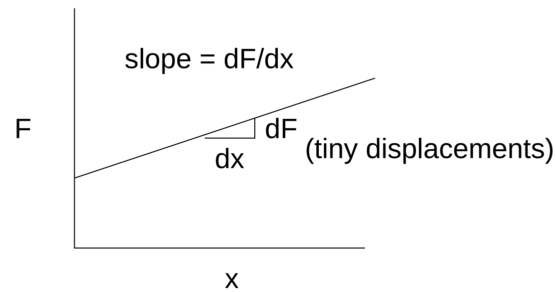
Quick Review of Partial Derivatives

1D Problems

First recall the use of derivatives in simple one-dimensional (one variable) problems. The equations for a line, for example, and its slope (the derivative) are

$$F = ax + b \quad \frac{dF}{dx} = a \tag{1}$$

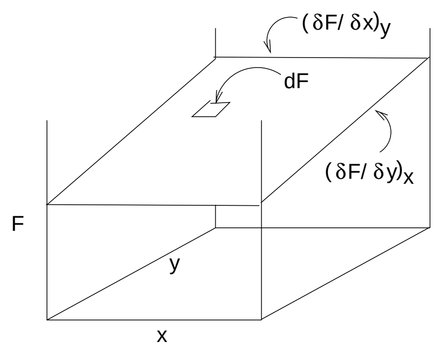


2D Example

Now image a simple two-dimensional (two variables) problem. The simple equation describing a plane can be used to illustrate the notion of partial derivatives.

$$F = ax + by + c \quad \left(\frac{\partial F}{\partial x}\right)_y = a \quad \left(\frac{\partial F}{\partial y}\right)_x = b \tag{2}$$

The slope of the function F along slices parallel to the x-axis at a fixed value of y is called a *partial derivative*, $(\partial F/\partial x)_y$. Another partial derivative of F can be written for the slope of slices parallel to the y-axis, $(\partial F/\partial y)_x$. The subscripts x or y simply serve as reminders that the one variable is treated as a constant when computing the derivative with respect to the other variable.



Note that for a two-dimensional plane, the partial derivative with respect to x happens to be independent of y, and the partial derivative with respect to y happens to be independent of x. This situation will not be true in general.

The *total derivative* of F is a tiny piece of the surface defined by the partial derivatives:

$$dF = \left(\frac{\partial F}{\partial x}\right)_y dx + \left(\frac{\partial F}{\partial y}\right)_x dy \tag{3}$$

It is straightforward to extend the two-dimensional case to problems with many dimensions.