Do trend traders tame chaos?

Feedback provides stability

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This paper investigates the role of feedback in financial markets. We start by reviewing the historic understanding of feedback in the field of economics. Then we focus on feedback effects in modern heterogeneous agent models which include trend traders. Studies of heterogeneous agent models typically describe the models' behavior in the very long run. We propose an explanation for the phenomenon that for certain parameter values computer simulations of these models regularly converge. Using insights from the theory of time-delayed feedback control in physics, we show that the way trend traders form their expectations serves as a feedback rule stabilizing the system. Extrapolating trends from the past then tames the chaos in the model.

Introduction

We investigate the role of feedback in the dynamics of a financial market. We concentrate on the foreign exchange market as captured in the heterogeneous agent model of De Grauwe et al (2005). This model includes fundamentalists and trend traders. Trend traders speculate on the continuity of past developments. Such trend traders are widely analyzed in the literature and are part of nearly every heterogeneous agent model, which makes our findings very generic.

We show how stability properties of a heterogeneous agent model are predetermined by the structure of the model. In particular, we exploit results from chaos theory to explain the fact that most deterministic financial heterogeneous agent models stabilize under certain conditions. We show that the behavior of trend traders contributes to the convergence of the system - extrapolating trends from the past tames chaos in the model.

Understanding why the outcome of many models converge after a certain time, would undoubtedly lead to a better understanding of the model itself and its underlying processes.

Feedback in the economy

The concept of a loop is known in social sciences under several names: interdependence, self-reference, knowledge of results, causal loops, recursion, self-reference, circular causality, circular and cumulative causation, feedback, and mutual causality (Richardson 1991: 3). But they all mean the same thing – a feedback loop.

Depicting causality by an arrow, one can graphically depict a feedback loop (Richardson 1991: 5-6) as in Figure 1. Here, A influences B and B influences A, forming a feedback loop. Positive polarity on arrows indicates that variables tend to change in the same direction, and negative polarity implies that variables change in the opposite directions. Polarity of causal links adds up to the polarity of a loop. In the case of Figure 1, the polarity of the feedback loop is negative. Positive feedback loops reinforce growth and negative feedback loops slow down growth.

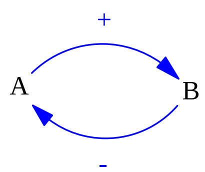

Figure 1: A simple feedback loop

Feedback in general

The concept of feedback was introduced in the social sciences in the 1940s (Richardson 1991: 12). Myrdal was one of the first economists to come close to formulating the concept of feedback in the mid-1940s (Richardson 1991: 81). He introduced the "principle of circular and cumulative causation" (Fujita 2007, Myrdal 1944; Myrdal 1957). Leibenstein (1950) wrote about the bandwagon effect, which is a positive feedback between the uptake and the number of users of a good. Merton (1948) discussed self-fulfilling prophecies, which form a circular structure similar to the one discussed in Myrdal (Myrdal 1944).

While feedback is implicit in works of many economists, according to Richardson (Richardson 1991), no one explicitly formulated the concept until Leijonhufvud. Leijonhufvud (1968) relied on the concept of feedback to interpret Keyne's General Theory. He later interpreted Walras's and Marshall's price adjustment mechanisms by explicitly using the notions of feedback loops (Leijonhufvud 1970). At about the same time, Day (1974) showed the feedback structure that underlies the Solow's growth model.

In general, diminishing returns form a negative feedback and increasing returns constitute a positive feedback (Arthur 1990). While economists have always recognized the importance of diminishing returns, current authors also emphasize that positive feedback is prevalent in the economy (Arthur 1994). Arthur reviews the competition between VHS and Beta VCR formats as an example of an economy that was governed by the presence of the positive feedback. As VHS gained the market share, it became increasingly more difficult for Beta format to compete – an example of a positive network externality that led to a positive feedback effect. Learning also forms a positive feedback – the more you do something, the better you know how to do it (Arthur 1990).

Feedback in financial markets

Several authors have emphasized the importance of feedback in asset markets. A typical point of analysis is the role of trend traders, also known as chartists, speculators, or feedback traders, on the stability of a financial system.

Arguing in the 1950s for flexible exchange rates, Friedman (Friedman 1953: 175) thought that speculation in foreign currencies would likely have a stabilizing effect on exchange rates. Kaldor (1939: 34) thought that speculators destabilize prices within a certain range of price-oscillations but speculation has a stabilizing effect outside of the range. He thought that each market has a specific range. De Long et al. (1990) suggest that feedback trading destabilizes asset prices. Cutler et al. (1990) support De Long's argument.

