequently, he may still be able to create an implicit ceiling on allowable bids. Such an implicit ceiling benefits the seller for the same reasons that an explicit one does.

To fix ideas, suppose that the seller is constrained to sell the object in a standard auction as in Section 2. However, whenever the auction concludes according to its rules the seller can either allocate the object according to the rules of the auction or he can cancel it and hold another auction identical to the first one. The same set of bidders participate in both auctions and there is no discounting between periods.[19]

Now suppose, before the first auction starts, the seller sends a message stating that he is offering up the object for sale at a fixed price c^* or the best–offer. We show that it is an equilibrium for buyers to interpret the fixed price c^* as an implicit ceiling and bid as in the c^*–auction. The seller does not cancel the auction unless he receives a bid p above c^*, whereupon he believes that the true value is at least p and expects to obtain at least that price in the second auction. Put differently, the cheap talk statement regarding the implicit ceiling c^* is credible.

Proposition 7 *When the seller can cancel the first auction, setting an implicit ceiling c^* is credible.*

Proof. We show that the following behavior constitutes a perfect Bayesian equilibrium of the dynamic game described above. The bidders bid in the first auction as they do in the c^*–auction. The seller's cancellation strategy is as follows. Whenever the auction concludes according to its rules at a price $P \geq c^*$ with at least one bidder active at some $p > c$, the seller concludes that the (expected) value of the good is at least equal to P, and cancels the first auction expecting to obtain at least that price in the second; otherwise, the seller expects to receive a price P in the second auction and does not cancel the first auction, allocating the object according to its rules. If the first auction is ever cancelled, informed bidders bid till the true value of V in the second auction, while uninformed bidders bid given their beliefs which are identical to those of the seller.

Consider first the behavior of any bidder. The critical deviation from the prescribed profile to check for is that where one bidder stays active beyond c^*. Given the behavior of other bidders,

[19] All results go through when the seller can cancel more than once and there is discounting, provided the discount factor is high enough. In view of our previous results we ignore the possibility of a reserve price throughout this section.

this bidder will be the winner of the auction and pay c^* if the seller observes the rules of the auction. Since the seller believes that he will receive a price of at least c^* in the second auction, he prefers to cancel the auction. In the second auction, the deviating bidder can win the object by paying a price that is at least c^* and so the deviation is not profitable for the bidder. Any other deviation by a bidder from his prescribed strategy is either ineffective as the seller does not cancel the auction, or irrelevant as it can only occur after multiple deviations from the prescribed strategies of the bidders.

It is easy to see that the seller's beliefs are correct on the path of play.[20] Finally, given that uninformed bidders share the seller's beliefs if the first auction is ever cancelled, it is optimal for all bidders to bid as specified in the second auction.■

The threat of cancellation and the possibility of a second auction restrains competition among informed bidders by adversely affecting their incentives to bid aggressively in the first auction and reveal their private information. Interpreting the seller's statement as an implicit ceiling, they bid as in the c^*–auction. However, this encourages uninformed bidders to bid aggressively, in particular, as in the c^*–auction. In equilibrium, the auction is never cancelled. Nevertheless, the threat of cancellation raises the seller's revenue over and above the standard auction, whenever the c^*–auction dominates the standard auction in expected revenues.

A fixed price or best offer mechanism supported by an implicit threat of cancellation is by no means the only possible instance of an implicit ceiling. In many situations, a seller may not be able to affect the terms of sale at all, possibly because of established historical norms or because the auction format is decided upon by an intermediary. However, before the auction starts, the seller may still be able to explicitly attach a contractually specified discretionary right to buy back the object from the winner (with the possibility of resale in a second auction in case the seller exercises his right). Such an option to buy–back the object can reasonably be viewed as an explicitly stated possibility of cancellation. This is easiest to see when the option to buy–back specifies that the seller will fully refund the winner, regardless of the price the winner pays—in such a case the right to buy–back is identical to the right to arbitrarily cancel the auction. Since an implicit threat to cancel the first auction can credibly create a ceiling for the seller when there is a possibility of resale, an explicitly stated threat in the form of such an option to buy–back the object can also do so.

[20]The seller's beliefs are also reasonable off the path of play. It is not only natural for the seller to have optimistic beliefs about V if any bidder violates the implicit ceiling, but such beliefs will also satisfy (suitable extensions of) forward induction notions such as the intuitive criterion.

An option to buy–back the object can also create an implicit ceiling even when the seller has to pre–specify the price at which he will buy back the object. In fact, it is not difficult to see that if the seller specifies that he will buy back the object at a fixed price c^*, then the arguments in the proof of Proposition 7 go through unchanged. In that proof, the seller is unwilling to cancel the auction whenever the auction concludes at a price p less than c^* as he expects to obtain a price p in the future. But then the seller will also be unwilling to buy–back the object when he has to pay the pre–specified price $c^* > p$. On the other hand, since the seller is willing to cancel the auction (i.e., provide a full refund) when some bidder is active at some price $p > c^*$, he is at least as willing to buy–back the object when he only needs to provide a partial refund c^*. Consequently, in equilibrium, the pre–specified buy–back price c^* acts as an implicit ceiling, the threat of buy–back is credible, but the right to buy–back is never exercised.

