A Multisectoral Micro-Macrodynamic Model [#]

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Abstract

This is a simulation model that intends to integrate macro dynamics with structural and sectoral features that shape and modify it. It is an attempt to analyze the macrodynamic effective demand effects of endogenous structural changes in the same setup. Micro-macro interactions result in the model from each level having its own dynamics dependent on the inputs it receives from the others. Each sector is modeled according to neo-Schumpeterian evolutionary microfoundations, with additional unorthodox micro behavioral assumptions. Together with exogenous foreign and government sectors, they are integrated into a multisectoral model. The main features of the model are: (1) simulated sectoral trajectories of a stylized economy derive from endogenous competitive dynamics as well as direct (input-output) and indirect (income, consumption) interactions; (2) sectors are distinguished according to their role in the productive structure and demand categories – consumption, intermediate and capital; (3) no equilibrium is assumed: dynamic interactions among firms' decisions (based on adaptive expectations) and their effects generate open-ended trajectories. Even though the simulation results presented in this paper only give a very brief idea of the trajectories generated by the model, a general robust result is obtained: the cyclical behavior of the GDP and its main aggregate components.

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1. Introduction and theoretical assumptions

This paper presents a micro-macro multisectoral evolutionary simulation model that combines neo-Schumpeterian evolutionary microfoundations with some post-Keynesian and Kaleckian assumptions and exhibits some preliminary results of simulation runs made on selected issues. The main objective of the model is to combine analytical elements that may be useful to investigate dynamic properties of capitalist economies, which depend mainly on micro-macro relations, with a special regard to the analysis of economic development. It is our belief that very important complementary insights and results can be drawn from combining these approaches[1]. As a starting point, both theoretical fields share the rejection of two neoclassical foundations: (i) substantive rationality; and (ii) equilibrium of agents and markets.

Concerning rational decision processes, both fields assume (or at least are compatible with) *bounded and procedural rationality*, as developed by Simon (1983), through which instrumental rationality may be reconciled with *hard uncertainty* (in the sense of Knight and Keynes). As is well known, the latter is supposed to be a feature of economic environments where irreducible information and competence gaps (in both cognitive and computational senses) can emerge. In such context,

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[#] This paper is part of an integrated research project supported by CNPq (Brazil's National Research Council). Ana Cristina Reif, a PhD student also working in the project, is co-responsible for all macro sections. The sectoral part of the model draws heavily on Possas, Koblitz *et al.* (2001).
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[1] For recent attempts see Verspagen (2002); Dosi, Fagiolo, Roventini (2005) and a survey by Llerena, Llorentz (2003).

rationality involves "satisficing" (*apud* Simon) kinds of sub-optimal solutions that may lead to strategies based on routines and conventions (Heiner, 1983; Nelson and Winter, 1982).

As for the rejection of the notion of equilibrium, neo-Schumpeterian and post-Keynesian approaches usually share the view that disequilibria and coordination failures are normal in a market economy. But this amounts to assuming, in more formal terms, a nonergodic and nonstationary economic environment, in which rational agents can make systematic forecasting mistakes, as opposed to the rational expectation hypothesis (Vercelli, 1991, p. 154-5). In particular, for the traditional neo-Keynesian economic growth and fluctuations theories up to the 60's, disequilibrium was essential in explaining capitalist economic dynamics, either in the more conventional interpretation as causing the propagation of fluctuations around a trend of moving equilibrium, or even when such trend is seen as irreducible to an equilibrium in any intelligible sense (Kalecki, 1954; Possas, 1983, 1999).

Both kinds of theories also admit that capitalist economies show regularities that may reduce uncertainty (without eliminating it) and allow long run decisions to be made, thus mitigating the effects of potential instability (Vercelli, 1991, ch. 5; Possas, 1993). But these regularities do not prevent capitalist economies from exhibiting nonlinearities originated from cumulative decisions and their structural effects (technological paths with technical progress and learning, synergies, etc.), which may cause strong structural instability. Technical progress and corresponding technological trajectories (Dosi 1982, 84) is probably the main dynamic process causing such effects in the long run, and not just through their direct innovative impacts. At the same time, they usually increase dependence on existing assets, acting as a source of increasing returns and sunk costs that create path dependence and lock-in effects in long run paths. Stable institutions may induce similar effects, although more complex and less studied by economists.

