Report on MSRI 1994 Summer Workshop on Hyperbolic Geometry and Dynamical Systems

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During the summer of 1994 the three of us gave a two-week workshop on hyperbolic geometry and dynamical systems at the Mathematical Sciences Research Institute (MSRI, pronounced "emissary") in Berkeley. The participants were about fifty graduate students chosen by their universities, most with no particular background in the subject. The workshop used nonstandard teaching methods and treated nonstandard topics. We describe here the thinking that led up to the workshop, our expectations and methods, and how things turned out in practice. Since there has been little discussion in the mathematical community about teaching and learning at the graduate level, we think that an account of our experience (mistakes and all) may be informative and useful in stimulating dialogue about graduate-level education.

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Hopes, Goals, and Philosophy

The workshop was an attempt to combine for a graduate audience our individual efforts in recent years to develop new approaches to teaching at the undergraduate level. The impetus for these efforts has been our dissatisfaction with the methods of pedagogy familiar to most of us. These methods tend to be of two types: collective, formal lectures on the one hand and self-study on the other. Many students profit from both, and we use both in our teaching, but neither stimulates interaction among students or between students and teachers. The result of this is that much of the energy latent in these relationships is never harnessed to further pedagogical aims.

Teaching mathematics need not be a choice between giving formal lectures and leaving the students to discover everything for themselves. One can consciously vary the style of one's interaction with a class and see what works best. Our workshop was an opportunity to try different approaches. We had some common ideas about what these approaches should be, and Bill and Jane had even tried out some of them in a team-teaching context. (We will use names to identify subsets of the authors.)

First, we wanted to avoid the over-emphasis prevalent in math education on formal definition and symbolic reasoning. Such reasoning is of course essential, but it is often difficult to communicate, and when presented too quickly or magisterially it can dampen a student's enthusiasm. In our workshop we led the students into the subject matter through hands-on experi-

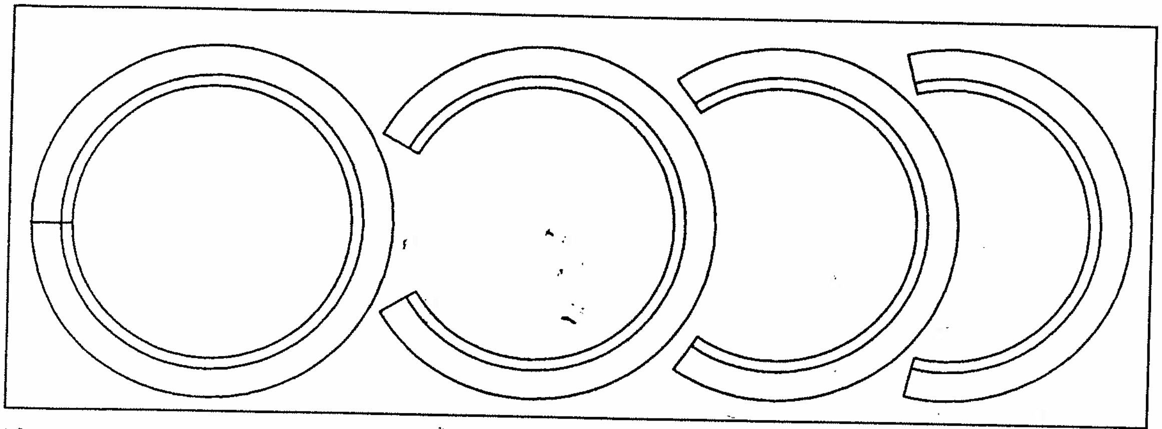

Figure 1. Gluing the outer edge of each strip to the inner edge of the one to its right causes the strips to warp. The result is paper of (approximately) constant negative curvature. The more strips are added, the more total curvature the surface acquires. Enlarge the figure and try it yourself.

ments that, we hoped, would encourage them to make connections between their own sensory observations and the symbolic representation of traditional teaching. This had been very successful at the undergraduate level: we've learned that theoretical knowledge takes on a more immediate and inviting importance for the students when it concerns an object that is "really there".

Another fundamental concern had been to lessen the ethos of competition among students. We prefer to emphasize cooperative learning, which reinforces the students' own capacity to discover ideas and tends to be more fun as well. Cooperative learning is hard in a lecture context, where members of an audience are expected to just absorb rather than find or share ideas. Because the finding and sharing of ideas was exactly what we wished to foster, our approach was to promote group learning and try to draw everyone into active participation. Our decision to team-teach was itself part of a conscious effort to provide a model of interaction for the students.

A third goal was to encourage students to explore in greater depth subjects of particular interest to them. Traditional courses, because of the pressure to cover material, often allow little opportunity for the type of spontaneous, in-depth exploration that nourishes a student's motivation to learn.

Finally, and perhaps most importantly, we wanted to share with students some of the beauty of mathematics, or rather some of what makes mathematics beautiful for us.

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Even with the strength of our shared teaching philosophy, it was a somewhat daunting prospect to lead a nonstandard workshop for a large, hand-picked graduate audience for the first time. Competition to get into the workshop had been stiff, and the students' expectations would be high. Another challenge for us was the heterogeneity of the class—some students had years of research experience already, some none.

We decided to assume no background in hyperbolic geometry, but to try to find material that would be new and interesting to as many students as possible. In the end, a large number of relevant mathematical topics were discussed in the workshop, including many not raised by us. We can't do justice to the latter, since they were too numerous and our knowledge of them came mostly from snatches of overheard conversations. Instead we mention the topics that we raised ourselves, and especially those that we approached in an unconventional way, at the same time that we try to give a feel for the dynamics of the workshop.

When students entered the MSRI lecture hall on the first day, they were each handed a glue stick and several pages of annular shapes to cut out (see figure above). Half of them were given scissors. They were told to begin cutting and pasting to make "hyperbolic paper" while other students arrived, office assignments were given out, and attendance and lunch orders were taken. They reacted well to the unusual demands we made of them and worked assiduously and with clear enjoyment on a task they might well have decided was more appropriate to a class of six-year-olds.

Students were then asked to discuss the following problem in small groups. Let S be the operation of adding one, and let D be the operation of doubling. How many integers does one obtain if one starts with 1 and applies all possible words in S and D of length at most n? Is there a connection between this problem and the shapes one gets by gluing together the annular cutout shapes?

Our goal in handing out paper, scissors, and glue sticks was to draw people in, equalize individuals with different backgrounds, and break the spell of symbolic representation, conven-