Feedback is important in the model of Hirshleifer et al. (2006). Hirshleifer et al. present a model in which irrational investors may increase trading activity thereby positively affecting cash inflows, which can lead to abnormal returns for irrational investors. This forms a feedback loop between prices and investments. They show that irrational investors benefit from the feedback effect even though the investors themselves are ignorant of the feedback.

Heemeijer et al. (2007) echo the Merton's (1948) view of the mechanism responsible for self-fulfilling prophecies: "in social systems individual expectations of beliefs can affect the aggregate outcome." Hence, they argue, "a market, like other social environments, may be viewed as an expectations feedback system: past market behaviour determines individual expectations which, in turn, determine current market behaviour and so on" (2007: 2). Heemeijer et al. (2007) distinguish positive and negative feedback between price and price expectation. Speculation on part of buyers plays out through the positive feedback: the expectation of high price by speculators causes a shift in demand, which produces a hike in price, thus confirming the speculators' expectations. The negative feedback, they suggest, would be due to the suppliers in the market: if suppliers expect a rise in prices, they crowd the market, thus shifting the supply curve to the left and causing a drop in actual prices. Heemeijer et al. (2007) design an experiment to test the effects of positive and negative feedbacks on the market outcomes. They find that in the case of the negative feedback, participants slowly coordinate their predictive strategies but the market price quickly converges to the equilibrium value. In the positive feedback case, market prices showed more oscillations and slow convergence to the steady state; the participants, however, coordinated their forecasting strategies very fast.

Owhadi (Owhadi 2004) confirms the importance of feedback for stability by showing that an asset market with very simple behavioral rules that do not include price feedback can lead to a market collapse if wealth concentration reaches a certain level.

Exchange rate model

In this section, we present the equations which establish the market exchange rate in the model of DeGrauwe et al. (2005). They can be classified into four subsections. First, the investors set their optimal portfolio. We will see that they work with a mean-variance utility scheme. Second, the agents use elementary rules in order to predict future exchange rate movements. The assumption of rational individuals is moderated as investors do not integrate the whole data set but reduce the complexity of decision by choice. Third, the agents review their forecasting formula and fourth, if necessary, they adapt it. This learning process makes part of their bounded rationality.

The optimal portfolio

The model of DeGrauwe et al. is micro-orientated: it is based on heterogeneous agents i in the market, who all have different ways of forming expectations about the future exchange rate. The common underlying calculus of all these investors is the maximization of their utility U. The utility function they optimize can be formally written as follows:

$$U\left(W_{t+1}^{i}\right)=E_{t}^{i}\left[W_{t+1}^{i}\right]-\frac{1}{2}\mu V_{t}^{i}\left[W_{t+1}^{i}\right] \qquad (1)$$

Utility depends on the expectations about the wealth in the following period $E_t^i\left[W_{t+1}^i\right]$ and is negatively influenced by the variance of their wealth portfolio $V_t^i\left[W_{t+1}^i\right]$. This variance term is weighted by μ, a coefficient for the risk aversion of the individuals: the less agents are willing to take risks the more negative they perceive uncertainty. The wealth of the individuals at time t+1 can be specified as:

$$W_{t+1}^i = \left(1+r^*\right)s_{t+1}d_t^i + \left(1+r\right)\left(W_t^i - s_t d_t^i\right) \tag{2}$$

Generally, the assets are divided into a foreign and into a domestic portfolio. Variable d_t^i represents the holdings of the foreign assets at time t. Multiplied with the (certain) foreign interest rate r^* and the exchange rate s_{t+1}, we get the value of the foreign holdings at time t+1 expressed in terms of the domestic currency. The second part of the right-hand side of the equation signifies the domestic assets. It is formed by the wealth W_t^i less the amount of money the agent invested in foreign assets before. The interest earned in the home country is defined by the (certain) domestic interest rate r. Substituting (2) into (1) yields:

$$\begin{aligned} &U\left(W_{t+1}^i\right) = E_t^i\left[\left(1+r^*\right)s_{t+1}d_t^i + \left(1+r\right)\left(W_t^i - s_t d_t^i\right)\right] \\ &-\frac{1}{2}\mu V_t^i\left[\left(1+r^*\right)s_{t+1}d_t^i + \left(1+r\right)\left(W_t^i - s_t d_t^i\right)\right] \end{aligned} \tag{3}$$

Using the standard rules for the transformation of expectation values and variances we can shorten this equation. After maximizing the utility with respect to d_t^i the optimal holding of foreign assets for the investors can be derived as:

$$d_t^i = \frac{\left(1+r^*\right)E_t^i\left[s_{t+1}\right] - \left(1+r\right)s_t}{\mu\left(1+r^*\right)^2 V_t^i\left[s_{t+1}\right]} \ , \tag{4}$$

with $\mu > 0$.