Main contributions from the neo-Schumpeterian *evolutionary* approach (Nelson and Winter, 1982; Dosi, 1984) are incorporated by including explicitly in the model its two theoretical cornerstones: (i) behavior *diversity* among agents, endogenously generated through search of opportunities to innovate; and (ii) a *selection* of firms, strategies and/or technologies basically through market competition, for which no reference to equilibrium is needed.

Feedback between strategies and selection through the market (or other institutions) entails an endogenous industrial dynamics. Industrial structure *and* performance emerge from this interaction across patterns of technological change that may shape technological trajectories (Dosi 1982, 84). A successful innovation allows a firm to reach competitive advantages and fetch larger profits and/or market shares, thus raising asymmetries not only in performance variables, but also in market structure (Dosi 1984, 88). Iterative processes based on change in parameters and/or in expectations by firms give place to open ended dynamic paths without any equilibrium trend, where not even self-organizing order or regularities are necessarily expected to be found.

In spite of being largely unpredictable, we believe that such long run trajectories can be successfully studied through simulations based on specific hypotheses concerning parameters and initial conditions. In fact, the performance of simulation exercises to investigate the basic dynamic properties of economic market processes of change has become a typical feature of the evolutionary neo-Schumpeterian research program, since one cannot expect analytical solutions usually to emerge for such complex system modelling - except under seriously restrictive assumptions, which can bring them close to irrelevance.

A surge of neo-Schumpeterian evolutionary models trying to analyze sectoral dynamics along these lines emerged in the last two decades[2]. The path breaking work of Nelson and Winter (1982) paved the way for a family of models of Schumpeterian competition. In the second round of these models, by the beginning of the 90's, there was a split within evolutionary/neo-Schumpeterian models: (1) "microdynamic models", related to industrial trajectories with technological change; and (2)

[2] For a survey see Dosi *et alii* (1988), Nelson (1994, 95) and Saviotti and Metcalfe (1991).

"endogenous growth models", more related to macroeconomic issues, clearly as a counterpoint to neoclassical endogenous growth models. But in spite of the effort to include important elements left out by neoclassical growth models, such models still present a serious flaw – there is no underlying economic structure, essential to any macroeconomic analysis, including the sectoral interrelation among consumption, investment and intermediate goods. The transition from micro to macro levels is done without macro-sectoral mediations between the firms and the economy as a whole, either through input-output relations or through income generation and final demand. In a few words, it means that these models have no macroeconomic level of analysis.

On the other hand, the main contributions from post-Keynesian economists[3] are related to the demand side and its implications at the macroeconomic level, in particular the effects of the decisions to spend in uncertain environments[4]. The demand side issues concern mainly agents' decisions to consume and to invest and their relation with current income and expectations formation. As highlighted by both Keynes[5] and Kalecki[6], and later on by post-Keynesians, the decisions to spend, according to the *principle of effective demand*, play an essential role in economic analysis, since they can be a distinct source of *causality* and equilibrium positions are no longer required as a methodological assumption to determine dependent variables. As shown by Kalecki and neo-Keynesian authors in multiplier-accelerator models, such induction effects over production and investment coming from demand may be responsible for major dynamic properties of the capitalist economy. Finally, since post-Keynesians are not very much interested in multisectoral models, and even less so in more elaborate micro analysis, an ample room is left for building a link by combining these two theoretical approaches, the neo-Schumpeterian and the post-Keynesian, in a multisectoral micro-macrodynamic framework.

2. Main features of the model

Multisectoral model. Input-output (sector x sector) matrices are employed together with other expenditure matrices – consumption (sector x personal income class) and incremental capital/output (sector x sector) - to endogenize main components of final demand. At the income generation side, matrices of income appropriation within classes (functional income class x sector) and of personal income appropriation (personal income class x functional income class) are defined. Other components of final demand, as exports and government expenses, are left exogenous. The importance of such sectoral interrelations is stressed by evolutionary authors: "the structure of input-output, as well as the untraded technological interdependencies of each economy, can be regarded as a huge feedback machine that amplifies, transforms or smoothes technological and demand impulses generated in any part of the economy, transmitting them to the rest of the system in ways which are both sector-specific and country- (or region-) specific" (Dosi, Pavitt and Soete, 1990, p. 108).

