GOLD

Interest rate = 5%
Spot price of an ounce of gold = $300

Q: What is the forward price for an ounce of gold deliverable one year from now?

A: $315

(forward price) = (spot price) (1.05)

F = S (1.05)

First Explanation

$S today » $S(1.05) a year from now

You can put the $S in the bank and earn interest.

having $S today for buying gold
» having $F a year from now for buying gold

—Buy an ounce for $S today and hold it for a year.
—Or contract today to buy an ounce a year from now for $F.

Forward price T years from now:

$$F = S\,(1 + r)^T$$

With continuously compounded interest:

$$F = S\,e^{rT}$$

$$F = S(1 + r)$$

If $S = \$300$ and $r = 5\%$, then $F = \$315$.

Second Explanation

If $F > S(1 + r)$ or $F < S(1 + r)$, then an arbitrageur can make money.

$F > S(1 + r)$

Say F = $340, S = $300, and r = 5%.

Borrow $300.
Spend the $300 to buy an ounce of gold.
Go short in a forward contract (agree to sell for $340 in one year).

Next year: sell the gold for $340, pay off the loan with $315.
Profit: $25.

$F < S(1 + r)$

Say F = $300, S = $300, and r = 5%.

Borrow an ounce of gold from Joe.
Sell the ounce of gold for $300 and put the $300 in the bank.
Go long in a forward contract (agree to buy for $300 in one year).

Next year: buy an ounce of gold for $300 and return it to Joe .
Profit: $15 interest

If you are holding gold for speculation, you can do this without finding someone else to borrow the gold from.

FOR THE GAME THEORIST

PROTOCOL OF THE GAME

Time 0

Market announces interest rate r.

$K_0 = \$0$

Time 1

Market announces prices S_1 and F.

Speculator announces investments ϕ and γ.

$K_1 = (\phi \text{ forward contracts}) + (\gamma \text{ ounces of gold}) - \$S_1\gamma$

Time 2

Market announces price S_2.

$K_2 = \$[(S_2 - S_1)\gamma - S_1\gamma r + (S_2 - F)\phi]$

K_n = Speculator's capital at time n

S_1 = spot price of gold at time 1

S_2 = spot price of gold at time 2

F = forward price at time 1 for gold deliverable at time 2

ϕ = # of forward contracts speculator buys (positive if he goes long; negative if he goes short)

γ = amount of gold he buys (positive if he borrows cash and buys gold; negative if he sells borrowed gold and banks the cash)

ARBITRAGE

$$K_2 = (S_2 - S_1)\gamma - S_1\gamma r + (S_2 - F)\phi$$
$$= S_2(\gamma+\phi) - S_1\gamma(1+r) - F\phi$$

Set $\phi = -\gamma$. (Short as many forwards as you buy gold.)

$$K_2 = - S_1\gamma(1+r) + F\gamma = [F - S_1(1+r)]\gamma$$

Unless $F = S_1(1+r)$, Speculator can choose γ to make K_2 positive.

OIL

Interest rate = 5%
Spot price of a barrel of oil = $19
Storage cost of oil = 2% (pay in advance)

Q: What is the forward price for barrow of oil deliverable one year from now?

A: $20.35

(forward price) = (spot price) (1.02) (1.05)

F = S (1.02) (1.05)

First Explanation

$S(1.02) today » $S(1.02)(1.05) a year from now

You can put the $S(1.02) in the bank and earn interest.

having $S(1.02) today for buying and storing oil
» having $F a year from now for buying oil

—Buy a barrel for $S today and store it for a year.
—Or contract today to buy a barrel a year from now for $F.

Forward price T years from now (interest rate r and storage cost u):

$$F = S\,[(1+u)(1+r)]^T$$

With continuous compounding:

$$F = S\,e^{(r+u)T}$$

$$F = S(1+u)(1+r)$$

If S = \$19, u = 2%, and r = 5%, then F = \$20.35.

Second Explanation

If F > S(1+u)(1+r) or F < S(1+u)(1+r), then an arbitrageur can make money.