The optimal amount of foreign currency depends on its expected excess return. As the investors are assumed to be risk averse, this term must be corrected for the risks involved. All individuals together establish the market demand D_t for foreign assets, where n_t^i is the number of investors of type i:

$$\sum_{i=1}^{N} n_t^i d_t^i = D_t \tag{5}$$

The market supply Z_t is assumed to be exogenous in this model. In the market equilibrium, demand equals supply. Hence, the market clearing condition is:

$$\begin{aligned} Z_t &= D_t \\ &= \sum_{i=1}^{N} n_t^i d_t^i \end{aligned} \tag{6}$$

We can employ equation (6) and (4) in order to rewrite the market equilibrium in dependence on s_t. Isolating the market exchange rate leads to

$$s_t = \left(\frac{1+r^*}{1+r}\right) \frac{1}{\sum_{i=1}^{N} \frac{\omega_t^i}{\sigma_{i,t}^2}} \left[\sum_{i=1}^{N} \omega_t^i \frac{E_t^i\left(s_{t+1}\right)}{\sigma_{i,t}^2} - \Omega_t Z_t \right] \tag{7}$$

with $\omega_t^i = \frac{n_t^i}{\sum_{i=1}^{N} n_t^i}$ as the portion of each type of agent and $\Omega_t = \frac{\mu}{(1+r)\sum_{i=1}^{N} n_t^i}$. This equation is determined by the fraction of domestic and foreign interest rate as well as by the expectations of the market participants. Owing to the fact that the investors are risk averse, the forecasts are weighted by their respective variance: if an agent expects the exchange rate to rise but has been mistaken several times in the past, he or she will be more careful in making investment choices in the future. Therefore, the influence of these undependable predictions on the market exchange rate is lower than if the forecasting revealed to be correct. It is important to notice that in the design of their model DeGrauwe et al. adhere to the concept by Brock/Hommes (1997), which was originally developed for stock markets. That is why the market exchange rate s_t does no express an exchange ratio but a fictive price.

Two types of investors: fundamentalists and chartists

There are two types of investors, that each use different methods in order to form expectations about the future exchange rate: fundamentalists and chartists. Their forecasting rules will be discussed in the following. It is important to underline that neither fundamentalists nor chartists take into account the whole amount of information available. They know about the intricacy of price building processes and about their incapability to deal with all details. Thus, they reduce their decision problem to two simplified rules. Fundamentalists rely on the value of the fundamental exchange rate s_t^*. This fundamental exchange rate is assumed to be exogenous and to follow a random walk without drift. It can be described as follows:

$$s_t^* = s_{t-1}^* + \varepsilon_t \tag{8}$$

where ε_t are normally distributed shocks. The fundamentalists compare the exchange rate on the market to the fundamental exchange rate. In the model at hand, they only consider the previous period when they are forming their expectations. The deviation of the market exchange rate from the fundamental is weighted by the factor ψ, where $0 < \psi < 1$ in order to prevent an explosive process. Then, fundamentalists establish their expectations by subtracting the weighted variation from the previous exchange rate:

$$E_t^f \left[s_{t+1} \right] = s_{t-1} - \psi \left(s_{t-1} - s_{t-1}^* \right) \tag{9}$$

This means that fundamentalists generally forecast a return of the future exchange rate to its fundamental value. The speed of this reversion is assumed to be determined by the velocity of adjustments on the goods market. Fundamentalists are supposed to know about these adaptation processes and about the value of fundamental variables. By comparing them with the market exchange rate, they take into account information directly. This process corresponds to a negative feedback rule. As we will see in the following, the mean reverting tendency sets a counterpart to the driving up behavior of the chartists.

Note, that the assumption of not fully informed agents has implications on the forecasting rule: the investors do not know the structure of the market. Therefore, they cannot keep track of the current market exchange rate. They only have data of time $t-1$ at disposition. As a result, there is a time gap between the instant the exchange rate is formed on the market and the moment in which it is integrated into the decision rules of the market participants.

Chartists, by contrast, only base their forecasting on the past market exchange rate. In considering past movements of s, they extrapolate past trends and transfer them into the future. Hence, they use information indirectly, i.e. through the exchange rate itself, and establish a positive feedback rule. Since they entirely neglect information concerning the fundamental variables they can be regarded as pure noise traders: Chartists do not rely on real economic data but trust in the message of the trading behavior of the other market participants. Their reliance on "market sentiments", i.e. general trends on the market, gives rise to herding behavior.

