Inflation Targeting:
a Framework for Communication∗

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Abstract
More than a monetary policy strategy, we interpret inflation targeting as a framework for communication. We model monetary policy as an information game between the Bank and private agents. Our analysis shows how the provision of an explicit numerical inflation objective overcomes potential information imperfections by providing a focal point for agents who form expectations. Furthermore, the combination of the target and the tolerance bands around it communicated, provide a very clear framework with which to evaluate monetary policy outcomes. A successful Central Bank then builds up credibility and a credible Central Bank is in a better position to be successful in subsequent periods. We show how (and when) inflation targeting exploits this self-reinforcing loop to help the Central Bank endure large and long-lasting shocks. Last, we show that a trade-off emerges when choosing the band-width: too narrow bands provide a focal point but reduce the likelihood of inflation being ‘successful’. Too wide bands on the other hand, lead easier to success but at the risk of failing to provide a clear focal point. We thus derive the optimal band-width for different scenarios.

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1 Introduction

It is often argued that the two distinct features of inflation targeting are the provision of first, an anchor for expectations and second, a transparent set of criteria with which to evaluate Central Banks (King 2002). In this paper, we concentrate not so much on what defines inflation targeting as a monetary policy strategy, but on how it operates as a communication framework that deals with information imperfections. With reference to the anchoring of expectations, we argue that when information available to agents is imperfect, the provision of clear signal, even if it is partial, can potentially provide a focal point at which agents coordinate. The deciding factor in the target’s capacity as a focal point is credibility and how it is updated. We examine how and why such a regime provides a clear framework for assessing monetary policy. In doing that we need to allow for two important mechanisms to enter explicitly in our methodology. The first is the inter-temporal self-reinforcing loop between credibility and success, which necessitates that credible Central Banks (CBs) are more successful, and successful Central Banks are easier credible. The second is modelling the choice of band-width around the announced target, in a way that captures the trade-off between providing a clear signal on the hand (narrow bands), but wishing to be successful as often as possible (wide bands), on the other. Allowing for these two mechanisms, we can then discuss the circumstances in which announcing an inflation target can be an effective communication framework.

They key variable that drives these mechanisms is expectations. Indeed, modern monetary policy has emphasized that maintaining a stable monetary environment depends crucially on the ability of the policy regime to control inflation expectations (Blinder et al, 2001, Woodford, 2003). Evidence of that is shown by Paloviita and Virén (2005) for inflation in the euro area and by Orphanides and Williams (2004) in their analysis of US monetary policy history. The latter authors argue that monetary policy failures are connected with changes in public sentiment about the future state of the economy. In other words, policy mistakes alone are not enough to produce long-term negative effects on monetary stability. Expectations will also have to deviate from long term objectives for these effects to materialize. As a means of preventing such expectations deviations, policy makers develop communication strategies that aim explicitly to align expectations with their own policy objectives. The provision of an explicit numerical inflation target is one such example of a communication strategy, the main advantage of which is arguably its ability to provide a focal point for private sector expectations. Empirical evidence appears to confirm that explicit quantitative targets for inflation succeed in that capacity, as shown by Mishkin and Schmidt-Hebbel (2001, 2007), Johnson (2002) and more recently by Levin
Central Bankers themselves emphasize the link between the two in their own evaluations of their respective monetary policies. Mervyn King claimed in 2002 (p.4) that for the UK case, inflation expectations had indeed been anchored to the pre-announced target. Similarly Issing et al (2005) emphasized the importance of announcing a clear inflation objective for helping coordinate expectations. Following this realization however, there are two questions that need to be addressed: first, why do expectations need to be coordinated? Second, how does communicating an inflation target actually help coordinate them? Demertzis and Viegi (2006) provide a model in which these questions are tackled explicitly. Monetary policy is portrayed as an information game in which, if private agents’ objective is to correctly forecast inflation at the relevant horizon, then to do so they need to both assess the Central Bank’s actions, as well as second-guess other agents’ views. It is the latter aspiration that, in effect, generates the motive to coordinate with other players. Communicating an inflation target then provides the option of coordinating at this level. The necessary and sufficient condition for coordination at the desired level is that the target announced is ‘sufficiently’ credible, where the sufficiency condition is derived endogenously in the set-up. This critical condition is a function of economic circumstances and the quality of all information available to the agents. The inflation target satisfies its role as a coordinator of expectations the more stable economic conditions. Similarly, this role is better fulfilled the more noisy all other public information available, implying that the target acts as a substitute for all other public information. Once expectations are tied down to the target, the Central Bank can then identify that policy that will help it meet its objectives and therefore, be ‘successful’.

The set-up so far allows for credibility to be the determinant of success in monetary policy. Equally important however, is the fact that only demonstrably successful monetary policies can earn Central Banks credibility\(^3\), which in turn would enhance their ability to be successful in the future. The opposite is also true; failing to achieve predefined objectives damages credibility and therefore, the Central Bank’s ability to also do so in the future. The important point here being that credibility and ‘success’ feed into each other in a self-reinforcing loop. We argue that the dynamic nature of this loop is what provides an inter-temporal link, crucial to the decisions Central Banks make (as they try to build up reputation), and is an essential component of modelling monetary policy in practice (see for example Blackburn and Christensen, 1989 and Bonfim and Rudebusch, 2000). The advantage of modelling monetary policy as an information game is that it provides an explicit measure for ‘sufficient’ credibility. At the same time, announcing an inflation target provides a very clear measure of success. Combining these two measures with a certain updating mechanism, we

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\(^1\)See also Leiderman and Svensson (1995) and Bernanke et al (1999) for earlier accounts of experiences with inflation targeting.

\(^2\)‘Success’ is identified here with meeting one’s objectives. Note that as a Central Bank’s objectives are chosen based on some societal welfare criteria, a successful central bank is beneficial to society.

\(^3\)In line with the ‘competence implies credibility’ argument presented by Moscarini, 2007.
can exploit the way credibility and success feed into one another. We will show how communicating a target exploits favorable circumstances better in terms of building credibility, that will then increase a Central Bank’s ability to withstand shocks when unfavorable conditions arise.

Last we are going to assume that credibility changes between periods are derived based on a Bayesian updating mechanism. The direction of change is determined by past inflation performance. The advantage of such a technique is that it can identify the exact band-width around the target that will enhance its efficiency. As success helps acquire credibility, there is a natural tendency to overestimate the size of the bands in order to increase one’s success record. However, the incentive to coordinate implies that wide bands are discounted as unclear signals that show no confidence in achieving pre-defined objectives. Our methodology will capture this trade-off and thus identify the optimal band-width.

The paper is organized as follows. Section 2 summarizes the way that a number of countries have experienced inflation targeting. Section 3 describes monetary policy as an information game and shows how the provision of an inflation target increases the options individuals have. However, the credibility of that target will be crucial for the individual’s decision. Section 4 then introduces the dynamic aspect to the game and links credibility with success inter-temporally. We thus derive how credibility improves or worsens depending on previous period’s inflation performance. Section 5 describes the results of Monte-Carlo simulations to identify the circumstances in which communicating an inflation targets provides the greatest gains in credibility and inflation performance. Section 6 then describes the case for an optimal band-width and shows how and why it varies with different assumed parameterizations. Section 7 offers a brief discussion of our results and concludes.

2 Inflation Targeting Experience

We describe here briefly how a number of countries have experienced inflation targeting in relation to the points raised above\(^1\). Has the provision of an explicit inflation target acted as a focal point for expectations? Are there discernible benefits to credibility and how do they manifest themselves? How have countries chosen to communicate the bands around the target and to what effect?

It is fair to say that most of the countries had experienced periods of relative high and volatile inflation prior to the introduction of explicit targets but have also seen, on the whole, significant improvements in inflation around the time of introducing it (Bernanke et al, 1999). In Figures 1-4 we summarize the way inflation, the target and the bands around it, and for some countries inflation expectations, have evolved over time.

A number of countries have introduced the targets at the bottom, or at relative low levels, of the downward trend (e.g. Sweden, UK, Switz.). Others waited

\(^1\)See Mishkin and Schmidt-Hebel (2001) for a comprehensive discussion of the way a number of countries have experienced inflation targeting.
till inflation had stabilized at relatively low levels (Norway, Australia, South Africa). Finally, a separate group have changed the inflation target bands progressively to bring inflation down (New Zealand, Canada), an approach that proved particularly popular in countries that were at the time facing relatively high levels of inflation (most Latin American countries, but also the Czech Republic, Hungary, Republic of Korea and Israel). For a number of countries in Figure 1 we see that expectations are simply disconnected from the level of inflation and are focused on the mid-point (Canada and Sweden being the prime examples, followed by Australia and New Zealand). The UK is also very interesting, especially since agents were very quick to internalize the change of inflation measure targeted in December 2003\(^5\). Figure 1 shows expectations

\(^5\)The Bank of England has been targeting the RPIX till December 2003 at a level of 2.5%. After that it has switched to targeting CPI at the level of 2%. We thus present data for the first till December 2003 and the second after that. The remit given to the MPC by the Chancellor states that the target is a point target. We report bands however of ±1% to reflect that the Governor must write an open letter to the Chancellor explaining why inflation is
moving to the new mid-point. But the EU (Figure 4) and Norway, who only have an upper limit rather than a symmetric band around a target, also appear to fall into this category with that limit acting effectively as the focal point.