If F > S(1+u)(1+r)…

Say F = \$25, S = \$19, r = 5%, u = 2%.

Borrow \$19.38.
Spend \$19 to buy a barrel of oil and \$0.38 to store it for a year.
Go short in a forward contract (agree to sell for \$25 in one year).

Next year: sell the oil for \$25, pay off the loan with \$20.35.
Profit: \$4.65.

If F < S(1+u)(1 + r)…

Say F = \$16, S = \$19, r = 5%, u = 2%.

Store someone's barrel of oil, for a payment of \$0.38.
Sell the oil for \$19.
Put the \$19.38 in the bank.
Go long in a forward contract (agree to buy for \$16 in one year).

Next year: buy a barrel of oil for \$16 and return it to the owner.
You have \$20.35 in the bank.
Profit: \$4.35

If $F > S(1+u)(1+r)$…

Say F = \$25, S = \$19, r = 5%, u = 2%.

Borrow \$19.38.
Spend \$19 to buy a barrel of oil and \$0.38 to store it for a year.
Go short in a forward contract (agree to sell for \$25 in one year).

Next year: sell the oil for \$25, pay off the loan with \$20.35.
Profit: \$4.65.

This works for oil.

If $F < S(1+u)(1 + r)$…

Say F = \$16, S = \$19, r = 5%, u = 2%.

Store someone’s barrel of oil, for a payment of \$0.38.
Sell the oil for \$19.
Put the \$19.38 in the bank.
Go long in a forward contract (agree to buy for \$16 in one year).

Next year: buy a barrel of oil for \$16 and return it to the owner.
You have \$20.35 in the bank.
Profit: \$4.35

This does not work for oil.

Not much oil is held for speculation. Companies storing oil usually have contracts that require its use.

Speculative commodities (gold): **F = S(1+u)(1+r)**

Consumption commodities (oil): **F £ S(1+u)(1+r)**

Example 1.1 (p. 10): Standard Oil's Bond Issue
In 1986, Standard Oil issued bonds from which the holder received no interest. At the bond's maturity, the company promised an additional amount based on the price of oil at that time. The additional amount was equal to the product of $170 and the excess (if any) of the price of a barrel of oil at maturity over $25. The maximum additional amount paid was $2,550 (which corresponds to a price of $40 per barrel). These bonds provided holders with a stake in a commodity that was critically important to the fortunes of the company. If the price of the commodity went up, the company was in a good position to provide the bondholder with the additional payment.

Exercise 1.31

Show that the Standard Oil bond is a combination of a regular bond, a long position in call options on oil with a strike price of $25, and a short position in call options with a strike price of $40.

Example 1.3 (page 11): Range Forward Contract

Range forward contracts or *flexible forwards* are popular in foreign exchange markets. Suppose that on January 20, 1998, a U.S. company finds that it will require sterling in three months and faces these exchange rates:

Spot 1.6273 *3-month forward* 1.6196

It could enter into a three-month forward contract to buy at 1.6196. A range forward contract is an alternative. Under this contract an exchange rate straddling 1.6196 is set. Suppose that the chosen band runs from 1.6000 to 1.6400. The range forward contract is then designed to ensure that if the spot rate in three months is less than 1.6000, the company pays 1.6000; if it is between 1.6000 and 1.6400, the company pays the spot rate; if it is greater than 1.6400, the company pays 1.6400.

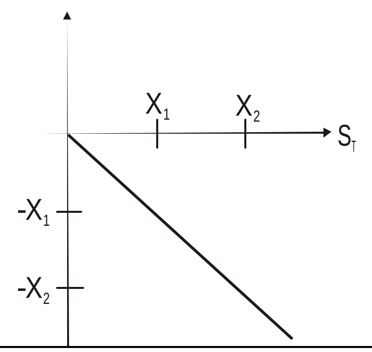
Exercise 1.26

Show that the range forward contract is a combination of two options. How can a range forward contract be constructed so that it has zero value?