The extent to which noise traders extrapolate past patterns into the future depends on the coefficient β, which measures the agents' inclination of using past changes. DeGrauwe et al. define β to be $0 < \beta < 1$, i.e. the future exchange rate will never be completely determined by past data. This assumption avoids an explosive process. It must be further pointed out that in the original model chartists integrate five lags into their forecasting rules (H = former exchange rates, here H = 5), whereas fundamentalists do not consider more than one period of the past. We follow this assumption. These elapsed movements of the exchange rate are multiplied by ρ_h, which are geometrically declining weights:

$$\rho_h = \frac{(1-\rho)\rho^{h-1}}{1-\rho^H}$$

where $1-\rho^H = \sum_{h=1}^{H}(1-\rho)\rho^{h-1}$. Thus, the expectations computed by the chartists can be written as:

$$E_t^c(s_{t+1}) = s_{t-1} + \beta\sum_{h=1}^{H}\rho_h \Delta s_{t-h} \tag{10}$$

Note, that H = 1 and $\rho_h = 1$ are special cases concerning the way of forming expectations: They correspond to investors with static expectations. As presented before, chartists also suffer from a time lag of information of one period. Thus, at time t, they only possess data of the market exchange rate at time $t-1$.

One may think that investors who neglect the whole data set about fundamental values cannot persist on the market. At this point, it is important to underline that there have been various studies on the use and the profitability of technical analysis, which have sustained the hypothesis of its widespread popularity in practice. Taylor and Allen (1992) were one of the first who documented the use of technical extrapolation methods among traders on the foreign exchange market. In their survey, approximately 90% of currency traders questioned in London answered that technical trading rules were an important component of their short-investment strategies. 60% of the interviewees judged charts to be at least as important as fundamentals. This result was corroborated by the inquiry conducted by Cheung et al. Using questionnaires, they systematically analyzed the North American, British and Asian exchange market. To the question "select the single most important factor that determines exchange rate movements" the currency traders responded that intraday and in the medium-run the exchange rate is mainly determined by non-fundamental factors. Obviously, most of the dealers think that irrational factors play a key role in the determination of exchange rates in the short and medium-term. In the long-run they indicate a larger impact of fundamental variables, but even then some irrational influences are suggested.

In the following, we will examine if chartists in our model also shape pricing processes on the foreign currency market. It will be seen that these technical traders indeed perform very well. Moreover, we will observe that the chartists' forecasting rule favors herding behavior and the development of bubbles.

Evaluation of the risk involved in the forecasting rules

As the individuals are risk averse, they have a stake in dependable expectations about the future. Consequently, the agents have to evaluate the risk involved in their forecasting rules. For the sake of simplicity, all of them measure the risk of their portfolio identically. In the model, every individual draws on the adjusted variance $\sigma_{i,t}^2$:

$$\sigma_{i,t}^2 = \left(1+r^*\right)^2 \sum_{k=1}^{K} \theta_k \left[E_{t-k-1}^i\left(s_{t-k}\right) - s_{t-k} \right]^2 \tag{11}$$

for $i = c, f$. This adjusted variance of the agents is mainly established by the weighted average of their squared forecasting errors in the past. In doing so, the variances of the previous periods lose the more influence the more distant they are. These geometrically declining weights are calculated as follows:

$$\theta_k = \frac{\theta\left(1-\theta\right)^{k-1}}{\sum_{k=1}^{K} \theta\left(1-\theta\right)^{k-1}}$$

In analogy to the forecasting rule of the chartists, five time lags are taken into consideration. Note, that this is true for chartists and fundamentalists.

Fluctuations between the two groups

In the model elaborated by DeGrauwe et al. the market participants select one of the two forecasting rules in order to streamline their decision process. At the end of each trading period, chartists and fundamentalists compare ex post the risk adjusted profitability of their rule to the one of the other group. Afterwards, they decide to keep their method or to skip to the more profitable one.

Therewith, they are close to the idea of evolutionary dynamics. Originally, Brock and Hommes applied the approach by making the weights (i.e. the quantitative proportions of the groups) dependent on their relative profitability. Its general mechanism is in tradition of a discrete choice framework, which involves three characteristics of the choice set: First, the alternatives must be mutually exclusive from the decision maker's perspective. Choosing one alternative necessarily excludes any of the other ones. In our model, this is guaranteed by the fact that an agent is either fundamentalist or chartist. Second, the choice set must be exhaustive, meaning all possible alternatives are included: Besides chartists and fundamentalists there are no other types of investors on our foreign currency market. Third, the number of alternatives must be finite.