But there are also big differences in the way countries have implemented the inflation targeting regime. For some countries, the target and bands around it have been seen, right from the start, as the long term objective. Countries like Australia, Sweden or Norway announced just the one band-width (and target). For others, there has been a certain degree of refining at the beginning, (see Canada, New Zealand and to a lesser extent the UK), eventually settling to a unique target and band-width after few adjustments. On the other hand, for a big number of countries, inflation targets and the bands around them were introduced progressively and were subject to changes in small and gradual steps.

Figure 2:

away from the point target, what actions the MPC will take to return inflation to target and the horizon over which they intend to do so.
This is typical for countries that were on a disinflationary path and were using the regime very much as a means of building up credibility. These countries set progressively ambitious objectives, thus allowing the public to evaluate their performance from one year to the next. A number of Latin American countries appear to be in such a regime (Chile, Colombia, Mexico and Peru).

Figure 3:

Once at a relatively low level of inflation, it will be very interesting to see whether the public in these countries, makes the switch in terms of now interpreting the inflation bands as a medium to longer-term objective and thus not penalize the Bank for short term misses. Preliminary evidence for Chile and the Czech Republic shows that expectations are either tied to the mid-point (Chile) or are following the mid-point as the band-width changes (Czech Republic, both in Figure 2). In Poland and South Africa on the other hand, expectations (1-year and 2-year ahead respective) appear to correlate highly with actual inflation, with little attention paid to the pre-specified bands. In both countries therefore,
inflation targeting has not be successful in terms of disconnecting expectations from inflation, even though, certainly in the case of Poland, inflation has been on a very successful downward trend\textsuperscript{6}.

Can we identify next instances where credibility allowed countries to deal more effectively with adverse shocks? There are two instances that are interesting: Australia in the period between 2000 and 2002 and Canada in 2003. Both countries were experiencing a period in which expectations had been at the midpoint of the target and therefore, arguably credible. In the periods in question, actual inflation was pushed outside the predefined limits but as credibility was in place for some time, this deviation was short-lived and did not present an obstacle to bringing inflation back to low levels\textsuperscript{7}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Inflation (CPI), Inflation Expectations Consensus Forecasts, 6-10 Years Ahead, Explicit Inflation Target (or objective)}
\end{figure}

Last, the issue of the band-width is naturally pivotal to the implementation of the regime. The argument, as explained above, is that wide bands reflect lack

\textsuperscript{6}\textit{Understanding the reasons behind this cannot be done without acknowledging the countries’ degree of openness.}

\textsuperscript{7}Sweden has a number of instances in which inflation was outside the limits (which were also not penalized but they entail being below the lower limit which might be deemed less of a problem. Interestingly, the UK is coming up to a test case, with inflation now being outside the upper band. Preliminary evidence shows that expectations are not unaffected. Will this last?}
of confidence and therefore fail to provide a focal point. Too narrow bands on the other hand, run the risk of being surpassed, which in itself penalizes Central Banks’ credibility. Mishkin and Schmidt-Hebbel (2001) explain that “...when Chile adopted inflation targeting in 1991 with inflation exceeding 20%, the inflation target was treated more as an official inflation projection rather than as a formal ‘hard’ target....Over time, the central bank put greater emphasis on the price stability objective and with its success in disinflating and meeting its inflation objectives, the public began to interpret those objectives as ‘hard’ targets for which the Central Bank could be made accountable” (italics added). In the beginning therefore, it is crucial to establish credibility. The bank then adopts relatively wide bands and puts an effort in achieving its announcement. As ‘success’ comes and credibility increases, the Central can then afford to both move its objective along the long-term objective of price stability, as well as refine the band that will focus expectations more effectively. The experience of most countries is that bands at the point of introduction are wider and only gradually do they become narrower as the mid-point is adjusted downwards. Countries that have instead experimented with the width along the way seem to be less vested against inflation (Brazil, Israel) although for the case of the Czech republic expectations appear to just follow the mid-point irrespective of where it is or the width around it. The case of SA on the other, is an example where a more gradual approach to the width may have been more beneficial to expectations. As it stands right now it appears that the communication of the target has not really helped disconnect expectations from the level of inflation.

We also present a graph for countries (Figure 4) that are slightly different altogether. The Euro area, although not an official inflation targeter, also has long-term expectations very close to the official quantitative objective for inflation of below but close to 2%. This is despite the fact that inflation appears to have stabilized just above the target. Japan and the US are two countries that have not communicated an explicit inflation target and have experienced monetary policy in very different ways. Expectations in Japan have been quite volatile and persistently above inflation. By contrast, in the US expectations are very monotonic and appear to follow the inflation trend, which up to 2000 was downward, whereas beyond 2000 stabilized at around 2.5%. If the break of correlation between inflation and expectations is a sign of credibility, then the US appears to compare very favorably.

3 Monetary Policy as an Information Game

The Central Bank has a standard loss function in which it chooses the rate of inflation $x$ to minimize the distance from the inflation objective set $x^T$ and close the output gap $y$,

$$L_{CB}|\xi = \frac{1}{2}E\left[(x - x^T)^2 + y^2\right]$$

subject to a standard Lucas supply function, $y = x - x^e + \xi$ where $\xi$ is a supply

9
shock with zero mean and constant variance, $\sigma^2_\xi$. Note that any Central Bank will have an objective $x^T$ irrespective of whether it communicates it to the public clearly, or even at all. We assume for simplification that the CB’s instrument is $x$. Optimization of (1) implies that

$$x|\xi = \frac{x^T}{2} + \frac{x^e}{2} - \frac{\xi}{2} \quad (2)$$

where $x$ is now the ex post inflation outcome conditional on the shock $\xi$ and $x^e$ is private sector expectations about the relevant rate of inflation. Representation (2) is of a structural form\(^8\) in the sense that expectations are not replaced (Leitemo, 2006). Svensson (2003) argues in favour of such a representation in order to indicate that factors like judgement that contribute to the way expectations are formed but cannot always be modelled, are an important contributor to monetary policy. In a typical commitment game, where the Central Bank communicates its target $x^T$ and commits to it, expectations formed by all individuals collectively are equal to the CB’s objectives, $x^e = x^T$ and the ex post outcome is

$$x|\xi = x^T - \frac{\xi}{2} \quad (3)$$

$$E(x) = x^T \quad (4)$$

Modeling monetary policy as an information game implies a departure from the assumption that expectations are by default equal to the objective of the Central Bank and analyses instead how individuals go about interpreting the information that is available to them when forming expectations. Rather than impose that expectations equal the target, the information game looks for the process followed by agents that leads to that being true. Every individual $i$ will be forming an expectation of inflation $x_i$, such that the collective outcome (for a continuum of agents) is $x^e = \int_0^1 x_j dj$, which is the expectation that is relevant to ex post inflation. The timing of the game assumed has the Central Bank deciding what its objectives are first, shocks occur next, then private agents

\(^8\)Note that (2) is specific to the underlying Lucas supply function assumed but demonstrates that the outcome will be a function of both the policy the Central Bank pursues as well as what the private sector anticipates. Analogously, had the model been of the standard Neo-Keynesian type based on Clarida Gali and Getler (1999),

$$x_t = \beta E_t x_{t+1} + k y_t + \epsilon_t \quad y_t = E_t y_{t+1} - \gamma (i_t - E_t x_{t+1}) + \eta_t$$

then the structural representation of the ex post inflation outcome would be

$$x_t = \frac{k^2}{1+k^2} x^T + \frac{1}{1+k^2} E_t x_{t+1} + \frac{\epsilon_t}{1+k^2}$$

Our point is to show that the ex post outcome is a function of both the CB objective as well as the expectations of the private sector.
form expectations based on information available about these shocks and policy objectives and finally the CB reacts to them.

We thus start by assuming that typically, every individual forms expectations based on two information sets, namely what is publicly available and therefore common to everyone, and what is available to them privately. Furthermore, every individual is aware of the fact that the ex post outcome of inflation $x$ will be determined by (2), in other words will be affected equally (given the model we assume) by the policy the Central Bank pursues to attain its objectives, as well as the average of expectations formed by the public.