[3] Stream of heterodox macroeconomists led by P. Davidson and H. Minsky who support Keynes' original ideas since the early 1970's, not to be confounded with neoclassical ones such as the so-called "new Keynesians" or post-keynesians related to the kaldorian approach.

[4] Of which some of the most original come from liquidity preference (Keynes, 1936, Davidson, 1972) and financial fragility (Minsky, 1975).

[5] "A decision to consume and not to consume truly lies within the power of the individual; so does a decision to invest and not to invest. The amounts of aggregate income and of aggregate savings are the results of free choices of individuals whether or not to consume and whether or not to invest; but they are neither of them capable of assuming an independent value resulting from a separate set of decisions concerning consumption and investment" (Keynes, 1936, p. 65).

[6] Commenting on the accounting equality, under some simplifying assumptions, between gross profits and the sum of gross investment and capitalist consumption, Kalecki (1954, p. 39) stated the same idea as follows: "…it is clear that capitalists may decide to consume and to invest more in a given period than in the preceding one, but they cannot decide to earn more. It is, therefore, their investment and consumption decisions which determine profits, and not vice versa".

Dynamic model. It generates trajectories in discrete time (periods). Since causality is based on decisions to produce and to spend (effective demand), no equilibrium positions are ever required. The use of given matrix coefficients do not prevent dynamic modeling, because it only requires such coefficients being fixed during each simulation period, while it is possible to change them between periods according to some established rules. Since periods are defined as a time lapse between consecutive decisions (production, investment, consumption), this assumption poses no consistency problem, given that decisions in any case could only be revised by the end of each period.

Firms are the basic units. Each firm belongs only to one sector. Structural changes in each sector are endogenously dependent on firms' behavior, especially as a result of technological and strategic diversity[7]. Conversely, firms try to adapt to market conditions through feedback mechanisms. Some basic features are: (i) prices are decided by firms according to expected markups, subject to endogenous change due to strategic market concerns; (ii) effective demand causality in production decisions and sales (*i.e.* absence of market supply and demand equilibrium) involves distinguishing between output and sales, as well as putting emphasis on short period expectations concerning sales, assumed to be endogenous (extrapolative); and (iii) investment decisions follow basically the same rule, but allowing for an important autonomous component related to technical progress, and imposing financial debt constraints. These mechanisms can be more deeply explored within the multisectoral structure, which allows treating as endogenous some important elements which otherwise could only be fixed exogenously.

More specifically, firms' strategies and decisions can be divided in three subsystems:

(i) *production*, *prices* and *profits*;

(ii) *investment*;

(iii) *technological search*.

In the first one, basic "effective demand" elements are drawn from Possas (1983, 84): production decisions are based on expected sales for the production period, extrapolated from the average of some previous periods[8]. As to prices, the present model assumes each sector to be an oligopoly with some degree of price competition as well as of product differentiation, following a version of Kalecki's price model (1954, ch. 1), in which actual price is a weighted average of the price corresponding to the expected markup and the industry average price, but subject to change according to a feedback from the firm's competitive performance.

Investment decision rules on new capacity are also drawn from Possas (1983, 84), based on extrapolated expected sales from some previous (investment) periods but limited by a financial constraint following Kalecki's principle of increasing risk (1954, ch. 8). Wider financial features are included in these decisions to capture the influence of assets and liabilities structure of the firm, represented by debt/equity ratio, retained profits and liquidity demand.

Lastly, technological search combines different approaches: both *innovative* and *imitative* searches follow a stochastic process as in Nelson and Winter (1982); and a learning process is also included drawing on the vintage model by Silverberg, Dosi, and Orsenigo (1988), from which a payback period criterion for equipment replacement decisions is also applied.