5 Conclusion

In a simple common value environment with ex–post hierarchically informed bidders, we show that setting a bid ceiling may be optimal for seller revenues. In particular, such a ceiling mechanism may dominate auctions with reserve prices as well as posted price mechanisms. Such price ceilings may be explicitly announced (and committed to) or implicit, arising in the latter case out of a credible threat to cancel the first auction after it is over and hold a second auction in its place. More broadly interpreted, our results suggest that it may be beneficial to impose an upper limit on the share of the pie one is willing to receive in multi–agent bargaining environments.

An investigation of similar questions in a richer information structure would be of interest. Furthermore, observe that in our common value framework there is no notion of allocational efficiency. In a framework with some private values however a ceiling may introduce some allocational inefficiency when the good is not allocated to the bidder with the highest value. Therefore, a ceiling may be less attractive in such circumstances. We leave this question also for future research.

6 Appendix

Proof of Proposition 2

We first define some basic objects in order to characterize the symmetric equilibrium of an auction with a reserve price $r > 0$. For $x \in [0,1]$, define

$$G(x) = F(x)\left[E[V|V < x] - x\right] = -\int_0^x F(u)du \tag{7}$$

and observe that it is a negative, decreasing and concave function. Let r^* be defined as the solution to

$$\pi(0, n-1)[\overline{v} - r^*] + (1 - \pi(0, n-1))G(r^*) = 0 \tag{8}$$

Note from the definition of $G(.)$ that such an $r^* \in (0, \overline{v})$ exists and is unique. The next lemma characterizes symmetric equilibrium in the auction with a reserve price $r > 0$. We will say that a bidder is present at the open whenever he belongs to the set $A(r)$ and say that he participates in the auction (with positive probability) if he is present at the open (with positive probability).

Lemma 1 *Fix $r > 0$. Then symmetric equilibrium strategies are as follows. Any informed bidder who knows $V = v$ participates in the auction if and only if $v \geq r$, and given that he participates, quits only at $p > v$, staying in otherwise, regardless of the history of quits. Furthermore,*

1. *if $r \geq r^*$ then uninformed bidders participate in the auction with probability* 0*;*
2. *while if $r < r^*$, then uninformed bidders participate in the auction with probability $\sigma_r \in (0,1)$ given by the solution to*

$$\pi(0, n-1)[1 - \sigma_r]^{n-1}[\overline{v} - r] + \sum_{k'=1}^{n-1} \pi(k', n-1)[1 - \sigma_r]^{n-1-k'} G(r) = 0; \tag{9}$$

and upon participation, an uninformed bidder quits as soon as anyone quits and otherwise quits only at $p > \overline{v}_k$, where $\overline{v}_k$ is the expected value of the object given all k bidders present at the open are uninformed, for $k \in \{1, ..., n\}$.

Proof of Lemma. The optimality of the behavior of informed bidders immediately follows using arguments analogous to the proof of Proposition 1. Given this consider the behavior of uninformed bidders in a symmetric equilibrium.

Fix $r > 0$ and let $\sigma_r \in [0,1]$ be the probability with which an uninformed bidder participates in the auction. Let $k \in \{0, 1, ..., n\}$ be the total number of bidders present at the open.

Observe first that given the behavior of informed bidders and conditional on participation and observing that $k > 1$, an uninformed bidder makes at most zero expected profits whenever at least one other bidder present at the open is informed. On the other hand, conditional on the event $k > 1$ and all other bidders present at the open are uninformed, an uninformed bidder must earn zero expected profits, in any symmetric equilibrium, at least whenever such an event has positive probability (i.e., $\sigma_r > 0$).

To see this, let $\overline{v}_k$ denote the expected value of the object given that all k bidders present at the open are uninformed (where we suppress the dependence of $\overline{v}_k$ on σ_r). If an uninformed bidder makes positive profits conditional on winning, given $k > 1$ and all other bidders are uninformed, then all uninformed bidders must quit with strictly positive probability at some price $p < \overline{v}_k$, conditional on a history of no quits. In such a case, the uninformed bidder wins the object with probability $\frac{1}{k}$ (for strictly positive profits $\overline{v}_k - p$) by quitting at p, when all other bidders are uninformed and also quit at p, and does not win the object in any other state (ignoring the zero probability case that $V = p$). If instead he continues at p and drops out immediately after he wins the object with higher probability whenever he wins by quitting at p, and earns the same expected profits in all other cases, a contradiction with equilibrium. Similarly, if an uninformed bidder earns negative profits conditional on $k > 1$ and all other bidders present are uninformed, all uninformed bidders must remain active with positive probability at some $p > \overline{v}_k$, conditional on a history with no quits. It is easy to see that by quitting with probability 1 at such a p an uninformed bidder strictly increases his expected profits. It follows that an uninformed bidder must earn zero expected profits conditional on being present at the open and observing $k > 1$, whenever $\sigma_r > 0$.