However, as the individual is interested in predicting the ex post level of inflation correctly\(^\text{9}\) (in order, for example, to base her wage negotiations on), she needs to interpret both components of (2) based on the information she has. Her objectives are captured by a standard expected dis-utility:

$$u_i (x^e, x^T) \equiv \frac{1}{2} \mathbb{E}_i (x_i - x)^2 \quad (5)$$

Note that subscript $i$ in the expectations operator indicates that the individual will be seeking to minimize her expected dis-utility, given her own perceptions. $x_i$ is individual $i$’s expectation of what inflation will be at the relevant horizon and $x$ is again the ex post inflation outcome. We use $x^e$ to refer to the expectations profile over all agents. The individual decides her action $x_i$ based on the first-order condition of (5). This is:

$$\text{arg min}_{x_i} u_i (x^e, x^T) = \mathbb{E}_i (x)$$

and from (2),

$$x_i = \mathbb{E}_i (x)$$

$$x_i = \mathbb{E}_i \left( \frac{x^T}{2} + x^e - \frac{\xi}{2} \right)$$

$$x_i = \frac{1}{2} \mathbb{E}_i (x^T - \xi) + \frac{1}{2} \mathbb{E}_i (x^e) \quad (6)$$

The optimal action for individual $i$ in (6) is thus a function of three things: the objectives of the Central Bank and hence the policy it will pursue, the shock that will have occurred and finally the average expectation formed by all individuals.

Moreover, in forming expectations $x_i$, individual $i$ needs to evaluate these three things, captured here by the expectations operator, subscript $i$. It follows that if $x_i = x_j \forall j$, then $x_i = x^e$ and individuals’ expectations are matched. However, although desirable, coordination between agents at any level of inflation is not sufficient; the optimal outcome occurs when agents coordinate at the objective pursued by the Central Bank, $x^T$. Coordination at any other expectation rate

still leaves agents away from the level of inflation that the CB aims to achieve. We will argue further down that knowledge of the CB objective is necessary but not sufficient for coordination at it. Following Morris and Shin (2002), we argue that information used by the agents is available in the form of a public signal common to all and a private signal which is specific to each agent in the economy. Individuals therefore, observe \( p \) and \( z_i \) where,

\[
\text{Public signal: } p = (x^T - \xi) + \eta \tag{7}
\]

\[
\text{Private signal: } z_i = (x^T - \xi) + \varepsilon_i \tag{8}
\]

Both \( \eta \) and \( \varepsilon_i \) are normally distributed with a zero mean and variance \( \sigma^2_\eta \) and \( \sigma^2_{\varepsilon_i} \) respectively. Furthermore, the two error terms are independent of \( x \) and of each other, such that \( \mathbb{E}(\varepsilon_i \varepsilon_j) = 0 \) for \( i \neq j \). The clarity of public information is not under the full control of the CB but is affected by a combination of the CB’s information strategy, the general market information available and noise. Based on these two types of signals, Morris and Shin (2002) show that agent \( i \)’s action (inflation expectation) then is

\[
x_i = \frac{2\alpha p + \beta z_i}{2\alpha + \beta} = x^T - \xi + \frac{2\alpha \eta + \beta \varepsilon_i}{2\alpha + \beta} \tag{9}
\]

where \( \alpha = \frac{1}{\sigma^2_\eta} \) and \( \beta = \frac{1}{\sigma^2_{\varepsilon_i}} \), the level of precision for the two information sets respectively.

**Definition 1**: We call (9) the "MS action".

We assume homogenous agents and calculate expectations across all agents as follows:

\[
x^e = \int_0^1 x_j dj = x^T - \xi + \frac{2\alpha \eta}{2\alpha + \beta} \tag{10}
\]

Equation (10) shows that the average expectation across all agents will be distorted by the (lack of) precision of the two signals as well as the preference for the ‘beauty term’ (\( r \) in Morris and Shin terminology), here equal to \( \frac{1}{2} \) (from 6).

### 3.1 Inflation Targets as Focal Points

Equations (9) and (10) show that expectations formed will be a function of the objectives of the Central Bank, the supply shock and the precision of information individuals have. However, if a Central Bank has communicated to the public what its inflation objective is, the individual effectively receives an extra signal in addition to (7) and (8), i.e.:

\[
\text{Central Bank signal: } h = x^T \tag{11}
\]
In turn the individual is faced with an option to either apply (9), or driven by her desire to coordinate with others, form expectations according to the target. In other words, the ‘action’ the individual takes could be either $a_i = x_i$ or $a_i = x^T$ and the ‘average’ action is respectively $\bar{a} = x_i$ or $\bar{a} = x^T$. The very provision of an inflation target therefore increases the number of options available to the individual and thus the number of potential outcomes\(^{10}\). Presenting the individual with this option is attractive because it overcomes the problem of having to guess what information everyone uses when forming expectations and is very much in line with the Morris and Shin (2002) observation that public information receives a greater weight in people’s action than is justified by its quality. However, the gain of bypassing information imperfections comes at the cost of getting only partial information ($x^T$) about the relevant set (state: $x^T - \xi$). The real trade-off therefore faced by the individual is more information for better precision. We examine then how the individual would go about evaluating this trade-off when forming a decision.

We re-write (5) to account for the multiplicity of actions and calculate the dis-utility attained when applying either of two actions, i.e.:

$$u_i(a_i, \bar{a}) \equiv \mathbb{E}_i (a_i - x)^2$$ (12)

Note that while the announcement of a target increases the number of options that the individual has, it only really offers the possibility of coordination at the target. A number of conditions need to be satisfied for this to happen, credibility being the most important, as we show next. Table 1 summarizes the pure form strategies available to individual $i$ and the dis-utility outcomes associated with them, based on (12):

<table>
<thead>
<tr>
<th>$a_i$ \ $\bar{a}$</th>
<th>$x^T$</th>
<th>$x^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>$\frac{\sigma_x^2 (2\alpha + \beta)^2}{\alpha (2\alpha + \beta)^2}$</td>
<td>$\frac{\sigma_x^2 (2\alpha + \beta)^2}{4\sigma_x^2}$</td>
</tr>
<tr>
<td>$x^T$</td>
<td>$\sigma_x^2 + \frac{\alpha}{(2\alpha + \beta)^2}$</td>
<td>$\frac{\alpha}{(2\alpha + \beta)^2}$</td>
</tr>
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</table>

Table 1 shows that for any given level of precision for public and private information, adopting the inflation target $x^T$ becomes a dominant strategy for individual $i$ if the variance of the supply shock is below a given threshold, i.e.:

$$\sigma_x^2 < \frac{\beta}{(2\alpha + \beta)^2}$$ (13)

As we assume homogenous agents (and by implication everyone adopts their dominant strategy) the game has then one Nash equilibrium, namely $E_1 \equiv$ \(^{10}Brazier et al (2006)\) allow also for this switch, only in their case it is between the target and past inflation, depending on which of the two had performed better in the past.
Individual $i$ ends up facing dis-utility equal to $\frac{1}{4}\sigma_\xi^2$. However, if condition (13) is not satisfied, i.e. $\sigma_\xi^2 \geq \frac{\beta}{(2\alpha + \beta)^2}$, then individual $i$’s optimal response in pure form strategies requires ‘matching’ the average action. In other words, $a_i = x_i$ is the best response to $\bar{a} = x^c$, and $a_i = x^T$ is the best response to $\bar{a} = x^T$. The game has now two Nash equilibria in pure form, namely $E_2 = (x_i, x^c)$ and $E_3 = (x^T, x^T)$.

The variable that is going to be pivotal to the individual’s decision, when faced with a matching game, is the extent to which the Central Bank is credible. A sufficiently credible, in the eyes of the individual, Central Bank will induce her to opt for $x^T$. Otherwise, the individual will resort to forming expectations based on all the information that she has, $x_i$. Our choice to use credibility as the decisive factor is based on the fact that modern monetary policy stresses it as the most important contributor to stable prices. This view is shared by professional economists and Central Banks alike, as shown in the surveys carried out by Waller and de Haan (2004) and Blinder (1999). Credibility is therefore what is going to allow Central Banks the flexibility to tackle shocks as they arise and what in itself will generate further monetary policy successes. Borrowing from Demertzis and Viegi (2006), appendix A explains in detail how credibility enters the individual’s choice-space and how she would go about deciding between the two actions. Given the representative nature of the individual assumed, this allows us to predict which of the two Nash equilibria, $E_2$ or $E_3$, will prevail. We present next the outcome of this process.

**Definition 2:** Let variable $v$ denote the degree of the inflation target’s credibility and $v \in [0, 1]$.\[11\]

Based on $v$ we apply an expected utility framework that will help the individual form a choice. This framework relies on Bacharach’s *Variable Universe Games* (1993) contribution on focal points. Based on this, Demertzis and Viegi (2006) derive the necessary and sufficient condition for agents (individually and collectively) to form expectations according to the target:

$$v \geq \frac{(2\alpha + \beta)^2 \sigma_\xi^2 - \beta}{4\alpha + (2\alpha + \beta)^2 \sigma_\xi^2}$$

Or in other words, (14) shows that the sufficient condition for individual $i$ to follow the target is when the target’s credibility is sufficiently high. Based on this condition, individual $i$ then forms expectations at time $t$ as follows:

$$a_i(t) = \begin{cases} 
  x_i (\text{and } x^c) & \text{if } v_t < \frac{(2\alpha + \beta)^2 \sigma_\xi^2 - \beta}{4\alpha + (2\alpha + \beta)^2 \sigma_\xi^2} \\
  x^T & \text{if } v_t \geq \frac{(2\alpha + \beta)^2 \sigma_\xi^2 - \beta}{4\alpha + (2\alpha + \beta)^2 \sigma_\xi^2} 
\end{cases}$$

Note that the condition for credibility $v$ depends on the variability of the shocks, and the precision of the two signals, public and private. It does not however,\[11\]We use the terms ‘inflation target credibility’ and ‘central bank credibility’ interchangeably.
depend on any of the current shocks, - supply, public or private information. This is the case because the agent evaluates the Central Bank in knowledge of the distribution properties of the three variables, but before their realizations occur. Inflation on the other hand depends also on the actual supply shock, and when expectations are formed according to the MS action, also on the (public) information shock observed every period.