Interactions at the sectoral level. Demand for each sector is in part determined endogenously by firms and household decisions to spend and in part exogenously by exports and government expenditure, and is divided among firms by a "*replicator*" dynamic equation. Production and investment decisions by each firm determine, respectively, the demand for intermediate and for capital goods, and household decisions determine demand for consumption goods.

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[7] Based on the sectoral evolutionary model in Possas, Koblitz, *et al.* (2001). But while in that case the only sector was modelled as a science based one, in the present case sectors are widely distributed across Pavitt's (1984) taxonomy.

[8] The exceptions are firms in the intermediate and capital goods sectors, which produce according to their current orders.

Consumption is a function of the average income of each income class, assumed to be linear and to have higher lags and lower propensity to consume for higher income classes. The income flowing to each class is calculated as a proportion of the total amount of wages and distributed profits. The distribution of the value added between wages and profits in each sector is a function of the average markup and the unit wage, which can be assumed to change across periods.

Exogenous blocks. In addition to the above endogenous core, the model also involves three partially exogenous blocks or "sectors" treated separately: foreign sector[9]; government (public expenditure, taxes and economic policy); and a financial sector (debts, capital investment and interest rates). This treatment allows an easier setting of specific simulation assumptions concerning strategic areas for macrodynamics and, in particular, for economic development. A multicountry model could be a future extension of the model, therefore endogenizing the foreign sector.

3. The model [10]

In its general specification, the model defines an economy with *m* income classes (1,…,*h*…,*m*), *p* sectors (1,…,*z*,…,*p*), at least three, *n* firms in each sector (1,…,*i*,…,*n*), each one initially containing *l* capital goods (1,…,*j*,…*l*). Following one of the present trends in neo-Schumpeterian evolutionary modelling, the model was built and the simulations were run on the software *Laboratory for Simulation Development* (LSD), details of which can be found in Valente (1999).

Block 1: Production

1A. Planned production

$$x^*_{i,t} = x^e_{i,t}(1+\sigma) - x^s_{i,t-1} \qquad (1),$$

subject to $0 \le x^*_{i,t} \le \overline{x}_{i,t}$,

where σ is exogenously fixed.[11] The production decision[12] $x^*_{i,t}$ at the beginning of period *t* is aimed at two goals: (i) to meet the expected demand for sales $x^e_{i,t}$ at the end of the production period beginning at *t*; and (ii) to keep the stock $x^s_{i,t}$ at a safe level to cope with unexpected demand fluctuations, which is assumed as a fixed proportion of sales, σ Production is limited by existing productive capacity $\overline{x}_{i,t}$, measured in production units. Since different equipment units have different productivities, we assume the most efficient ones to be used first.

In the capital good sectors each firm produces based on current orders, ${}^k e_{i,t}$, its planned production being equal to the previous orders[13], subject to the same capacity constraint:

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[9] In this version the foreign sector includes only the trade balance, an important source of inflow and outflow of demand in any economy.

[10] Although the complete model involves all the equations described below, not all will be used in every simulation run.

[11] In the simulations, σ = 0,1. This and other fixed parameters were defined by "educated guesses" and are assumed to be the same across firms, except when otherwise stated.

[12] The production decision follows Metzler (1941); see Gandolfo [(1985), p.90], and Possas (1983).

[13] Since planned production is the basis on which every sector decides its demand for intermediate goods, planned production of intermediate goods also has to be based on past orders, and effective production on current orders.

$$ {}^{k}x^{*}_{i,t} = {}^{k}e_{i,t} \qquad (1'), $$

subject to $0 \leq {}^{k}x^{*}_{i,t} \leq \overline{x}_{i,t}$.

1.B. Expected sales

The way expectations are formed has an essential role in micro-macro interactions, since it is one of the main channels through which the uncertainty present at micro level decisions affects the macro outcomes. As highlighted by Simon, "in investigating and describing the production decision rules used by business firms, one aspect of particular interest would be the use or non-use of forecasts of future sales, and the procedures for arriving at those forecasts" (Simon, 1984, p. 31). In this version, firm's sales expectations follow an extrapolative rule[14] from past effective orders e_i:

$$ x^{e}_{i,t} = e_{i,t-1}\left(1 + \gamma \frac{\Delta e_{i,t-1}}{e_{i,t-2}}\right) \qquad (2), $$

with γ exogenously fixed.[15] This is an important parameter[16] related to the stability (or amplitude) of the cycle and it reflects both the state of confidence on the expectation formation by the agents and the competitive conditions of the sector.