Consider next the profits of the uninformed bidder, conditional on being the only bidder present at the open (i.e., $k = 1$) and so winning at the reserve r. Letting $D = \sum_{k'=0}^{n-1} \pi(k', n - 1)[1 - \sigma_r]^{n-1-k'}$, this is given by

$$\frac{1}{D}\left[\pi(0, n-1)[1-\sigma_r]^{n-1}[\overline{v} - r] + \sum_{k'=1}^{n-1} \pi(k', n-1)[1-\sigma_r]^{n-1-k'} G(r)\right]. \qquad (10)$$

The first term reflects the probability that all other bidders are uninformed and not present at the open times the expected profits from winning in this case. Each term in the sum reflects the probability that all k' informed bidders as well as all uninformed bidders are not present at the open, times the expected profits from winning in this case.

Observe first that $\sigma_r < 1$. For if $\sigma_r = 1$, then the expression in (10) reduces to $G(r) < 0$, so that an uninformed bidder will be strictly better off from not participating in the auction at all. Next notice that if $r \geq r^*$, then $\sigma_r = 0$. For if not the expression in brackets in (10) is strictly less than

$$\pi(0,n-1)[\overline{v}-r]+\sum_{k'=1}^{n-1}\pi(k',n-1)G(r)\leq 0$$

since $r \geq r^*$. Conversely, if $r < r^*$ then we must have $\sigma_r > 0$. For if not an uninformed bidder will strictly prefer to participate in the auction with probability 1 and quit immediately after r. It follows that when $r < r^*$, σ_r is chosen so that the expression in (10) exactly equals zero, i.e., σ_r is the solution to (9).

Furthermore, since conditional on participating and observing k, an uninformed bidder must always earn zero expected profits, he cannot do better than by quitting as soon as he observes a quit and quitting above $\overline{v}_k$ otherwise, where for $k \in \{1,...,n\}$, $\overline{v}_k$ is defined as the solution to the zero expected profit condition conditional on winning at $\overline{v}_k$ and all k bidders are uninformed:

$$\pi(0,n-1)\xi_r(k-1,n-1)[\overline{v}-\overline{v}_k]+\sum_{k'=1}^{n-k}\pi(k',n-1)\xi_r(k-1,n-k'-1)F(r)[E[V|V<r]-\overline{v}_k]=0$$

where for $0 \leq k \leq K \leq n$, $\xi_r(k,K)=\binom{K}{k}\sigma_r^k(1-\sigma_r)^{K-k}$ is the probability that k out of K uninformed bidders are present at the open. Observe that $\overline{v}(1)=r$ and $\overline{v}_k \leq \overline{v}_n = \overline{v}$. This concludes the proof of the lemma.■

For the proof of the Proposition, recall first that in the standard auction a bidder makes positive profits when he is the only informed bidder and $V > \overline{v}$, in which case he pays a price equal to $\overline{v}$. Using Lemma 1 observe next that when the seller imposes a reserve price $r \in (0,\overline{v}]$ such a bidder makes weakly higher expected profits as with positive probability he wins the object at a price weakly less than $\overline{v}$. Furthermore, an informed bidder with a realized value $v \in (r,\overline{v}]$ makes strictly positive profits when he is the only informed bidder, whereas in the standard auction such a bidder makes zero profits. Since uninformed bidders make zero profits in both auctions, it follows that each bidder makes weakly higher ex-ante expected profits in the auction with a reserve price $r \leq \overline{v}$. Since the object is not sold with strictly positive probability, it follows that the seller is strictly worse off from imposing a reserve price $r \in (0,\overline{v}]$, compared to the standard auction.

Consider next $r > \overline{v} > r^*$. Since uninformed bidders do not participate by Lemma 1, the seller's expected revenue is

$$\pi(1,n)(1-F(r))r+(1-\pi(0,n)-\pi(1,n))(1-F(r))E[V|V>r]$$

Using the monotone hazard rate condition, observe that for the optimal choice of r to be greater than $\overline{v}$, the derivative of the last expression above must be positive at $r = \overline{v}$. This yields

$$\frac{\overline{v}f(\overline{v})}{1 - F(\overline{v})} < \frac{\pi(1,n)}{1 - \pi(0,n)} = \frac{n\alpha(1-\alpha)^{n-1}}{1-(1-\alpha)^n}$$

For n large enough, this inequality is violated so that the optimal choice of a reserve r cannot be greater than $\overline{v}$. This completes the proof.■