4 Evaluating Monetary Policy

Having announced a target, we examine next how this target helps the Central Bank gain credibility. The main assumption behind what follows is that credibility is solely determined by a Central Bank’s previous success, or in other words, by how well it has managed to achieve its objectives in the past (Blackburn and Christensen, 1989). However, ‘success’ itself is in turn affected by two things: the ability to tie down expectations to its target (credibility), but also the size of the supply shocks. Observing then the Central Bank’s track record, agents update their beliefs about its abilities and accordingly affect the inflation outcome in the next period. There is therefore an inter-temporal loop between success and credibility which is reinforcing in both directions and is essential to the monetary policy outcome.

This approach is very similar in spirit, to that of Bomfim and Rudebusch (2000) with two important differences. First, expectations in our case are discrete, in that the switch between the two ‘expectations states’ depends on how current credibility compares to the critical condition in (14). Bomfin and Rudebusch (2000) instead have expectations being formed in a continuous manner, depending partially on the target and its credibility (which is also updated given past success) and partially on past performance. The discrete switching applied here is the direct result of the individuals’ incentive to coordinate, which induces them to look for opportunities to converge to focal points. Second, our updating mechanism is also slightly different to that of Bomfim and Rudebusch (2000), in that the individual rewards a successful Central Bank in terms of increasing the level of trust she puts in it, but she also penalizes an unsuccessful Central Bank by reducing credibility. The game is organized in such a way that at a given period, the Central Bank operates with a given ‘stock’ of credibility, very much in the Barro-Gordon (1983) sense. This implies that within that period, the Central Bank can no longer affect its credibility. This is a necessary feature in our view, in order to capture the fact that credibility is intrinsically the result of past performance only, and that at any point in time when the Central Bank takes a decision, it reckons with the fact that it has to operate within the confines of its own reputation. However, today’s actions will affect next period’s reputation, the Central Bank’s credibility and ultimately also its ability to be successful. With the help of numerical simulations we will show in the next section how the Central Bank’s success rate increases with a provision of a numerical target and under which conditions. We find the following: it is easier to build up credibility when the economic environment is stable or
put the other way around, it is a lot more difficult to improve reputation when economic circumstances are unfavorable. Within given economic circumstances however, when the CB is credible (i.e. expectations are successfully tied down to the inflation objective), its ability to achieve that objective is enhanced. In all circumstances however, if reputation is linked to performance and performance is linked to exogenous shocks, reputation and credibility can be gained but can also be lost from one period to the next. This emphasizes the ephemeral nature of credibility and the importance of capitalizing on favorable circumstances in order to withstand unfavorable ones. We explain how this updating occurs next.

4.1 Credibility Gained, Credibility Lost

We define first the terms ‘success’ and ‘credibility’ in monetary policy.

Let variable $S \in \{s, \bar{s}\}$ denote whether the Central Bank is successful or unsuccessful and $\Pr(S = s)$ the probability of a Central Bank being successful. We define $\rho$ as the radius of tolerance around the target.

**Definition 3:** A successful Central Bank ($S = s$) is one for which $|x_t - x^T| < \rho$ at a given $t$; by implication an unsuccessful Central Bank is one for which $|x_t - x^T| \geq \rho$.

Definition 3 above implies that a Central Bank announces an inflation target $x^T$ and a band around it $(2\rho)$. It can easily be interpreted as the band-width around an inflation target, as used by most inflation targeting Central Banks in practice. Naturally, as appearing to be successful is of importance to the Central Bank, one could use the band-width to maximize success. However, there is an important trade-off between providing a focal point (narrow band) and being successful (wide band) that leads to the existence of an optimal width. We will discuss this in section 6.

Let variable $C \in \{c, \bar{c}\}$ denote whether the Central Bank (and therefore its target) is credible or not. From definition 2 above, $v = \Pr(C = c)$.

**Definition 4:** A credible Central Bank ($C = c$) is one for which $\bar{a} = x^T$; a non-credible Central Bank ($C = \bar{c}$) is one for which $\bar{a} = x^e$.

Agents form views about the Central Bank’s credibility $v$ based on past period’s performance. For any period $t$, the timing of the game is as follows:

$$v_t | x_{t-1} \rightarrow \xi_t \rightarrow \bar{a}_t = \begin{cases} 
  x^e & \text{if } v_t < \frac{(2\alpha + \beta)^2\sigma^2 - \beta}{(2\alpha + \beta)^2\sigma^2 - \beta} \\
  x^T & \text{if } v_t \geq \frac{(2\alpha + \beta)^2\sigma^2 - \beta}{(2\alpha + \beta)^2\sigma^2 - \beta}
\end{cases} \rightarrow x | \xi_t, a_t \rightarrow x_t$$

16
The Central Bank begins with a certain level of credibility $v_0$, that is common knowledge. The private sector observes the outcome $x_t$ and updates its confidence at the Bank based on Bayes’ rule:

$$
\begin{align*}
\text{If } S_t = s & \text{ then } v_{t+1}(s) \equiv \Pr(c|s) = \frac{\Pr(s|c)}{\Pr(s)} \Pr(c) \\
\text{If } S_t = \bar{s} & \text{ then } v_{t+1}(ar{s}) \equiv \Pr(c|\bar{s}) = \frac{\Pr(\bar{s}|c)}{\Pr(\bar{s})} \Pr(c)
\end{align*}
$$

**Corollary 1** An implication of this updating is that as success increases credibility $v$, it will be easier to satisfy (14) in the next period and therefore tie down expectations. The opposite is also true: if monetary policy is unsuccessful, then it becomes increasingly more difficult to succeed in the period after that.

Given the new level of credibility $v_{t+1}$, the sequence of events at period $t + 1$ is identical to above, i.e.:

$$v_{t+1|x_t} \rightarrow \xi \rightarrow a_{t+1} \rightarrow x_{t+1} \rightarrow \ldots,$$

the private sector evaluates the outcome and updates again.

### 4.2 The Inflation Distribution

Based on (2), we can now derive the first and second moments of inflation, given expectations.

**First Moment:** the expected inflation outcome is always $x^T$ irrespective of how expectations are formed.

$$
\mathbb{E}(x_{1|a}) = \begin{cases} 
\mathbb{E} \left( \frac{x^T}{2} + \frac{x^T}{2} - \frac{\xi}{2} \right) = x^T & \text{for } \bar{a} : x^T \\
\mathbb{E} \left( \frac{x^T}{2} + \frac{x^T}{2} - \frac{\xi}{2} \right) = x^T & \text{for } \bar{a} : x^e = x^T - \xi + \frac{2\alpha}{2\alpha + \beta}
\end{cases}
$$

**Second Moment:** The variance however, is different for the different expectations formed$^{12}$.

$$
\sigma^2(x_{1|a}) = \begin{cases} 
\text{var} \left( \frac{x^T}{2} + \frac{x^T}{2} - \frac{\xi}{2} \right) = \frac{\sigma^2}{2} & \text{for } \bar{a} : x^T \\
\text{var} \left( \frac{x^T}{2} + \frac{x^T}{2} - \frac{\xi}{2} \right) = \frac{\sigma^2}{2} + \frac{\alpha}{(2\alpha + \beta)} & \text{for } \bar{a} : x^e = x^T - \xi + \frac{2\alpha}{2\alpha + \beta}
\end{cases}
$$

It is straightforward to see that the variance of inflation is smaller if expectations are tied to the target $x^T$. We can now calculate the probabilities of success given

$^{12}$Demertzis and Hughes Hallett (2007) observe a very similar result (theoretically as well as empirically), whereby greater degrees of transparency (and in this context inflation targeting is a more transparent regime) do not affect the level of inflation, but do affect its variability.
a certain distribution for the shocks and assuming that inflation is normally distributed, i.e. $x_t \sim N \left[ x^T, \sigma^2(x_t|\alpha) \right]$. The probability of success when $\alpha = x^T$ (i.e. $Pr(s|c)$) is as follows:

$$Pr \left( x^T - \rho \leq x_t \leq x^T + \rho \right) = \Phi \left( \frac{x^T - \rho - x^T}{\sigma(x_t|x^T)} \leq z_t \leq \frac{x^T + \rho - x^T}{\sigma(x_t|x^T)} \right)$$

$$= \Phi \left( \frac{-\rho}{\frac{\sigma^2}{2}} \leq z_t \leq \frac{\rho}{\frac{\sigma^2}{2}} \right) \quad (15)$$