Block 2: Total sectoral demand and firms' demand

2A. Total sectoral demand

Total demand is specified differently for each sector, so it will be presented separately. Note that in the case of both capital and intermediate goods, total demand will determine planned production[17].

(i) Intermediate goods sectors

The effective production of each firm in these sectors is constrained both by its productive capacity and by the stock of intermediate goods it holds. The amount of inputs produced by sector z necessary to meet the planned production of firm i is given by fixed technical coefficients:

$$ x^{in}_{z,i,t} = x^{*}_{i,t}\left(a^{i}_{z,i,t} + a^{m}_{z,i,t}\right) \qquad (3), $$

or, in matrix form:

$$ X^{in}_{t} = (A^{i}_{t} + A^{m}_{t})\hat{x}^{*}_{t} \qquad (3'), $$

[14] Gandolfo (1985), p, 95, suggests that this equation was originally proposed by Goodwin (1947). For a taxonomy of simple expectation formation rules see Williamson (1989), p.214.

[15] In the benchmark simulations, γ = 0.3.

[16] The role of this parameter is even more important in the investment decisions. As will be explained later, the expected sales are also relevant to define the amount of desired induced investment.

[17] In capital goods sector, current production depends on past orders, since its production period - the investment period - is largest than any other.

where X_t^{in} is the (p x pn) matrix of required inputs; A_t^i and A_t^m are the input-output (p x pn) matrices of domestic, $a^i_{z,i,t}$, and imported, $a^m_{z,i,t}$, technical coefficients; and $\hat{x}_t^*$ is the diagonal matrix (pn x pn) of planned production of each firm i, $x^*_{i,t}$.

If the required amount of each intermediate good is available, the quantities spent will be those indicated on the columns of the matrix X_t^{in}; otherwise, only a proportion[18], $\rho_{i,t,}$ of these quantities will be used for each firm i. Therefore, the total demand for domestic intermediate goods at t is determined by each firm's expected planned production for t+1, $\hat{x}^{*e}_{t+1}$, again calculated by an extrapolation rule, deducting its previous unused stock of intermediate goods[19], x^{isr}_{zit} :

$$ {}^{i}e_t^i = \left(A_t^i \hat{x}^{*e}_{t+1} - X_t^{isr} \right) \bar{u} \qquad (4), $$

with

$$ x^{*e}_{i,t+1} = x^*_{i,t} \left(1 + \gamma_i \lambda_{i,t} \right) \qquad (5), $$

where ${}^{i}e_t^i$ is the vector (p x 1) of domestic demand for intermediate goods sectors, X_t^{isr} is the (p x pn) matrix of the unused stock of intermediate goods[20] and $\bar{u}$ is a column unitary vector.

(ii) Consumption goods sectors

The domestic demand for consumption goods is determined by households' income, according to their income class, and government expenses. The consumption of each class is assumed to be a linear function, with increasing lags[21] and decreasing marginal propensity to consume, of the average real income of each class, $\bar{y}_h^r$, plus a fixed autonomous consumption:

$$ {}^{c}e_t^i = C\bar{y}^r + C^A \bar{u} + c_t^g \qquad (6), $$

where C and C^A are (p x m) matrices of the marginal propensity to consume and autonomous consumption, respectively, $\bar{y}^r$ is the vector (m x 1) of the average real income of each class; and c_t^g is the vector (p x 1) of the government consumption measured in output units of each sector.

(iii) Capital goods sectors

The investment decisions on fixed capital (for *expansion* and *replacement* of productive capacity) have two major components: expected sales and the financial risk of increasing indebtedness[22]. Firms take these decisions at every investment period, which by assumption consist of *six* production periods each [23], corresponding to the time lag needed to produce, install, and start operating the new equipments. In order to be implemented, such decisions must be financially feasible, *i.e.*, the firm must

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[18] This proportion is equal to lowest ratio for the firm j of the available intermediate goods of sector i, $x^{id}_{i,j,t}$, and the amount required, $x^{in}_{i,j,t} : \rho_{j,t} = \min_i \left(\dfrac{x^{id}_{i,j,t}}{x^{in}_{i,j,t}} \right)$.