Proof of Proposition 3

For any posted price $p \in [0, 1]$, an informed bidder will participate only if $v \geq p$. Let $\sigma_p \in [0, 1]$ be the probability with which an uninformed bidder participates in a symmetric equilibrium. For $0 \leq k \leq K \leq n$, let $\xi_p(k, K) = \binom{K}{k}\sigma_p^k(1-\sigma_p)^{K-k}$ be the probability that k out of K uninformed bidders participate. Consider the expected profits of an uninformed bidder from bidding at the posted price p. This is given by

$$\begin{aligned} &\pi(0, n-1) \textstyle\sum_{k'=0}^{n-1} \xi_p(k', n-1)\frac{1}{k'+1}[\overline{v} - p] \\ &+ \textstyle\sum_{k=1}^{n-1} \pi(k, n-1) \sum_{k'=0}^{n-1-k} \xi_p(k', n-1-k)[\frac{1}{k'+1}G(p) + \frac{1}{k+k'+1}H(p)] \end{aligned} \tag{11}$$

where the functions H and G have been defined in (1) and (7) respectively. The first term above is the probability that all other bidders are uninformed times the expected probability of winning in such a case times the expected payoff conditional on winning; the second term has the same interpretation, but when at least one bidder is informed. The bidder makes zero profits when he is not the winner.

A symmetric equilibrium exists for all p. To see this observe that if the expression in (11) is non–positive for $\sigma_p = 0$, then it is an equilibrium for uninformed bidders not to participate. On the other hand, if the expression in (11) is positive for $\sigma_p = 0$ but negative for $\sigma_p = 1$, then by continuity there exists $\sigma_p \in (0, 1)$ for which the expression in (11) is exactly equal to 0 and uninformed bidders are willing to participate with positive probability. Finally, if the expression in (11) is non–negative for $\sigma_p = 1$, then it is an equilibrium for uninformed bidders to participate with probability 1.

Note next that from the definitions of H and G, for all $x \in [0, 1]$

$$H(x) + G(x) = \overline{v} - x \tag{12}$$

Using this in (11), we can rewrite an uninformed bidders expected profits from participating as

$$\sum_{k=0}^{n-1} \pi(k, n-1) \sum_{k'=0}^{n-1-k} \xi_p(k', n-1-k) \left[\frac{1}{k'+1}(\overline{v} - p) + \frac{k}{(k+k'+1)(k'+1)}G(p)\right] \tag{13}$$

Suppose first that $p > \overline{v}$. Then the expression (13) is negative for any $\sigma_p > 0$ and so we must have $\sigma_p = 0$. From Lemma 1, if the seller holds an auction with a reserve price equal to p, then uninformed bidders will still not participate, but informed bidders will bid and pay a price above p whenever $v > p$ and at least two bidders are informed. As a result, the seller will earn higher revenues from such an auction with a reserve price p compared to offering up the object at a fixed price p. However, from Proposition 2, for $n \geq n_\alpha^r$ the seller earns even higher revenues from the standard auction compared to the revenues from the auction with a reserve price of p so that the standard auction dominates all such posted prices $p \geq \overline{v}$ for $n \geq n_\alpha^r$.

Next consider $p \leq \overline{v}$. Conditional on winning, an informed bidder's expected profit is given by $H(p)$. Furthermore, an informed bidder wins with probability at least $1/n$ when he bids at p. Since uninformed bidders earns non–negative profits, it follows that the sum of the expected profits of all bidders is at least $\alpha H(p)$. Recall that the analogous expression for the standard auction is $n\alpha(1-\alpha)^{n-1}H(\overline{v})$. Since H is a decreasing function and $p \leq \overline{v}$ it follows that the seller is strictly better off from the standard auction compared to any posted price $p \leq \overline{v}$, for any n large enough satisfying $n(1-\alpha)^{n-1} < 1$.■

Proof of Proposition 5

The ceiling dominates the standard auction iff

$$n(1-\alpha)^{n-1}H(\overline{v}) > H(c^*) \tag{14}$$

For $x \in [0,1]$ define

$$U(x) \equiv \pi(0, n-1)[\overline{v} - x] + (1 - \pi(0, n-1))H(x) \tag{15}$$

and observe from the definition of c^* in (4) that $U(c^*) = 0$. Using this last fact in (14), we obtain the following necessary and sufficient condition for the ceiling to dominate the standard auction:

$$c^* < \overline{v} + n\left(1 - (1-\alpha)^{n-1}\right) H(\overline{v})$$

Since $U(x)$ is easily checked to be a decreasing function, we conclude that a necessary and sufficient condition for this to hold is that

$$U[\overline{v} + n(1 - (1-\alpha)^{n-1})H(\overline{v})] < 0,$$

Some straightforward manipulation yields,

$$\alpha < 1 - \left[\frac{H(\overline{v} + n(1 - (1-\alpha)^{n-1})H(\overline{v}))}{nH(\overline{v})}\right]^{\frac{1}{n-1}} \tag{16}$$

as our necessary and sufficient condition. For fixed n, the right–hand side is increasing in α, strictly positive at $\alpha = 0$ and strictly less than 1 at $\alpha = 1$. It follows that there exists a unique α_n^*, such that a ceiling dominates the standard auction iff $\alpha < \alpha_n^*$, establishing part 1 of the Proposition.