Naturally the probability of success when $\alpha = x^e$ (i.e. $Pr(s|\bar{c})$) is smaller (as the variance is larger):

$$Pr \left( x^T - \rho \leq x_t < x^T + \rho \right) = Pr \left( \frac{x^T - \rho - x^T}{\sigma(x_t|x^e)} \leq z_t \leq \frac{x^T + \rho - x^T}{\sigma(x_t|x^e)} \right)$$

$$= \Phi \left( \frac{-\rho}{\sqrt{\frac{\sigma^2}{2} + \frac{\alpha}{(2\alpha + \beta)}}} \leq z_t \leq \frac{\rho}{\sqrt{\frac{\sigma^2}{2} + \frac{\alpha}{(2\alpha + \beta)}}} \right) \quad (16)$$

### 4.3 A Numerical Example

Assume the following values for relevant parameters including a level for initial credibility ($v_0$):

$$x^T = 2 \quad \rho = 0.5 \quad \sigma^2_x = 0.25 \quad v_0 = 0.5 \quad \alpha = 4 \quad (\sigma^2_\eta = 0.25) \quad \beta = 4 \quad (\sigma^2_\epsilon = 0.25)$$

The variance of private and public information is equal to that of the shocks, and the Central Bank is successful if $(1.5 \leq x \leq 2.5)$ in each period with the midpoint being the target announced. Moreover, we assume that the distribution of inflation is $x_t \sim N \left[ 2, \sigma^2(x_t|\alpha) \right]$, so that we can apply the standard normal tables for updating $v$. The Central Bank is assumed to start with credibility equal to 50 per cent. The joint probability density function for the two variables $C$ and $S$ is shown in table 2:

<table>
<thead>
<tr>
<th>$C :$</th>
<th>$Pr(c \cap s)$</th>
<th>$Pr(c \cap \bar{s})$</th>
<th>$Pr(c) = v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>$Pr(c \cap s)$</td>
<td>$Pr(c \cap \bar{s})$</td>
<td>$Pr(c) = v$</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>$Pr(\bar{c} \cap s)$</td>
<td>$Pr(\bar{c} \cap \bar{s})$</td>
<td>$Pr(\bar{c}) = 1 - v$</td>
</tr>
</tbody>
</table>

| $S :$ | $Pr(s)$ | $Pr(\bar{s})$ | 1 |

Table 2: Credibility and Success
We these probabilities we can then calculate the probability of Central Bank monetary policy being successful. From (15), (16) and the standard normal tables we calculate the following:

\[ \Pr(s | c) = \Pr(-0.71 \leq z \leq 0.71) = 0.5205 \]
\[ \Pr(s | \bar{c}) = \Pr(-0.49 \leq z \leq 0.49) = 0.3781 \]

Accordingly, we can fill in the rest of the table:

<table>
<thead>
<tr>
<th>IT</th>
<th>$s$</th>
<th>$\bar{s}$</th>
<th>$C:\rho = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.26</td>
<td>0.24</td>
<td>0.5</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>0.19</td>
<td>0.31</td>
<td>0.5</td>
</tr>
<tr>
<td>$S:\rho = 0.45$</td>
<td>0.55</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Based now on Bayesian updating, we calculate the level of credibility for the next period, for a successful Central Bank:

\[
v_1 = \Pr(c | s) = \frac{\Pr(s | c) \Pr(c)}{\Pr(s)} = \frac{0.5205 \times 0.5}{0.45} = 0.57
\]

We observe therefore, that the private sector will increase the level of confidence that it has about the Bank, from 0.5 to 0.57. If however, the Central Bank is unsuccessful, then it will be penalized in the sense that $v$ will reduce:

\[
v_1 = \Pr(c | \bar{s}) = \frac{\Pr(\bar{s} | c) \Pr(c)}{\Pr(\bar{s})} = \frac{0.3781 \times 0.5}{0.55} = 0.44
\]

Credibility reduces from 0.5 to 0.44.

## 5 Communication and Success

We can now show how the announcement of an inflation target can help the Central Bank stay within the range of values that constitute a successful monetary policy. We will show this first through illustrative numerical simulations for 20 periods and second through Monte Carlo simulations to summarize our observations. We assume the same parameterization in the simulations:

\[ x^T = 2, \quad \rho = 0.5, \quad \beta = 4 \quad \text{(or } \sigma_e^2 = 0.25) \]
Two parameters are now subject to uncertainty and these are supply shocks $\xi$ and public information $\eta$, which are drawn from their distributions in every period. Both parameters have a zero mean and respective variances $\left(\sigma^2_{\xi}, \sigma^2_{\eta}\right)$ equal to 0.25. Private information is also subject to noise, but it does not affect the decision on average from period to period, and is therefore, fixed in this exercise. In the absence of shocks, if expectations are equal to the target, then the CB achieves its inflation objective and welfare is maximized. If on the other hand, expectations are equal to the MS action, then inflation will not be equal to the target. Irrespective of how expectations are formed however, the presence of shocks can seriously hamper the CB’s ability to be successful. This inevitably affects the way private agents update credibility. We demonstrate this next.

Following the parameterization assumed, the condition (14), for inflation expectations to be equal to the target is 0.62. We assume a starting value for $v = 0.6(< 0.62)$, implying that in the first period, $t = 1$, expectations will follow the MS rule. Random numbers are drawn for each period for both the supply shock $\xi$ as well as the shock to public information $\eta$, for 20 consecutive periods. Figure 5 demonstrates how successful the CB is under IT ($x_t$ and $x_e$) and non-IT ($x_t^{MS}$ and $x_e^{MS}$), and how credibility evolves based on the success of the two regimes ($v_t$ and $v_t^{MS}$, respectively). Note that credibility in the MS case does not feed back into the decision of the individual and is therefore a counter-factual. By contrast that for the IT does feed back into the game. The difference between the two paths for credibility demonstrates first and foremost, the importance of allowing for credibility and success to enter in a loop. However, it also accounts for the greater success rate of IT and illustrates the importance of becoming credible, or in other words of having a communication strategy that is capable of tying down expectations.

Figure 5 shows that despite the lack of credibility in the first period, the shocks drawn do not prevent the CB from being successful in maintaining inflation within the specified bands. This success helps reward the CB in the next period by increasing $v$, helping it go over the 0.62 mark. For the IT regime this implies that expectations are now tied down to the target. In turn, this helps control inflation in the period after that and given the size of the shocks that occur, still hold inflation within the bands. The same is true for the MS regime even though expectations do not equal the target. However, after the sixth period, the shocks occurring are large enough to throw inflation under the MS regime outside the bands and therefore induce a reduction in credibility in the subsequent period. By contrast, the fact that credibility was sufficiently high for expectations to be equal to target implied that the same shock was easier to handle with IT and therefore prevented inflation from coming out of the bands. This process reinforces itself in all the periods that follow and accounts for the divergence in credibility observed, as well as for the fact that the success rate for IT is 95%, whereas that for MS is 50%.
However, it is also possible that for the same parameterization, the shocks drawn are unfavorable enough for the coordinating feature of IT to never come into operation. This is shown in Figure 6 where the two regimes overlap with each other.

A third possibility is that illustrated in Figure 7. In this example, acquiring credibility through inflation targeting is not a permanent characteristic of the regime: if a series of negative shocks hit the economy, the credibility gained can also be lost. What inflation targeting does however achieve is that it makes the system more robust to unfavorable circumstances. In our example, although credibility starts decaying after period ten, the focal point characteristic lasts for an extra five periods before expectations revert to the MS formation.

The question is then how often can IT improve the success rate, and under which conditions are these improvements the greatest? Simulations will demonstrate the general results implied for a variety of shocks, based on 1000 repetitions.

5.1 Simulations

We generalize some of these results by performing Monte-Carlo simulations. Figures 5-7 show a block of 20 consecutive simulations. Given the formula for updating credibility applied, twenty periods are sufficient for credibility to converge to either one or zero. In evaluating the results from these Monte Carlo simulations, it is important therefore to rely on multiples of 20-period blocks. We will run 1000 (larger numbers of draws do not change the results) of 20-period blocks rather than the alternative of 20000 consecutive simulations.
Parameterization will be identical to what is shown above, unless otherwise stated. We investigate two issues: first, how often the announcement of a IT improves credibility; and second, what this in turn implies for the success of monetary policy.

5.1.1 Does Announcing a Target Always Improve Credibility?

We first ask whether the announcement of a target always leads to an increase in credibility. In what follows we show the percentage of times for which credibility at the 20th period was higher than credibility assumed at the start. This does not account for oscillations in credibility during the 20-period block but provides an indication of what level credibility converges to. We do this for two different levels of initial credibility and for a variety of different assumptions for the two shocks drawn.