In their model, Brock/Hommes use past realized net profits π_t^i as the publicly available performance. These profits are defined as the entire earnings on the optimal foreign portfolio. DeGrauwe et al. slightly adapt the method and outline profits as the one-period earnings of investing one monetary unit in the foreign asset:

$$\pi_t^i = \left[s_{t-1}\left(1+r^*\right) - s_{t-2}\left(1+r\right) \right] \operatorname{sgn}\left[\left(1+r^*\right) E_{t-2}^i\left(s_{t-1}\right) - \left(1+r\right) s_{t-2} \right] \tag{12}$$

with

$$\operatorname{sgn}[x] = \begin{cases} 1 & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x = 0 \\ -1 & \text{for} \quad x < 0 \end{cases}$$

The sign function guarantees that the agents only realize a profit if they correctly predict the direction the market exchange rate moves into. For instance, if an investors expects the exchange rate to rise and this forecast is realized indeed, the algebraic sign of the bracket term is positive (sgn[x] = 1). Thus, the profit of this agent equals the observed increase (which must be corrected for the interest differential). If, by contrast, the forecast reveals to be wrong the sign gets negative (sgn[x] = -1) and the investor suffers a loss identical to the change in the exchange rate. Only if the market exchange rate remains at its previous level the agents do not earn or lose money. Then, the relative profitability of the two forecasting rules are defined as a weighted average of past realized profits: $\pi_t^{'c} = \pi_t^c - \mu\sigma_{c,t}^2$ and $\pi_t^{'f} = \pi_t^f - \mu\sigma_{f,t}^2$. These are the net profits corrected for their respective variance. Remember, the agents in the market are unwilling to take risks. That is why the variance is weighted by the risk aversion of the traders. Here, this risk aversion is assumed to be a constant factor, which is identical for everybody.

When the number of agents n_t^i in each group tends towards infinity, the fractions of the groups are determined by the Logit discrete choice model probabilities. The Logit concept is derived under the assumption that the unobserved factors (i.e. the fundamental shocks ε_t in our model) are uncorrelated over alternatives, as well as having the same variance for all of them. Using the Logit transformation rules, the fractions of the two groups can then be expressed in the following way:

$$\omega_t^c = \frac{\exp\left[\gamma\pi_t^{'c}\right]}{\exp\left[\gamma\pi_t^{'c}\right] + \exp\left[\gamma\pi_t^{'f}\right]} \tag{13}$$

$$\omega_t^f = \frac{\exp\left[\gamma\pi_t^{'f}\right]}{\exp\left[\gamma\pi_t^{'c}\right] + \exp\left[\gamma\pi_t^{'f}\right]} \tag{14}$$

As in Logit models the fractions ω_t^c and ω_t^f add up to one, the latter can be rewritten as

$$\omega_t^f = 1 - \omega_t^c \tag{15}$$

A crucial feature of the concept by DeGrauwe et al. is that the investors are boundedly rational. This means that on the one hand they do not bear in mind all data available and use simple mental models as decision rules: they are not completely rational. On the other hand they check the fitness of their elementary forecasting rules ex post and switch if advantageous. Hence, the majority of agents choose the predictor which yields most. For instance, if $\pi_t^{'c} > \pi_t^{'f}$ more investors will follow a technical trading rule; thus, ω_t^c increases.

However, profits are not the only factor which determines the weight of each group. In fact, it is common knowledge that habits are important to human beings. In everyday life we constantly rely on psychological patterns that have been formed in the past. Therefore, to a certain extent, the force of habit can entail sluggishness among agents even if the other group of investors earns more money. The coefficient γ specifies this (un)willingness of the individuals to switch their forecasting rule, with $0 < \gamma < \infty$. A low parameter γ reflects a certain inertia of the investors concerning the way they form their expectations. If γ takes a value of zero this means that nobody ever changes a pattern of prediction that has been chosen once: the fractions of chartists and fundamentalists remain fixed. If the agents, on the contrary, react strongly to the relative profitability of the forecasting rules, γ is high. The special case of limiting γ to infinity leads to the neoclassical deterministic choice model, where in each period all agents choose the optimal predictor. In the following, however, particularly the more interesting constellations of high but finite values for the intensity of choice γ will be considered.

To sum up, the model integrates psychological elements of decision making.
Cognitive psychologists claim that human beings look for patterns and subsequently generalize them in order to structure the real world. This inductive behavior is represented by chartists as well as by fundamentalists. Secondly, the individuals apply these general rules to future special cases (deductive proceeding). Ex post, they receive a feedback about the accuracy of their expectations. If this response is coherent with their forecasting rule, they keep it; if not, they try to reduce the mental discrepancy by switching their forecasting rule.

Feedback in the exchange rate model

Based on mathematical description of the De Grauwe et al. model, the causal structure of the exchange rate model is as in Figure 2. There are 22 loops passing through the market-clearing exchange rate. Some of these loops are positive and some are negative (the signs are not shown in Figure 2).