Next, for fixed α consider how the right–hand side of (16) behaves as n becomes large. Since V has compact support in $[0,1]$, for n large enough $\overline{v} + n(1-(1-\alpha)^{n-1})H(\overline{v}) \geq 1$ and so the numerator of the expression in the right–hand side of (16) is equal to 0, while the denominator is positive. From (16), we conclude that this establishes part 2 of the result.■[21,22]

Proof of Proposition 6

We proceed in steps. In Step 1 we characterize the symmetric equilibrium with two bidders for every reserve and ceiling combination in the set Z. In Step 2, we show that a positive reserve price is not optimal. In Step 3, we show that either the simple ceiling c^* is optimal or a ceiling is not optimal at all.

Step 1

We begin by defining a few objects. For $\{r, c\} \in Z$, let

$$U_1(r,c) \equiv (1-\alpha)\frac{1}{2}[\overline{v} - c] + \alpha[G(r) + \frac{1}{2}H(c)]$$

and

$$U_0(r,c) \equiv (1-\alpha)[\overline{v} - r] + \alpha[G(r) + \frac{1}{2}H(c)]$$

and

$$W(r,c) \equiv (1-\alpha)[1 + \frac{\alpha G(r)}{(1-\alpha)[\overline{v} - r]}][\overline{v} - c] + \alpha H(c)$$

Using (12), notice that since $r \leq c$, whenever $U_1(r,c) \geq 0$ we have $U_0(r,c) > U_1(r,c)$. We will show below that symmetric equilibria have the following structure. Informed bidders participate iff $v \geq r$ and continue till $\min[v,c]$ regardless of drop–outs. Uninformed bidders participate with probability $\sigma_r \in [0,1]$, continuing till $\overline{v}$ with probability 1, continuing at $\overline{v}$ with probability $\sigma_{\overline{v}} \in [0,1]$, and continuing beyond till c with probability 1, quitting at any point immediately

[21]When instead, V has unbounded support, it can be shown that the same limit of the right–hand side of (16) equals $\alpha_\infty^* \equiv 1 - \exp[-H(\overline{v})h^*]$ where $h^* = \lim_{v\to\infty} h(v)$. Thus for distributions with unbounded support where in addition $h^* = \infty$, we obtain the same conclusion as in part 2 of the Proposition, whereas if $h^* < \infty$, then the ceiling dominates for large n iff $\alpha < \alpha_\infty^*$. Note that part 1 of the Proposition does not need a compact support for V.

[22]It is not difficult to verify that the ratio of information rents in the c^*–auction to that in the standard auction converges to 0 as n becomes large.

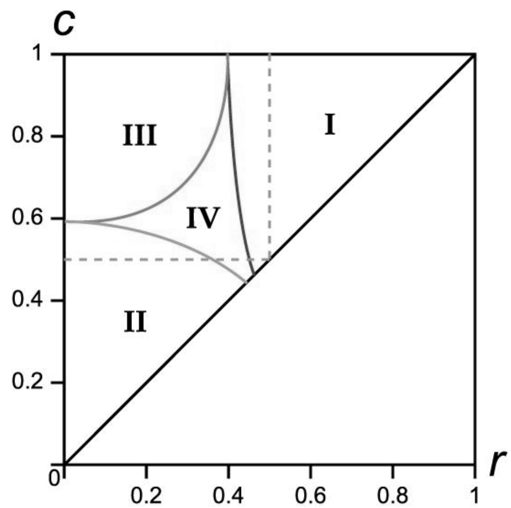


Figure 1: Equilibrium zones

after observing a quit. For $\{r, c\}$ such that $U_0(r, c) \leq 0$, $\sigma_r = 0$. This is represented by zone I in Figure 1.[23] For $\{r, c\}$ such that $U_1(r, c) \geq 0$, $\sigma_r = 1 = \sigma_{\overline{v}}$ (zone II in the figure). For $\{r, c\}$ such that $U_1(r, c) < 0 < U_0(r, c)$, $\sigma_r \in (0, 1]$ with $\sigma_r < 1$ if $r > 0$ (zones III and IV). Furthermore, in such a case, $\sigma_{\overline{v}} \in [0, 1]$ with $\sigma_{\overline{v}} > 0$ iff $c < 1$ and $\sigma_{\overline{v}} < 1$ if $W(r, c) < 0$ (zone III) while $\sigma_{\overline{v}} = 1$ if $W(r, c) \geq 0$ (zone IV).

The behavior of informed bidders is immediate, for every $\{r, c\}$. As for uninformed bidders, conditional on participating and observing that both bidders are present, uninformed bidders must continue with probability 1 for all $p \in [r, c]$, with $p \neq \overline{v}$. For if they drop–out with positive probability at any $p < \overline{v}$, then by continuing at p and dropping out immediately after such a bidder makes strictly higher expected profits than from dropping out at p. On the other hand, if they drop–out with positive probability at $p > \overline{v}$, then the expected payoff is negative, whereas by dropping out at $\overline{v}$ such a bidder can guarantee zero expected profits. It follows that uninformed bidders must drop–out anytime they observe a drop–out at a $p \neq \overline{v}$.