Assume relative low initial credibility: $v(0) = 0.5$.

Starting with low initial credibility and values for the shocks $\sigma_\xi^2 = 0.25, \sigma_\eta^2 = 0.25$, IT will lead 20 percent of the times to an increase in final credibility levels. This improvement occurs 49 percent of the times when public information precision declines.

$$
\begin{array}{c|c|c}
\sigma_\xi^2 &=& 0.25 \\
\sigma_\eta^2 &=& 0.25 \\
\% &=& 20 \\
\sigma_\xi^2 &=& 0.25 \\
\sigma_\eta^2 &=& 0.5 \\
\% &=& 49
\end{array}
$$
However, the presence of unstable economic conditions (i.e. relatively high supply shocks), irrespective of the quality of public information, prevents IT from improving credibility. A barely 0.02 percent of the times will credibility have increased (and 0.05 per cent when public information is more unclear).

\[
\begin{align*}
\sigma^2_{\xi} &= 0.5, \quad \sigma^2_{\eta} = 0.25 & 0.02 \\
\sigma^2_{\xi} &= 0.5, \quad \sigma^2_{\eta} = 0.5 & 0.05
\end{align*}
\]

Assume relative high initial credibility: \(v(0) = 0.7\).

Starting now from relatively high levels of credibility can make a difference to the extent of improvement brought by the application of IT. Under normal shocks, inflation targeting will cause an improvement to the initial 0.7 level of credibility, 69 percent of the times. When public information is imprecise, the "focal point" argument is almost always (83 percent of the time) helping the Central Bank improve its credibility.

\[
\begin{align*}
\sigma^2_{\xi} &= 0.25, \quad \sigma^2_{\eta} = 0.25 & 74 \\
\sigma^2_{\xi} &= 0.25, \quad \sigma^2_{\eta} = 0.5 & 83
\end{align*}
\]

But starting from high credibility does not guarantee convergence to the target and any gain in credibility, if the economy is subjected to significant supply shocks. Again it is the size of the shocks that will determine accumulation.
versus decumulation of credibility. Inflation targets work as good coordinating mechanisms only when supply shocks are low, as shown below:

\[
\sigma_c^2 = 0.5, \quad \sigma_e^2 = 0.25 \quad 0.15 \\
\sigma_c^2 = 0.5, \quad \sigma_e^2 = 0.5 \quad 0.29
\]

5.1.2 Monetary Policy Regime and Monetary Policy Success

We now evaluate the effectiveness of the two regimes in terms of the rates of success for a variety of model parameterizations. Table 4 presents the success rates for the two regimes, IT and MS, or in other words the number of times that following the two shocks and the CB’s reaction, inflation ends up being between 1.5 and 2.5 percent. We show this under different parameterizations for the shocks and the initial level of credibility assumed.

| Table 4: IT and Successful Monetary Policy |
|-------------------------------|----------------|
|                                | Success IT % | Success MS % |
| Normal: \( \sigma_c^2 = 0.25, \sigma_e^2 = 0.25 \) | 88 66 | 57 50 |
| \( v_0 = 0.7 \) | 71 66 |
| \( v_0 = 0.5 \) | 51 50 |
| Large Supply Shock: \( \sigma_c^2 = 0.5, \sigma_e^2 = 0.25 \) | 99 84 | 99 84 |
| \( v_0 = 0.7 \) | 91 65 |
| \( v_0 = 0.5 \) | 80 65 |
| Small Supply Shock: \( \sigma_c^2 = 0.1, \sigma_e^2 = 0.25 \) | 75 66 | 67 66 |
| \( v_0 = 0.7 \) | 75 66 |
| \( v_0 = 0.5 \) | 67 66 |
| Large Information Shock: \( \sigma_c^2 = 0.25, \sigma_e^2 = 0.5 \) | 88 66 | 57 50 |
| \( v_0 = 0.7 \) | 71 66 |
| \( v_0 = 0.5 \) | 51 50 |
| Small Information Shock: \( \sigma_c^2 = 0.25, \sigma_e^2 = 0.1 \) | 99 84 | 99 84 |
| \( v_0 = 0.7 \) | 91 65 |
| \( v_0 = 0.5 \) | 80 65 |

Our first observation from Table 4 is that initial levels of credibility matter both in terms of the success of IT itself but also in terms of bringing big improvements by comparison to MS. With reference to (relatively) large supply shocks, the two regimes are almost identical and IT will not be able to help coordinate expectations (except in very extreme cases). When shocks are small on the other hand, although IT helps in that respect, MS is also capable of providing successful results (gains of 15% independently of initial credibility assumed\(^{13}\)). It is when shock are mediocre (“normal”) that IT can be beneficial, especially

\(^{13}\)This is because with this parametrization the credibility threshold for the target to become focal point is very low (0.34), i.e. even if the target is not credible there is no implied cost in focusing on it, as the shocks are very low on average.
when credibility is relatively high to start with (22% gains). When it comes to the precision of public information, IT is not generally advantageous if information is generally good (small information shocks). However, when all other public information is relatively poor, then the provision of a clear monetary objective can improve the success rate by 26 percent. Here the level of initial credibility matters such that the greater the credibility to begin with, the bigger the increase in IT monetary policy success by comparison to MS.

6 Optimal Inflation Targeting Bands

We discuss next how the choice of band-width affects the trade-off between precision and success\textsuperscript{14}. One the one hand, while a relative width band increases naturally the probability of success, at the same time the provision of a midpoint target is less effective in terms of acting as a focal point. So the inflation target looses its meaning as the bands increase. On the other hand, while a target and its known range help solve the coordination term in the agents’ objective function, it is also true that the sheer provision of a clear criterion also exposes failure. Calculating the relevant conditional probabilities illustrates this. For the parameterization assumed in section 4.3, the following hold:

\[
\begin{align*}
\Pr(c/s) &= 0.58 & \Pr(c/\bar{s}) &= 0.44 \\
\Pr(\bar{c}/s) &= 0.42 & \Pr(\bar{c}/\bar{s}) &= 0.56
\end{align*}
\]

In other words, if success is observed, it is 58% likely that this was due to credible policies. If failure is observed then the probability that this is due to lack of credibility is 56%. Now lets see how these two probabilities change as the band-width increases. We assume now that $\rho = 2$. The joint probability distribution is shown in Table 5:

<table>
<thead>
<tr>
<th>IT</th>
<th>$\rho = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s$</td>
</tr>
<tr>
<td>$c$</td>
<td>0.49</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>0.97</td>
</tr>
</tbody>
</table>

The first observation is that the probability of success is 0.97 which is natural as the bands are now wider. However, while it is very difficult to assign the

\textsuperscript{14}See Mishkin and Westelius (2006) for an attempt to examine how the band-width deals with time-inconsistency problems. In their attempt the authors introduce an explicit cost in the CB’s utility function for landing outside the bands. In our case this is already incorporated in the mechanism for updating credibility, as we show next. However, their approach has the attractive feature that costs from deviating from the target are a function of the distance from the bands, although graphs in section 2 illustrate that in terms of pinning down expectations that does not seem to matter very much.
cause of success when it is observed, (0.51 vs 0.49), once failure is observed (and it will be observed very rarely, \( Pr = 0.03 \)), then it is almost certain (0.91) that this failure is the result of lack of credibility. This is intuitive, as failing to achieve inflation within a relatively wide band indicates lack of ability on the part of the Central Bank. So, in this respect, the announcement of the target has worked against the Central Bank, as it provides a very obvious criteria by which to identify its failure.

\[
\begin{align*}
Pr(c/s) &= 0.51 & Pr(c/\bar{s}) &= 0.09 \\
Pr(\bar{c}/s) &= 0.49 & Pr(\bar{c}/\bar{s}) &= 0.91
\end{align*}
\]

The width of the bands works also in the opposite direction. We assume next very narrow bands, i.e.: \( \rho = 0.1 \).

<table>
<thead>
<tr>
<th>IT</th>
<th>( \rho = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( s )</td>
</tr>
<tr>
<td>( c )</td>
<td>0.06</td>
</tr>
<tr>
<td>( \bar{c} )</td>
<td>0.04</td>
</tr>
<tr>
<td>( S : )</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Success is now rather unlikely (0.10 from Table 6), but once you do observe it, it is more likely that it is the CB’s credibility that has achieved it (0.59). By contrast, failure is a lot more likely (0.90), but the source of this failure is a lot less easy to discern (0.49 versus 0.51).

\[
\begin{align*}
Pr(c/s) &= 0.59 & Pr(c/\bar{s}) &= 0.49 \\
Pr(\bar{c}/s) &= 0.41 & Pr(\bar{c}/\bar{s}) &= 0.51
\end{align*}
\]

But this then points to the fact that there is an optimal band-width for the Central Bank which encapsulates the following trade-off between the enhancing the probability of being successful and the provision of a clear and precise signal. We apply numerical methods to identify next what the optimal band-width is for a number of different parameterizations in the shocks.