[19] In the case of intermediate goods sectors, since they produce based on these orders, the remaining stock of inputs is not deducted since it is not yet determined.

[20] The columns corresponding to intermediate sectors in the matrix $X^{isr}{}_t$ are zeros.

[21] The simulations assume m=4. For class A, a 4 period lag is assumed; for class B, 3; for class C, 2; and, for class D, 1.

[22] That is, the debt/capital (or debt/equity) ratio.

[23] This version of the model assumes the same investment period for all sectors, although decisions are not simultaneous.

be capable of paying for the new capital goods either with its own and/or with borrowed resources, as it will be explained bellow. A proportion[24] of the aggregate demand for capital goods is for imported ones; thus, the orders received by the domestic capital good sector are determined deducting imports and adding government investments:

$$ {}^{k}e_t^i = {}^{k}X_t\left(\overline{u} - {}^{k}m\right) + \hat{p}_{t-1}^{-1} I_t^g \qquad (7),$$

where ${}^{k}e_t^i$ is the (p x 1) vector of orders received by the domestic sectors; ${}^{k}X_t$ is the (p x pn) matrix formed by capital goods total demand vectors; ${}^{k}x_{i,t}$; ${}^{k}m$ is the (pn x 1) vector of import coefficients for capital goods; I^g_t is the (p x 1) vector of government investment expenditure and $\hat{p}_{t-1}^{-1}$ is the (p x p) diagonal matrix of the inverse of last period average prices[25].

(iv) Effective orders

Finally, effective orders for each sector are determined by the sum of domestic and foreign demand[26].

$$ e_t = {}^{c}e_t^i + {}^{k}e_t^i + {}^{i}e_t^i + e_t^x \qquad (8),$$

where e_t^x is the (p x 1) vector of each sector's export orders, defined below at the foreign sector block.

2B. Firms' demand: replicator dynamic equation and competitiveness

The discrete formulation presented here was developed by Kwasnicki and Kwasnicka (1996), based on Silverberg's (1987) adaptation to firms' competition of the original equation developed by Fisher. The only difference here is an adjustment parameter μ[27].

$$ s_{i,t} = s_{i,t-1}\left[1 + \mu\left(\frac{E_{i,t}}{\overline{E}_t} - 1\right)\right] \qquad \text{or} \qquad \frac{s_{i,t} - s_{i,t-1}}{s_{i,t-1}} = \mu\left(\frac{E_{i,t}}{\overline{E}_t} - 1\right) \qquad (9),$$

such that $0 \le \mu \le 1$ and $\overline{E}_t = \sum_{i=1}^{n} E_{i,t} s_{i,t-1}$,

where E_i is a competitiveness index for firm i, based on price and delivery delay.

In the model, each firm's competitiveness is defined as:

$$ E_{i,t} = \frac{1}{p_{i,t}^{\varepsilon 1} . dd_{i,t}^{\varepsilon 2}} \qquad (10),$$

where p_i is the price and dd_i is the delivery delay for firm i, and ε_1 and ε_2 are respectively the firm's competitiveness elasticities relative to price and to delivery delay.

The above definition is consistent with Silverberg's [(1987), p. 121] comment that relative, not absolute, price differences may drive a customer away from a seller to another. Silverberg introduces the price logarithm in his definition to keep up with that observation. In our model this device is unnecessary since a different replicator equation is used: while in Silverberg's model market share

[24] The ratio is sector specific; however, in the standard simulations it is set to 5% for all sectors.
[25] Given technical indivisibilities, the effective level of government investment is determined by the integer component.
[26] In general the sectors are specialized, which means that only one of the three domestic components is positive.
[27] This parameter is specific to each sector. A discussion on the consistency of this equation is presented in Possas, Koblitz *et al.* (2001).

depends on the *absolute* difference between individual and average competitiveness[28], here it is the *ratio* between individual and average competitiveness that fulfills this role. Therefore, in defining competitiveness as a function of price, the market share for each firm will be determined by relative prices.