Observe next from Propositions 1 and 2, that symmetric equilibrium behavior is as specified above in the case $c = 1$. As a result, in what follows we focus on the case where $c < 1$. Note that whenever $c < 1$ we must have $\sigma_{\overline{v}} > 0$— for if not, an uninformed bidder by continuing till c

[23]The figure uses a uniform distribution and $\alpha = \frac{1}{2}$.

competes only against an informed bidder and makes strictly positive profits, whereas he makes zero profits from dropping out at $\overline{v}$.

Consider first a symmetric equilibrium where $\sigma_r = 0$. Since no uninformed bidder wants to participate we must have that the expected profits from participating be non–positive, or $U_0(r, c) \leq 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone.

Consider next a symmetric equilibrium where $\sigma_r = 1 = \sigma_{\overline{v}}$. Since uninformed bidders must earn non–negative expected profits, we must have $U_1(r, c) \geq 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone.

Next suppose that in a symmetric equilibrium $\sigma_r = 1$ but $\sigma_{\overline{v}} \in (0, 1)$. Conditional on reaching $\overline{v}$ and dropping out an uninformed bidder must make zero expected profits and since $\sigma_{\overline{v}} \in (0, 1)$ must also make zero expected profits from continuing till c. This implies that $\sigma_{\overline{v}}$ must solve:

$$(1-\alpha)\sigma_{\overline{v}}\frac{1}{2}[\overline{v} - c] + \alpha\frac{1}{2}H(c) = 0 \tag{17}$$

It follows that $c > c^*$. Notice next that the ex–ante expected profits of an uninformed bidder is equal to a weighted sum of his expected profits when he is the only bidder present at the open and his expected profits when both bidders are present at the open. Due to arguments used to establish (17), the second expected value is equal to zero, while since $\sigma_r = 1$, the first expected value is negative if $r > 0$ and zero otherwise. It follows that $r = 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone and furthermore that $U_1(r, c) < 0 < U_0(r, c)$.

Consider now a symmetric equilibrium where $\sigma_r \in (0, 1)$ and $\sigma_{\overline{v}} \in (0, 1)$. Using arguments similar to those used in establishing (17) we must have that uninformed bidders must earn zero profits so that (given σ_r), $\sigma_{\overline{v}}$ must solve

$$(1-\alpha)\sigma_r\sigma_{\overline{v}}\frac{1}{2}[\overline{v} - c] + \alpha\frac{1}{2}H(c) = 0 \tag{18}$$

It follows that $c > \overline{v}$. Furthermore, since $\sigma_r \in (0, 1)$, uninformed bidders must earn zero profits from participating, i.e., σ_r must solve

$$(1-\alpha)(1-\sigma_r)[\overline{v} - r] + \alpha G(r) = 0 \tag{19}$$

It follows that $r \in (0, \overline{v})$. Furthermore, using the solution for σ_r obtained in (19) in (18), we obtain

$$(1-\alpha)[1 + \frac{\alpha G(r)}{(1-\alpha)[\overline{v} - r]}]\sigma_{\overline{v}}[\overline{v} - c] + \alpha[H(c)] = 0$$

Since $c > \overline{v}$ and $r \in (0, \overline{v})$, we must then have $W(r, c) < 0$ and, using (18), $U_1(r, c) < 0$ and furthermore, using (19), $U_0(r, c) > 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone.

Finally, consider a symmetric equilibrium where $\sigma_r \in (0, 1)$ but $\sigma_{\overline{v}} = 1$. Since uninformed bidders randomize participation, we must have that the ex–ante expected profits of an uninformed bidder equals zero, i.e.,

$$(1-\alpha)[(1-\sigma_r)(\overline{v}-r) + \sigma_r \frac{1}{2}(\overline{v}-c)] + \alpha[G(r) + \frac{1}{2}H(c)] = 0 \tag{20}$$

It follows that $r < \overline{v}$. For if not, since $c \geq r$ and by (12), $G(r) + \frac{1}{2}H(c) \leq \frac{1}{2}G(r) + \frac{1}{2}(\overline{v} - r) < 0$, we obtain a contradiction with (20). Furthermore, the expression in (20) is monotonically decreasing in σ_r. It follows that it is strictly less than $U_0(r, c)$ and strictly greater than $U_1(r, c)$, yielding $U_1(r, c) < 0 < U_0(r, c)$. Furthermore, uninformed bidders must make non–negative profits conditional on reaching $\overline{v}$ and continuing, i.e.,

$$(1-\alpha)\sigma_r[\overline{v} - c] + \alpha[H(c)] \geq 0$$

From (20) it follows that

$$\sigma_r \geq 1 + \frac{\alpha G(r)}{(1-\alpha)[\overline{v} - r]} \tag{21}$$

so that, substituting the term on the right–hand side of the last expression for σ_r in the left–hand side of (20) and using the monotonicity of the expression in (20), we obtain $W(r, c) \geq 0$. It is straightforward to check that the specified behavior is an equilibrium in this zone. This completes the description for symmetric equilibria for all $\{r, c\} \in Z$.