### 6.1 Grid-Search for the Optimal Band-Width

We plot the radar graphs for the social loss contours (captured here by 5, the representative individual) based on different values for credibility, supply shocks and information shocks. The radius \( \rho \) around the target is depicted along the circumference of the radar graphs and losses are minimized at the centre of the circle. Losses are therefore increasing as the ray of each contour increases. We start by addressing how the optimal band-width is affected by the initial level of credibility.
Figure 8: Welfare Effect of Band Width (Variation in Initial Credibility)

Figure 8 displays losses for four different values of initial credibility. We see that as the level of initial credibility increases, losses become smaller. However, for any given level of credibility, losses are minimized at a certain band-width shown by the shortest ray from the centre to the respective contour. These are also depicted in the graph. For example, for \( v_0 = 0.5 \) the optimal band width is equal to 0.45. As the level of initial credibility increases the optimal band-width also increases (0.7 and 1 for \( v_0 = 0.6 \) and \( v_0 = 0.7 \) respectively). This points to the fact that Central Banks that are not credible need to be tighter in formulating their ambitions.

Figure 9 shows next how the band-width changes for a variety of supply shocks. For small supply shocks, losses are very small as well. As the size of the shocks increases then the Central Bank can afford to increase the band-width a little from 0.5 to 0.6. However, as Figure 9 shows the losses are fairly constant across the band-width.

This is not the case when considering different shocks to public information, (information of different quality) which could benefit from the identification of the optimal band-width. Figure 10 plots losses for three different information shocks. The width of the band plays a role in terms of the ranking of losses, such that for very high information shocks \( (\sigma^2_0 = 0.7) \), relatively narrow bands generate the smallest level of losses, whereas relatively wide bands generate the highest losses. When looking however at the just the optimal levels, we see that when information is very imprecise, then any signal can benefit welfare.
by helping to distill information and coordinate the game. Similarly, when information is by itself very precise, then for a signal to be helpful it has to be very precise (narrow band) to induce agents to switch forming expectations from MS to IT.

Between the three cases (Figures 8-10), we see that differences in credibility require big differences in the bands around the target adopted. The same holds for differences in the quality of public information available although to a smaller degree. Differences in the shocks on the other hand, do not require big differences in the width of the bands.

6.2 Optimal Band-Width Simulations

The following two tables show the optimal band for different parametrizations and the possible gain in applying the optimal band-width in relation to worst outcome.

Table 7 shows the results of our grid search for the optimal band-width when public information noise is assumed to be constant, i.e. $\sigma^2_{\eta} = 0.25$. The first two rows show the specific parametrization for the standard error of the supply shock, and initial credibility. For each parametrization $\rho_{\text{Min}}$ gives the optimal band width (the band width that minimizes social welfare losses, presented in the next column), while $\rho_{\text{Max}}$ gives the losses for the worst case. The last row shows the difference in losses between best and worst case, which gives an
indication of the curvature of the loss function and the ‘risk’ implied by getting the band wrong.

Table 7: Supply Shock Variability (Constant $\sigma_\eta^2 = 0.25$)

<table>
<thead>
<tr>
<th>$\sigma_\xi^2$</th>
<th>$\sigma_\xi^2$</th>
<th>$\sigma_\xi^2$</th>
<th>$\sigma_\xi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$ = 0.7</td>
<td>$\nu$ = 0.5</td>
<td>$\nu$ = 0.7</td>
<td>$\nu$ = 0.5</td>
</tr>
<tr>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>0.75</td>
<td>1.25</td>
<td>0.65</td>
<td>1</td>
</tr>
<tr>
<td>8.6</td>
<td>10.6</td>
<td>10.4</td>
<td>1.3</td>
</tr>
<tr>
<td>0.65</td>
<td>1</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>10.4</td>
<td>1.25</td>
<td>10.6</td>
<td>2.6</td>
</tr>
<tr>
<td>0.7</td>
<td>0.2</td>
<td>0.25</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.2</td>
<td>5.5</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>4.8</td>
<td>5.5</td>
<td>2.6</td>
<td>0.7</td>
</tr>
</tbody>
</table>

As noted earlier supply shock dominance renders inflation targeting superfluous. The only situation in which inflation targeting provides a significant improvement in macroeconomic performance is the one showed in the last column where low supply shocks is coupled with low initial credibility. In this case a very narrow band might provide a quick way to earn credibility, by exploiting favorable conditions to their full.

In table 8 we look instead at scenarios that are subjected to different information shocks. As previously noted, low credibility (meaning credibility lower than the threshold) requires setting a narrow target band, in order to ensure that success, when observed, is the result of the central bank’s competence and not of luck. This effect is particularly magnified if the quality of public information
is particularly poor ($\sigma^2_\eta = 0.5, v = 0.4$). On the other hand, we also see that when a central bank is relatively credible ($v = 0.7$), then tightening the band (from 1% width to 0.25%) can forego potentially significant gains in welfare ($\sigma^2_\eta = 0.25, v = 0.7$).

Table 8: Public Information Shock Variability (Constant $\sigma^2_\xi = 0.25$)

<table>
<thead>
<tr>
<th>$\sigma^2_\eta$</th>
<th>$\sigma^2_\eta$</th>
<th>$\sigma^2_\eta$</th>
<th>$\sigma^2_\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v = 0.7$</td>
<td>$v = 0.4$</td>
<td>$v = 0.7$</td>
<td>$v = 0.4$</td>
</tr>
<tr>
<td>$\rho$ Losses</td>
<td>$\rho$ Losses</td>
<td>$\rho$ Losses</td>
<td>$\rho$ Losses</td>
</tr>
<tr>
<td>Min</td>
<td>1.1</td>
<td>1.9</td>
<td>0.55</td>
</tr>
<tr>
<td>Max</td>
<td>0.45</td>
<td>1.3</td>
<td>0.95</td>
</tr>
<tr>
<td>Diff</td>
<td>0.6</td>
<td>1.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

7 Conclusions

Critics often argue that inflation targeting as a monetary policy regime puts far too high a weight on inflation to the detriment of output and growth. Friedman, (2003) argues that “...the language in which that debate takes place exerts a powerful influence on the substance of what the participants say, and eventually even over what they think”. He then goes on to say that “...a powerful motivation for adopting this framework, at least in some quarters, is the hope that if the explicit discussion of the central bank’s policy is carried out entirely in terms of an optimal inflation trajectory, concerns for real outcomes may somehow atrophy or even disappear from consideration altogether”. Mervyn King (1997) has objected to this argument by arguing that inflation targeters are not synonymous to ‘inflation nutters’. To this, our analysis adds that the strength of the ‘use and meaning of words’ argument notwithstanding, the potential benefits of inflation targeting arise from its ability to tackle information imperfections and not necessarily from the monetary policy choices it implies. We argue that the emphasis is on the communication of certain choices, not the choices themselves. The underlying monetary policy strategy (preferences and objectives) is then not necessarily uniquely identified. Indeed, countries have implemented and also experienced inflation targeting in very different ways.

Our analysis also shows under which conditions inflation targeting can make a difference, but by consequence also when it cannot. It is important therefore to note that no regime seals the Central Bank from external shocks totally. There will be occasions when economic circumstances will just prevent good outcomes from occurring. What good and effective communication can achieve however, is help build up the Central Bank’s ability to withstand unfavorable shocks when they arise. And the provision of a clear signal will be of the greatest value-added when all other information available is unclear, because it then provides a focal point for expectations. Countries for which information is abundant and clear, and for which Central Banks are either credible already, or are faced with small shocks, will see no discernible benefits from dedicating resources to improving their communication.
In fact one needs to examine whether providing a clear signal may even be harmful. One of the implications of very precise communication is that both success as well as failure are clearly defined for the public to see. The private sector observes where current inflation is vis-à-vis the objective announced and therefore rewards or penalizes the Central Bank in terms of credibility. In our analysis, the default monetary policy regime (MS) does not allow for credibility and success to feature in the game. In every period, the Central Bank and private agents decide on their action to the best of their abilities, without reviewing performance and credibility. The expectations formation process is therefore independent of past performance and by comparison, inflation targeting can only improve success. However, one could conceivably compare inflation targeting to other regimes that do allow for this credibility-success loop, but then defined less tightly. We could then compare how these alternative regimes affect a Central Bank's credibility in periods of adverse shocks. Our discussion on how the width of the band affects success is a clear indication that other less clear definitions might prove less harmful (although also less effective in providing a focal point).

We examine the merits of inflation targeting in communicating monetary policy choices. The clear criteria for evaluating outcomes that it entails, as well as the ability to provide focal points, maximize the way the credibility-performance loop is exploited. Coupled with sound policies, our analysis shows that a clear communication strategy can improve monetary policy performance.
A Variable Universe Games

When expectations formation becomes a matching game, individual $i$ is unable to differentiate between the two Nash equilibria in pure form, $E_2 \equiv (x^T, x^T)$ and $E_3 \equiv (x_i, x^e)$. To decide on an action therefore, she needs to compare the expected (rather than actual) dis-utility that each of them delivers; in other words $E[u_i(a_i, \bar{a})]$, where $a_i \in \{x_i, x^T\}$ and $\bar{a} \in \{x^e, x^T\}$. This in turn depends on the probability with which the average action takes one of its own two respective values.