Block 3: Firms' orders, actual production and sales

3A. Firms' orders

Effective orders received by a firm depend on total sector demand, e_t, and on the firm's market share s_{it}, determined by the replicator dynamic equation, under the effect of firm's competitiveness:

$$e_{i,t} = s_{i,t} e_t \qquad (11).$$

However, orders actually received by an individual firm might differ from this level if some firm is unable to meet its orders and other firms in the same sector have excess supply. The resulting excess demand is divided among remaining firms on the basis of their market share ranking.

3B. Actual production and sales

Actual production is planned production subject to the intermediate goods constraint:

$$x_t = \hat{x}_t^* \times \rho_t \qquad (12),$$

where x_t is the (pn x 1) vector of the effective production, ρ_t the (pn x 1) vector of elements $\rho_{i,t}$ and $\hat{x}_t^*$ was defined before.

In the case of firms belonging to an intermediate sector, actual production is not based on planned production, but on actual orders, as in capital goods sector. In this case, however, the aim is not only to meet the domestic and foreign current orders, but also to keep the stock $x_{i,t}^s$ at an acceptable level, in face of unexpected demand fluctuations (as in the consumption goods sectors):

$${}^i x_{i,t}^* = {}^i e_{i,t} \times (1 + \sigma_i) - x_{i,t}^s \qquad (13),$$

subject to $0 \le {}^i x_{i,t}^* \le \overline{x}_{i,t-1}$.

Actual sales x_t^v are determined by the effective orders, which may or may not correspond to the expectations that previously defined the level of production. This interaction between sales and production over time creates a mechanism of dynamic induction over the subsequent production decisions, via changes on the expected behavior of future sales. Obviously sales level cannot exceed the production level plus existing stocks:

$$x_t^v = e_t, \qquad (14),$$

subject to $0 \le x_t^v \le x_t + x_{t-1}^s$.

Finished goods inventories are, by definition:

$$x_t^s = x_{t-1}^s + x_t - x_t^v \qquad (15).$$

The stock of intermediate goods available for next period production is given by the amount not used in the current period, $x_{i,j,t}^{isr}$, plus the amounts purchased domestically and imported [29]:

[28] The equation used by Silverberg is $\frac{df_i}{dt} = A(E_i - \overline{E}) f_i$, where f_i is the market share of firm i.

[29] Some degree of substitutability between domestic and imported goods of the same sector is assumed.

$$x^{is}_{i,j,t} = x^{isr}_{i,j,t} + {}^{i}e^{i}_{i,j,t} + x^{mi}_{i,j,t} \quad (16),$$

where $x^{mi}_{i,j,t}$ are sector j imports of sector i goods. If effective orders (in the first round) received by a firm exceed the sum of its planned production and available stocks, the firm will incur in a delivery delay, which will have a negative impact on its competitiveness in next period. The delivery delay figure compares effective orders with effective sales:

$$dd_{i,t} = \frac{e_{i,t}}{x^{v}_{i,t}} \quad (17).$$

Block 4: Prices and costs

4A. Price decisions

The price equation used here, as shown elsewhere[30], is a discrete version of Silverberg's, consistent with the version specified before for the replicator equation and it is also identical to the price equation used by Kalecki (1954, ch. 1) in his analysis of the "degree of monopoly" of a firm under imperfect competition:

$$p_{i,t} = \theta p^{d}_{i,t} + (1-\theta)\,\overline{p}_{t-1} \quad (18), \quad \text{or}$$

$$k_{i,t} = \theta k^{d}_{i,t} + (1-\theta)\frac{\overline{p}_{t-1}}{u_{i,t}} \quad (18'),$$

where $p^{d}_{i} = k^{d}_{i} u_{i}$ is the firm's desired price for each period, *i.e.*, the price that results from applying the desired markup k^{d}_{i} over the unit variable cost u_i; and k_i is the effective markup corresponding to the effective price p_i.