<u>Step 2</u>

We will show that each auction $\{r, c\} \in Z$ with a reserve price $r > 0$ is strictly dominated in expected revenues by some auction $\{0, c'\} \in Z$, that has no reserve.

Consider first the expected revenues of the seller for the auction with reserve and ceiling pair $\{r, c\} \in Z$ such that $r \geq \overline{v}$. From step 1, using (12) observe that then $U_0(r, c) < 0$ so that $\sigma_r = 0$. Given the behavior of informed bidders it follows immediately that the seller will do better by instead holding the auction $\{r, 1\}$, which by the assumed conditions of the proposition is dominated by the standard auction $\{0, 1\}$. The same logic also eliminates all auctions $\{r, c\}$ where $U_0(r, c) \leq 0$, allowing us to focus throughout what follows on the case where $U_0(r, c) > 0$ so that $\sigma_r > 0$.

Next consider the auction $\{r, c\}$ with $0 < r < \overline{v}$ and $c^* < c < 1$ and $U_0(r, c) > 0$. From step 1, we must have $\sigma_r \in (0, 1)$ and $\sigma_{\overline{v}} \in (0, 1]$, as in such a case $U_1(r, c) < 0$. We compare the expected revenues such an auction with that from the auction $\{0, c\}$ that has the same ceiling but no reserve. From step 1, in the auction $\{0, c\}$ uninformed bidders participate with probability 1 and continue at $\overline{v}$ with probability $\sigma'_{\overline{v}} \in (0, 1)$. In either auction an informed bidder makes positive profits only when he is the only informed bidder or when $V > c$ while uninformed bidders make zero expected profits. Furthermore, the object is sold with probability 1 in the auction $\{0, c\}$, while it is not sold with positive probability in the auction $\{r, c\}$. It thus suffices to show that informed bidders earn higher expected profits in the auction $\{r, c\}$ compared to the auction $\{0, c\}$. Suppose first that $\{r, c\}$ is such that $W(r, c) \geq 0$. Then $\sigma_{\overline{v}} = 1$ in the auction $\{r, c\}$. Furthermore, letting $\Pi(r, c|v)$ denote the expected profits of a bidder conditional on being informed and conditional on the realized value of $V = v$, it is seen that

$$\Pi(r, c|v) = \begin{cases} 0 & \text{when } v < r \\ (1-\alpha)(1-\sigma_r)[v-r] & \text{when } r \leq v < c \\ (1-\alpha)(1-\sigma_r)[v-r] + ((1-\alpha)\sigma_r + \alpha)\frac{1}{2}[v-c] & \text{when } v \geq c \end{cases} \quad (22)$$

In contrast, in the auction $\{0, c\}$, conditional on being an informed bidder and the realized value of v of V, the expected profits of an informed bidder is

$$\Pi(0, c|v) = \begin{cases} 0 & \text{when } v < \overline{v} \\ (1-\alpha)(1-\sigma'_{\overline{v}})[v-\overline{v}] & \text{when } \overline{v} \leq v < c \\ (1-\alpha)(1-\sigma'_{\overline{v}})[v-\overline{v}] + ((1-\alpha)\sigma'_{\overline{v}} + \alpha)\frac{1}{2}[v-c] & \text{when } v > c \end{cases} \quad (23)$$

Comparing state by state, we see that the $\{0, c\}$ auction dominates the $\{r, c\}$ auction if $\sigma_r \leq \sigma'_{\overline{v}}$. But, from step 1, $\sigma'_{\overline{v}}$ solves (17). Since $c > c^* > \overline{v}$, the expression in (17) is decreasing in $\sigma_{\overline{v}}$, so that if $\sigma_r > \sigma'_{\overline{v}}$, then the expression in (17) is negative if we replace $\sigma_{\overline{v}}$ with σ_r in it. Since, again from step 1, σ_r solves (20), it follows that σ_r must satisfy

$$(1-\alpha)(1-\sigma_r)(\overline{v}-r) + \alpha G(r) > 0$$

which, since $r < \overline{v}$, yields the desired contradiction with (21).