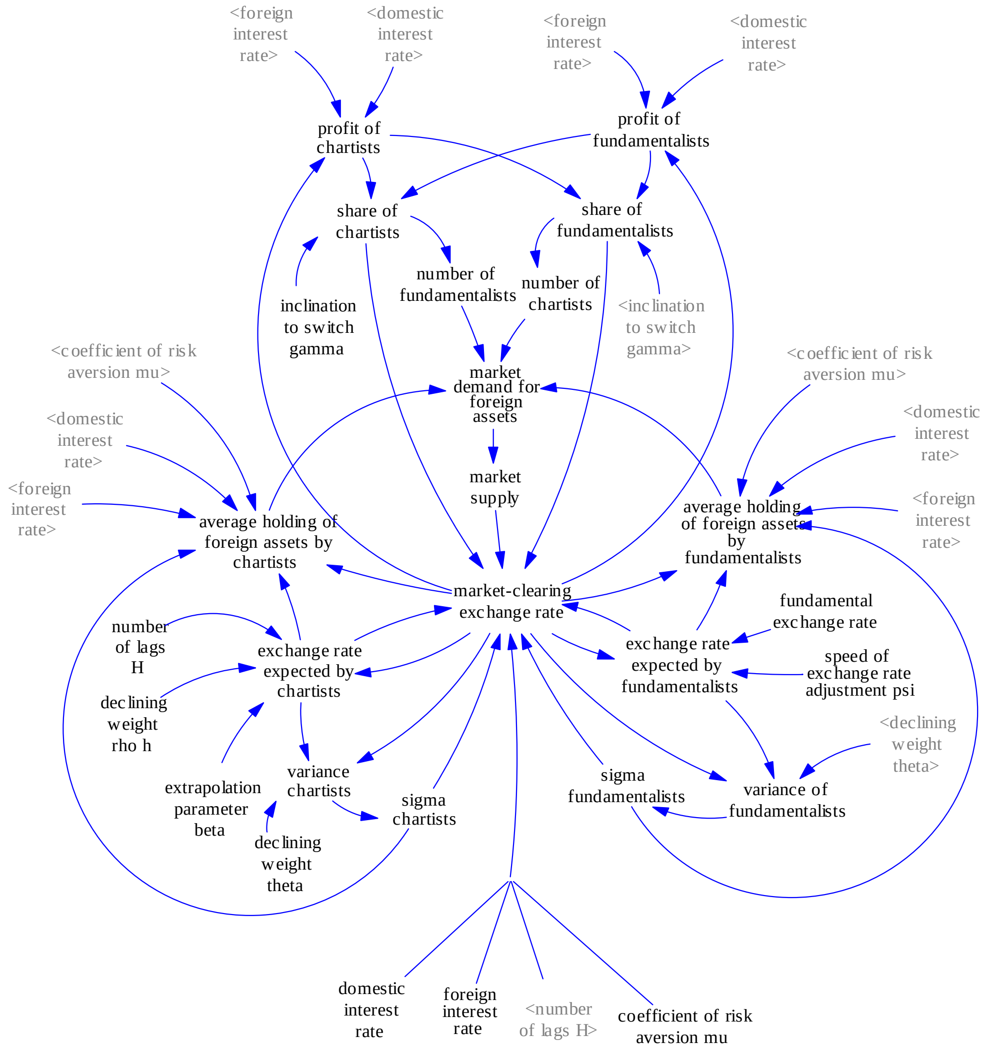

Figure 2: feedback structure of the exchange rate model

In the following section we concentrate on a specific feedback loop from Figure 2, namely, the loop between market-clearing exchange rate and the exchange rate expected by chartists.

The feedback control

The meaning of chaos control

Before turning to the analysis, let us briefly clarify the notion of "chaos". Even though there is no standard definition of "chaos" in the literature, there are several features a "chaotic system" generally exhibits. A chaotic system is nonlinear and its outcome crucially depends on the initial conditions. The behavior of such a model appears to be random, even though the model itself is deterministic in the sense that it does not contain any random parameters. If an outside observer does not know the initial conditions or the laws of motion, exact prediction of the system's long-run trajectory is impossible.

During the last decade, researchers not only described the chaotic behavior of chaotic systems but also studied ways of to stabilize them. This is the domain of the research area of chaos control. The origin of this prominent exploratory field in physics lies in the observation that on the one hand chaotic motion provides a huge number of unstable states and that on the other hand each of these states can be stabilized by extremely small control forces. Insights of physics may help a better understanding of the stability in heterogeneous agent models.

Physicists developed three general concepts in order to control chaotic systems. First, such systems can be controlled by means of a continuous external perturbation, which constantly forces the varying variable back on a smooth path. Second, it is possible to stabilize them by a time-discrete conditioned intervention, where deviations are corrected once in a while. Third method is the so-called time-delayed feedback control. This method may induce a better understanding of heterogeneous agent models which are used to represent financial markets. Therefore, it is presented in more detail in the following section.