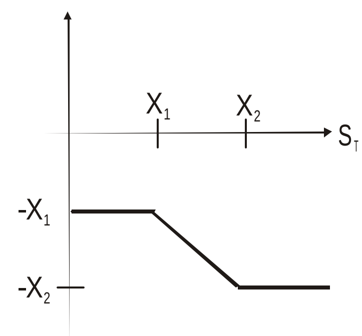
Geometric Solution of Exercise 1.26

We want to buy one unit of a foreign currency at time T.

This is our position if we plan to buy one unit of the foreign currency at time T. This position is entirely in negative territory. Our "payoff" is $-S_T$, where S_T is the price of the currency.

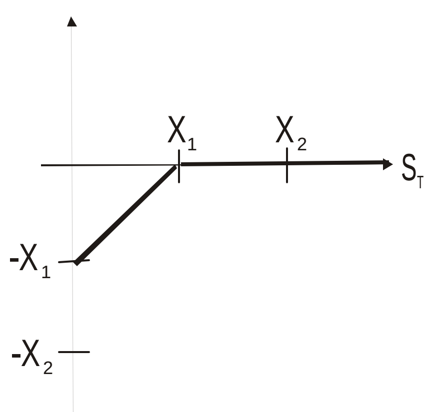


This is the position we want to be in (the range forward contract). Here we pay at least X_1 no matter how low the market cost S_T is at time T. But we pay no more than X_2.

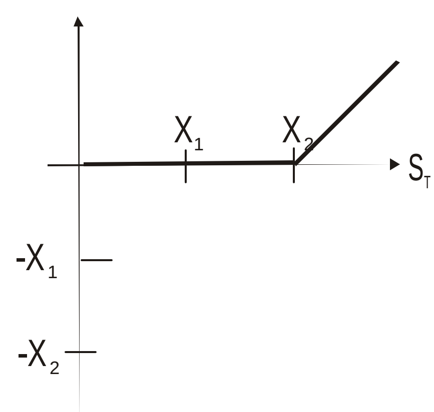


How can we convert the left-hand side to the right-hand side? The answer is that we add two option positions:

Short position in a put with strike price X_1.



Long position in a call with strike price X_2.



Can you see geometrically that when you add the two option positions, you get the range forward position?

The next page gives the algebraic explanation, from the solutions manual.

Algebraic Solution of Exercise 1.26

(From the solutions manual, available at New Jersey Books).

Consider a range forward contract to buy one unit of a foreign currency. Suppose that ST is the final exchange rate (value of one unit of the foreign currency) and the contract is structured so that

(a) If $S_T < X_1$, a price of X_1 is paid.
(b) If $X_2 < S_T$, a price of X_2 is paid
(c) If $X_1 \le S_T \le X_2$, the spot price S_T is paid.

The range forward contract can be regarded as a short position in a put option with exercise price X_1 combined with a long position in a call option with exercise price X_2. This is demonstrated by the following table:

	Cost of currency	Terminal value of put position	Terminal value of call position	Net cost
$S_T < X_1$	$-S_T$	$-(X_1 - S_T)$	0	$-X_1$
$X_1 \le S_T \le X_2$	$-S_T$	0	0	$-S_T$
$X_2 < S_T$	$-S_T$	0	$S_T - X_2$	$-X_2$

The range forward contract is normally set up so that the initial value of the call equals the initial value of the put, that is, so that it costs nothing to set up the range forward contract. Note that when $X_1 = X_2$ a regular forward contract is obtained.

Exercise 1.32

Use the DerivaGem software to calculate the value of the range forward contract considered in Example 1.3 (page 10) on the assumption that the exchange rate volatility is 15% per annum. Adjust the upper end of the band so that the contract has zero value initially. Assume that the dollar and sterling risk-free rates are 5.0% and 6.9% per annum, respectively.

Solution

Exchange rate = 1.6237
volatility = 15%
risk-free rate = 5.0%
foreign risk-free rate = 6.9%
time to exercise = 0.25 years

- Call with strike 1.6400 is worth 0.03874.
- Put with strike 1.6000 is worth 0.03851.
- So portfolio (short in put and long in call) is worth 0.03874 –0 0.03851 = 0.0023.

Use trial and error (increase strike for call slightly) to make the portfolio value zero.