Suppose next that $\{r, c\}$ is such that $W(r, c) < 0$. Then $\sigma_{\overline{v}} \in (0, 1)$ in the auction $\{r, c\}$ and solves (18) whereas $\sigma'_{\overline{v}}$ (corresponding to the $\{0, c\}$ auction) solves (17) so that we obtain $\sigma_r \sigma_{\overline{v}} = \sigma'_{\overline{v}}$. Furthermore conditional on being an informed bidder and the realized value v of V, the expected profits of an informed bidder in the $\{r, c\}$ auction is equal to 0 when $v < r$, equal to $(1-\alpha)(1-\sigma_r)[v-r]$ when $r \leq v < \overline{v}$, equal to

$$(1-\alpha)[(1-\sigma_r)(v-r) + \sigma_r(1-\sigma_{\overline{v}})(v-\overline{v})]$$

when $\overline{v} \leq v < c$ and equal to

$$(1-\alpha)[(1-\sigma_r)(v-r)+\sigma_r(1-\sigma_{\overline{v}})(v-\overline{v})]+[(1-\alpha)\sigma_r\sigma_{\overline{v}}+\alpha]\frac{1}{2}H(c)$$

when $v \geq c$. Since $\sigma_r\sigma_{\overline{v}} = \sigma'_{\overline{v}}$, comparing state by state with the expression for $\Pi(0, c|v)$ it follows that the $\{0, c\}$ auction strictly dominates the $\{r, c\}$ auction.

Next, consider the auction $\{r, c\}$ with $0 < r < \overline{v}$ and $r \leq c \leq c^*$ and $U_0(r, c) > 0 > U_1(r, c)$. It is immediate from step 1 then that $W(r, c) \geq 0$ so that $\sigma_r \in (0, 1)$ and $\sigma_{\overline{v}} = 1$. We compare such an auction with the c^*–auction $\{0, c^*\}$. Conditional on being an informed bidder and the realized value v of V, the expected profits of an informed bidder in the $\{0, c^*\}$ auction is equal to $\frac{1}{2}(v - c^*)$ when $v \geq c^*$ and 0 otherwise. Since $c^* \geq c$, comparing with the expression for $\Pi(r, c|v)$ in (22), we see that informed bidders earn strictly higher profits in the $\{r, c\}$ auction compared to the $\{0, c^*\}$ auction. Since uninformed bidders earn zero expected profits in both auctions and the object is sold with probability 1 in the $\{0, c^*\}$ auction but not in the $\{r, c\}$ auction, it follows that the former strictly dominates the latter in expected revenues.

It remains to consider auctions with $\{r, c\}$ such that $0 < r < \overline{v}$ and $r \leq c \leq c^*$ and $U_1(r, c) \geq 0$. From step 1, $\sigma_r = \sigma_{\overline{v}} = 1$ so that conditional on being informed and the realized value v of V, the profits of an informed bidder equals $\frac{1}{2}[v - c]$ when $v \geq c$ and 0 otherwise. This is higher than the corresponding expression in the $\{0, c^*\}$ auction, state by state. Uninformed bidders earn non–negative expected profits in the $\{r, c\}$ auction and zero expected profits in the $\{0, c^*\}$ auction. Furthermore, since $r > 0$, the object is not sold with positive probability in the $\{r, c\}$ auction but sold with probability 1 in the $\{0, c^*\}$ auction. It follows that the latter strictly dominates the former in expected revenues. This completes step 2.

Step 3

From step 2, it follows that the revenue maximizing choice of a reserve and ceiling pair $\{r, c\}$ must satisfy $r = 0$. Then, the object is always sold with probablity 1, for any c. Furthermore, it is easy to see that the optimal ceiling c is at least c^*— from step 1, compared to an auction $\{0, c\}$ with $c < c^*$, both informed and uninformed bidders earn lower profits in the auction $\{0, c^*\}$.

Let $\Pi(0, c)$ be the unconditional expected information rents that the buyers earn in any auction with $r = 0$ and $c \in [c^*, 1]$. Since all these auctions are efficient (from Step 1), in order to compare revenues across these auctions it suffices to compare $\Pi(0, c)$ for different values of c. Observe first that for $c \in [c^*, 1]$,

$$\Pi(0, c) = \alpha[(\alpha + (1-\alpha)\sigma_{\overline{v}})\frac{1}{2}H(c) + (1-\alpha)(1-\sigma_{\overline{v}})H(\overline{v})]$$

Using the solution (17) for $\sigma_{\overline{v}}$, let

$$\Delta(c) \equiv \Pi(0,c) - \Pi(0,1) = \frac{1}{2}\frac{\alpha H(c)}{c-\overline{v}}[c + H(c) - \overline{v} - 2H(\overline{v})] \tag{24}$$

Note that $\Delta(c) \leq 0$ iff

$$c + H(c) \leq \overline{v} + 2H(\overline{v}) \tag{25}$$

The left–hand side of (25) is monotone increasing in c, implying that if the standard auction dominates the c^*–auction (i.e., if (25) is violated for $c = c^*$), then it also dominates all ceilings $c < 1$. Conversely, for c such that (25) holds, the derivative with respect to c of $\Delta(c)$ is

$$\Delta'(c) = [c + H(c) - \overline{v} - 2H(\overline{v})]\frac{d}{dc}[\frac{\alpha H(c)}{c-\overline{v}}] + \frac{\alpha H(c)}{c-\overline{v}}[1 + H'(c)] > 0$$

implying that when the c^*–auction dominates the standard auction, it also dominates all ceilings $c > c^*$. This completes the proof.■

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