Time-delayed feedback control in physics

Among physicists conducting research on chaos control, it had been common knowledge for a long time that time delay does not increase but reduce the efficiency of a control scheme. This is straightforward as intermittent corrections are naturally less precise and thus less powerful than continuous ones. However, contrary to what one would expect intuitively, time delay can also be used to stabilize chaotic movements. Such a time-delayed feedback control was first suggested for physical problems by Pyragas (Pyragas 1992).

Pyragas' approach uses a measurable output signal s_t for stabilization purposes. On the basis of this signal at different times, a feedback variable $\Delta s_{t-\tau}^{feed} = s_t - s_{t-\tau}$ is defined. This feedback is the difference between the current state of the system s_t and its state some τ time units ago, $s_{t-\tau}$. The feedback variable, in effect, collects information about how strongly and in which intervals the system fluctuates. Hence, observing the chaotic movements for a sufficiently long learning time is enough to collect all data that is needed to stabilize the fluctuations. For simplicity, a linear relationship is assumed. There is experimental evidence for this assumption.

Once it is clear how the system evolves, this feedback $\Delta s_{t-\tau}^{feed}$ is linearly amplified by a model specific parameter value σ, which is chosen exogenously. Parameter σ weights the fluctuation pattern in order to generate the actual control force

$$F_t = \sigma \Delta s_{t-\tau}^{feed} = \sigma \left[s_t - s_{t-\tau} \right] \quad (16)$$

Figure 3 illustrates the mechanism of the time-delayed feedback control.

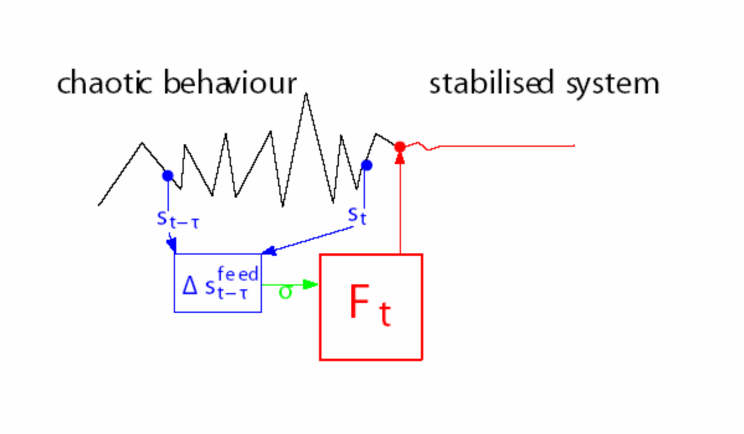

Figure 3: Time-delayed feedback control

Pyragas showed that by re-introducing F_t into the chaotic system, its chaotic behavior is swept away[1]. In fact, his method forces the fluctuations on a constant path. Thus, the concept of time-delayed feedback control suggests how an experimental set-up or a model may be modified to obtain a stabilizing outcome.

One advantage of the application of time-delayed feedback control is that it necessitates knowledge about the chaotic system. For instance, it is worthwhile pointing out that the model-specific time-difference τ and the multiplier σ are sufficient for the stabilization of chaotic motions[2]. Remarkably, stabilization does not require any analytical knowledge of the system's dynamics.

Trend traders as hidden chaos control

Why does this abstract concept of chaos control matter for economists? In order to get to the bottom of this question, we will exploit the knowledge of time-delayed feedback control within the scope of the exchange rate model by De Grauwe et al. (2005). Their paper includes a deterministic and a stochastic setting of the model. In the following, we will refer to the deterministic version. This model includes different types of agents who are boundedly rational and forecast future exchange rate movements using different

[1] Pyragas' finding is based on experimental evidence. His experimental results are supported by a numerical analysis of the Lyapunov exponent λ, which is a well-established quantitative measure of the sensitivity to initial conditions in chaos theory.

[2] Technically, this delay has to coincide with the period of the unstable periodic orbits of the system.

information. One trader group are trend traders.[3] As only the presence of trend traders is important for our hypothesis, we will concentrate on this type of agents. Trend traders make their investment decisions according to exchange rate developments in the past. This means that they extrapolate past price changes into the future. Empirical studies showed that in the short-run this behavior is pre-dominant among professionals who trade financial assets[4]. Trend traders are hence widely used in the literature and as such they are part of nearly every heterogeneous agent model. In this sense, the model by De Grauwe et al. is typical.

As not every individual in heterogeneous agent models is a trend trader, the agents' forecasts of future price developments diverge. Upon these differing expectations, agents mutually sell and buy financial assets (within the setup by De Grauwe et al., this asset is a fictitious currency). As a result of the trading, dynamic chaos evolves.

De Grauwe et al. show that the developing deterministic chaos converges for certain parameter constellations. This is also true for other heterogeneous agent models in finance that include trend traders.