**Definition A1:** Define: $q \equiv \Pr(\bar{a} = x^T)$ and $(1 - q) \equiv \Pr(\bar{a} = x^e)$, $q \in [0, 1]$.

Expected dis-utility for individual $i$ for each of her two actions is then:

\[
\mathbb{E} \{u_i [x_i, (x^e \text{ or } x^T)]\} = (1 - q)u_i (x_i, x^e) + q u_i (x_i, x^T) \tag{17}
\]

\[
\mathbb{E} \{u_i [x^T, (x^e \text{ or } x^T)]\} = (1 - q)u_i (x^T, x^e) + q u_i (x^T, x^T) \tag{18}
\]

The difficulty with this however, is that probability $q$ is unknown to individual $i$ and she is thus unable to evaluate which action to take based on (17) and (18). She needs therefore, to identify an alternative framework that provides her with sufficient information that she can then base her choice on. We will argue that rather than evaluate $q$, it will suffice for individual $i$ to simply know how the target is perceived (and not necessarily applied) on average. Otherwise put, it will suffice for the individual to know the proportion of people that consider the target credible. We believe that in the context of monetary policy this information is publicly available and easier for the individual to deduce. Based on that, we will show that she can then identify under which conditions following the target provided by the Central Bank is to her advantage.

As $q$ is not known to the individual, we need to identify an other prior that captures information that is more readily available to the individual. The regime of inflation targeting implies the following\(^{15}\):

1. First, the option of following the target is available to everyone. We believe this to be an adequate representation of the fact that inflation targets are widely announced and are therefore, seen and understood by everyone. In that respect, it is reasonable to assume that any random individual would understand that the options shown in Table 1 under IT are now available to them.

\(^{15}\)Note that our description of what the provision of an explicit quantitative target implies draws heavily from the common notion that IT constitutes "...a simple yardstick by which to judge policy. Given lexicographic preferences over inflation and other goals, an inflation target range, and a fixed horizon, inflation targeting becomes very easy to monitor", Faust and Henderson (2004), p127.
2. Second, not necessarily everyone believes that the target is credible. In other words, not everyone believes that the CB can actually achieve the target, or even that the notion of "price stability" and the value \( x^T \) announced are necessarily equivalent. This is in our view an important reflection of the fact that revealing a quantitative objective is not automatically sufficient for attaining it.

3. Finally, while credibility is by no means certain, there is nevertheless a common perception as to how credible such a target might be. In other words, everyone knows and understands how the target is perceived by others. More specifically, one could interpret this likelihood as the proportion of people, say \( v \), who consider the target credible. We assume that \( v \) is common knowledge and sufficient for the individual to base her choice on.

Based on Bacharach’s (1993) paper, we can rely on this likelihood \( v \) to evaluate individual \( i \)'s expected dis-utility \( \mathbb{E}[u_i(a_i, \hat{a})] \) of following action \( a_i \) for a given collective action \( \hat{a} \). This has the advantage that the priors used are now known to her. However, as \( v \) is not the actual probability with which the collective action is equal to the target (\( v \neq q \)), we need to proceed in two steps\(^{16}\) in order to provide a comprehensive framework that accounts for all possible outcomes.

**Case 1:** First, we consider the case when the collective choice is always the default MS action. The target is therefore never applied, irrespective of whether it is deemed credible. This may happen either because the average perception is that the CB’s target is not credible, or because the average individual does not believe that about others. The default MS action is then applied on average and inflation expectations equal \( x^e \). Are there then circumstances in which individual \( i \) is still better-off following the target? Her expected dis-utility of following either of her two options is then:

\[
\mathbb{E}[u_i(x_i, x^e)] = (1 - v)u_i(x_i, x^e) + vu_i(x_i, x^e) \\
= u_i(x_i, x^e)
\]

\[
\mathbb{E}[u_i(x^T, x^e)] = (1 - v)u_i(x^T, x^e) + vu_i(x^T, x^e) \\
= u_i(x^T, x^e)
\]

From Table 1, these equal respectively

\[
\mathbb{E}[u_i(x_i, x^c)] = \frac{\alpha + \beta}{(2\alpha + \beta)^2}
\]

\[
\mathbb{E}[u_i(x^T, x^c)] = \sigma^2 + \frac{\alpha}{(2\alpha + \beta)^2}
\]

\(^{16}\)Actually there are three but the third is trivial as we show in footnote 14 further down.
It follows that

\[
E[u_i(x^T, x^e)] < E[u_i(x_i, x^e)] \quad i f f \\
\sigma^2 < \frac{\beta}{(2\alpha + \beta)^2} \quad (19)
\]

Expression (19) illustrates the condition for which individual \( i \) would effectively ignore the fact that everyone else applies the MS default action, and still follow the target, even though she is aware she would be the only one. But this is only rational if following the target constitutes her dominant strategy. It is therefore, no surprise that (19) is identical to the condition shown earlier on under dominance, i.e. (13).

**Case 2:** Second, we then relax the stringency of the first assumption and assume now that the proportion of people who believe that the target is credible, actually apply it. When should then individual \( i \) choose the target? This is a somewhat less restrictive case than the one above and expected dis-utility for player \( i \) of pursuing either of her two options is now,

\[
E\{u_i [x_i, (x^e \text{ or } x^T)]\} = (1 - v)u_i (x_i, x^e) + vu_i (x_i, x^T), \\
E\{u_i [x^T, (x^e \text{ or } x^T)]\} = (1 - v)u_i (x^T, x^e) + vu_i (x^T, x^T)
\]

and from Table 1,

\[
E\{u_i [x_i, (x^e \text{ or } x^T)]\} = (1 - v) \frac{\alpha + \beta}{(2\alpha + \beta)^2} + v \left[ \frac{1}{4}\sigma^2 + \frac{4\alpha + \beta}{(2\alpha + \beta)^2} \right], \\
E\{u_i [x^T, (x^e \text{ or } x^T)]\} = (1 - v) \left[ \sigma^2 + \frac{\alpha}{(2\alpha + \beta)^2} \right] + v \frac{1}{4}\sigma^2
\]

It follows that,

\[
E\{u_i [x^T, (x^e \text{ or } x^T)]\} < E\{u_i [x_i, (x^e \text{ or } x^T)]\}, \quad i f f \\
\sigma^2 \leq \frac{\beta + v4\alpha}{(1 - v)(2\alpha + \beta)^2} \quad (20)
\]

Again, the size of the shock needs to be below a given ratio before the individual applies the target. However, as the condition is less restrictive than before, (since \( \frac{\beta}{(2\alpha + \beta)^2} < \frac{\beta + v4\alpha}{(1 - v)(2\alpha + \beta)^2} \)), it is also the sufficient condition for individual \( i \) to follow that target. Interestingly, we can re-write this condition in terms of \( v \)

\[
v \geq \frac{(2\alpha + \beta)^2 \sigma^2 - \beta}{4\alpha + (2\alpha + \beta)^2 \sigma^2} \quad (21)
\]
This now shows that the sufficient condition for individual $i$ to follow the target is when a suitably large proportion of the public believe the target to be credible.

**Case 3**: The average action is now assumed to always be the target. One could envisage a situation (admittedly unlikely) in which everyone thinks for themselves that the target is not credible but choose nevertheless to follow it, because they believe they are alone in thinking that. It is trivial then to show that following the target is individual $i$’s preferred strategy.

$$E \{ u_i \left[ x_i, x^T \right] \} = \frac{1}{4} \sigma^2 + \frac{4\alpha + \beta}{(2\alpha + \beta)^2}$$

$$E \{ u_i \left[ x^T, x^T \right] \} = \frac{1}{4} \sigma^2$$

$$E \{ u_i \left[ x^T, x^T \right] \} < E \{ u_i \left[ x_i, x^T \right] \}$$

**B DATA**

Data compiled in section 2 is based primarily on Mishkin and Schmidt-Hebbel (2001), Levin, Natalucci and Piger (2004) and national Central Bank websites. Series for inflation (quarterly, y-o-y, 1990q1-2007q2) are taken from DATAS-TREAM.

**Comments**

**Chile**: Inflation Expectations - one-year ahead
→ www.bcentral.cl

**Euro Area**: From 1999-mid2003 - EU two-year ahead inflation forecasts

**South Africa**: Bureau for Economic Research - CPI 2-years ahead

**Poland**: Official Inflation Targeting from 1998 only; prior to 1998, eclectic monetary policy.
Inflation Expectations: IPSOS SURVEY DATA
→ mean of the expected rate of inflation over next 12 months
→ www.nbp.pl

**Czech Republic**: Inflation Expectations -12 months ahead
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As from January 2010, the point target will become 2%
→ www.cnb.cz
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