INTEREST RATES:

The analysis of interest rates over time is complicated because ***rates are different for different maturities***.

"Interest rate for borrowing money for the next 5 years" is ambiguous, because it does not specify how to average rates for different periods within the 5 years.

Zero rate: The rate if you pay back the principal and all interest on February 14, 2007.

Yield: The discount rate that makes the present value of the cash flow (coupons and principal) from a bond equal to the price of the bond.

Forward rate: Rate on February 14, 2002, for borrowing money on February 14, 2006 to be paid back with interest on February 14, 2007.

Coupon rate: Annual amount of coupons as a fraction of the principal.

Other complications:

- All these rates can be stated at any compounding (annual, semi-annual, continuous, etc.).
- All these rates are different in different markets (U.S. treasury, gilts, LIBOR, etc.).

Zero rate: The rate if you pay back the principal and all interest on February 14, 2007.

We have to start somewhere, and Hull starts with the zero rate.

Assume that zero rates are known. (Zero-coupon bonds are priced by the market.). Then we can calculate:

- **bond price,** the price of a bond with given coupons. (Hull's slide 4.5)
- **par yield,** the coupon rate at which a bond's face value is its price. (Hull's slide 4.7)

These are arbitrage prices if zero-coupon bonds are traded.

(In practice, if zero-coupon bonds are not traded, then we go the other way: we calculate zero rates from the yields of bonds actually traded. But let's leave that for later.)

Two concepts of yield:

Bond yield: Discount rate that equates the bond's cash flows to the bond's market value. (Hull's slide 4.6)

- Concept you learned in your introductory financial analysis course.
- The cash flows (coupons and principal repayment) are given.
- The input from the market is merely the market price of this particular bond. (You do not use the term structure of market interest rates directly.)
- The **bond yield** is an average of the interest rates you get if you buy the bond.

Par yield: The coupon rate that makes a given face value of a bond equal its market price. (Hull's slide 4.7)

- Only the maturity and face value of the bond is given (how much money the issuer wants to raise).
- The input from the market is the term structure of interest rates.
- The **par yield** tells you the coupon you have to pay to issue the bond at the given face value.

We usually state bond yield with continuous compounding and par yield with semi-annual compounding. *But this is only a convention.*

Three ways of representing the term structure of interest rates:

Zero rates: The rates for different maturities if you pay back the principal and all interest at maturity.

Forward rates: The rates at which you can contract now to borrow money at different future times.

Par yields: The coupons for bonds at different maturities.

All three are determined by today's market.

- The **forward rate** gives the purest information about today's market price for money at a particular time in the future.
- The **zero rate** averages the forward rates over the period to maturity.
- The **par yield** averages in a different way, so that times closer to maturity count for less.

See the picture on p. 94.

Continuously compounded rates of interest over successive intervals of time can be averaged.

$$e^a e^b = e^{a+b} = e^{(a+b)/2 \text{ x } 2}$$

Example 1

Suppose your money grow by the continuously compounded rates

- 10% in year 1
- 12% in year 2

What is the continuously compounded rate over the two years?

At the end of year two, your $1 has become

$$\$e^{.10}e^{.12} = e^{.10 + .12} = e^{.11 \text{ x } 2} = = e^{rT},$$

where r = 11% and T = 2 years.

Example 2:

If you have 3 years at 10% and 1 year at 12%, the continuously compounded rate for the whole 4-year period is

$$(3 \text{ x } 10 + 1 \text{ x } 12) / 4 = 11.5\%.$$

With continous compounding,

- average to get zero rates from forward rates.
- undo the average to get forward rates from zero rates.

Question 4.28. Assume that a bank can borrow or lend money at the same interest rate in Eurodollar markets.

Rates with continuous compounding:
90-day: 10%
180 day: 10.2%

Eurodollar futures contract price:
Contract maturing in 90 days: 89.5

What arbitrage opportunities are open to the bank?

HINTS

- What is the forward rate for borrowing money for 90 days 90 days from now?
- Use the standard formula to convert the Eurodollars contract price into an interest rate. Then convert this interest rate (which is an interest rate with quarterly compounding) into continuous compounding.

Question 4.30.

Hints:

(b) "10-basis point increase in yields" means that the continuously compounded interest rate goes from 10% to 10.1%.

(c) You should find that the change of interest rate from 10% to 15% lowers the price of Portfolio A less than that of Portfolio B.

(d) Look at the equation at the bottom of page 113. It says that a larger convexity C will keep the bond price from decreasing so much when y increases.