Now let us have a closer look at the precise rule that trend traders use to build their forecasts. As stated above, trend traders assume that the future evolves according to the past. Hence, they extrapolate past price trends into the future.

For reasons of simplicity, we assume that these agents only take into account one past period (L = 1), i.e. they compare the exchange rate at two different moments in time. In the model of De Grauwe et al., these agents then forecast fluctuations of the exchange rate in the following way:

$$E_t\left(\Delta s_{t+1}\right)^{L=1} = \beta \Delta s_{t-1} = \beta\left[s_t - s_{t-1}\right] \tag{17}$$

The extent to which trend traders base their forecast on extrapolated patterns of the past depends on the coefficient β, which measures the agents' inclination to pay tribute to past changes when forming their prediction about subsequent exchange rates. De Grauwe et al. define β to be $0 < \beta < 1$. Thus, the forecasts of this group are partly determined by past data. These cautious extrapolations of past trends avoid an explosive forecast.

After having presented how trend traders form their forecasts (equation (17)), we compare this rule to the time-delayed feedback control (equation (16)). Juxtaposing the two formulae, it can be easily seen that the mechanisms are structurally identical.

[3] Heterogeneous agent models include at least one other trader type: Fundamentalists, who compare the current exchange rate to a hypothetical exchange rate based on fundamental values such as inflation rate, growth etc.

[4] Cheung2000a, Cheung2000, Cheung2001

First of all, time-delayed feedback control and trend traders' forecasting rule are both based on lagged differences $\Delta s_{t-\tau}^{feed}$ and Δs_{t-l} respectively. Second, in both cases the feedback variable is linearly amplified by a parameter, namely σ and the extrapolation parameter β. Third, both variables, F_t and $\Delta E_t\left(s_{t+1}\right)^{L=1}$ are re-introduced into the system (see Figure 2). As trend traders make their investment decisions according to their predictions about future market developments, these expectations are affecting the market-clearing exchange rate.

Hence, one can conclude that the incorporation of trend traders may not be innocent to the model's stabilization properties. In fact, trend traders, who rely on past movements of the market exchange rate, introduce a time-delayed feedback. Thus, it is the presence of trend traders' extrapolative forecasting rule contributes to the stabilization of the market exchange rate.

Enhancement to several lags L

This result is robust if trend traders consider price changes with regard to several points in time[5]. If these agents form their predictions relying on the past L periods, their forecasting rule formally looks as follows:

$$\Delta E_t\left(s_{t+1}\right) = \beta \sum_{l=1}^{L} \rho_l \Delta s_{t-l} \tag{18}$$

where the elapsed movements of the exchange rate Δs_{t-l} are multiplied by geometrically declining weights ρ_l. These are generated by

$$\rho_l = \frac{\left(1-\rho\right)\rho^{l-1}}{\left(1-\rho^L\right)}$$

with $1-\rho^L = \sum_{l=1}^{L}\left(1-\rho\right)\rho^{l-1}$ and $0 < \rho < 1$.

While the extrapolation parameter β applies to the whole sum of past differences, ρ_l assigns individual weights to the differences depending on the point in time they occurred. Recent observations are more influential when trend traders form their predictions about the future.

[5] For instance, in the simulations published by De Grauwe et al. trend traders integrate five lags into their forecasting rules (L=5).

Even though the original time-delayed feedback control scheme and the trend traders' extrapolation pattern differ concerning the number of time-delayed differences, this does not distort the general results. Since the original publication by Pyragas in 1992, the limitation of using one lag only has been overcome and control models have been proposed which use multiple delay times[6].

Another difference concerns the geometrically declining weight ρ_l given to exchange rate movements which have occurred earlier in the past. On this point the economic model simply conducts some internal weighting of the fluctuation patterns in the past. This parameter therefore remains without impact on the general result of stabilization.

Conclusion

We investigate the impact of the feedback structure of the De Grauwe et al. (2005) model on the dynamics of the model. In particular, we concentrate on the feedback loop formed by the expectation formation rule of the chartists.

We adapt the results from the latest research on chaos control in physics. The physics-borne concept of time-delayed feedback control suggests that the heterogeneous agents in the De Grauwe et al. model contribute to the convergence of the trajectories to the steady state after a certain time.

Our work suggests that the convergent behavior of a popular heterogeneous-agent model that includes trend traders is closely linked to the way these traders form their forecasts. The trend traders' forecasting rule, which compares market exchange rates at different moments of time, corresponds to time-delayed feedback control. This result generalizes to all models which include trend traders. Whether one or more lags are taken into account is irrelevant.

[6] Just/Benner/Schöll (Just2003) provide a comprehensive overview of experimental and theoretical time-delayed feedback control.

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