As mentioned above, the latter equation is exactly the one used by Kalecki (1954, ch. 1). Both Kalecki and Silverberg look at their equations as simple extensions of the so-called "full cost principle" to oligopolistic conditions, where it is impossible for firms to ignore each other's prices. Alternatively, it can be understood as one of the determinants of markup in oligopoly: as a sort of compromise between the desired markup by a firm (or its long run strategic markup) and current competitive conditions. While low cost firms enjoy the advantage of making additional profits in the short run, in excess of what would result from applying the strategic markup, high cost firms sacrifice their desired markup for keeping their market share (Silverberg, 1987, p.130).

Another behavioral implication of this equation is that, since the average price is weighted by market shares, larger firms will have a greater influence on average market price, thus playing a kind of price leadership, while small firms can substantially reduce prices without producing a large impact on market price (to an amount required, for instance, to start a price war).

Each firm's unit variable cost in a given period, $u_{i,z,t}$, is the sum of unit input costs $m_{i,z,t}$ and unit labor cost, which depends on the nominal wage rate $w_{z,t}$ and on the firm's average productivity, $\overline{\pi}_i$ (see below):

$$u_{i,z,t} = m_{i,z,t} + \frac{w_{z,t}}{\overline{\pi}_{i,z,t}}$$

(19),

with w changing according to the sector average productivity[31]; and

[30] Possas, Koblitz *et al.* (2001).
[31] More details in block 7.

$$m_t = A_t^i p_{t-1} + er_t A_t^m p_t^m \qquad (20),$$

where p_t^i and p_t^m are (p x 1) domestic (sector average) and foreign price vectors, respectively, and er_t is the exchange rate[32]. As commented before, unit input cost is one of the channels that spreads technological progress between sectors, through input-output relations.

4B. Technological routines and productivity

Average labor productivity for each firm varies over time as a function of (i) the investment on fixed capital and the degree of productive capacity utilization; (ii) the R&D strategy adopted; and (iii) the efficiency of the learning-by-doing process.

Fixed capital stock at any period is heterogeneous, composed of equipments requiring different labor productivity to operate, so that the firm's average productivity depends on which capital goods are being used and on their degree of utilization. Each equipment's productivity at a given period, on its turn, results from the combination of the outcome of the firm's technological search at the moment it was ordered (more details later) and of the improvements obtained while using it, associated with the adjustment processes that must be done, along with the mentioned learning by doing process[33]. These advantages[34], however, are balanced by two other factors also present in the model: (i) the learning-by-doing effects were realistically assumed to be limited; and (ii) they are specific to each equipment/technology, so that when the latter is replaced, the firm enters into a different "learning curve"[35].

(a) Productivity of each equipment/technology:

$$\pi_{i,j,t} = \pi^0_{i,j,t} h_{i,j,t} \qquad (21),$$

where $\pi^0_{i,j,t}$ (initial productivity of equipment j of firm i) is determined in Block 6 below, and $h_{i,j,t}$ is defined afterwards.

(b) Learning effect (*learning-by-doing*):

$$h_{i,j,t} = 1 + z(1 - \exp(-\tau \sum_t x^*_{i,j,t})) \qquad (22).$$

The parameters z and τ of this equation represent respectively the growth rate of the equipment's initial productivity that can be reached through *learning-by-doing*, and the speed with which it can reach this level[36].

Block 5: Investment decisions and financial constraint

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32 For simplification, the exchange rate is being kept constant.

33 Scherer and Ross (1990), ch. 4, pp. 97-98. Specific product economies of scale are those associated with the amount produced and sold of only one product.

34 The difference of these advantages among sectors will be taken into account. According to Scherer and Ross, *op. cit.*, "...some of the product lines in which learning-by-doing is most important (such as semiconductors, aircraft, and computers) are also characterized by rapid technological obsolescence of product designs. The development of a completely new design often permits an initially handicapped producer to jump to a new learning curve in a position of equality or even superiority" (p.372).

35 More details in Scherer and Ross (1990) and Possas, Koblitz *et al.* (2001).

36 Both parameters are given as initial conditions.

Investment decisions determine both the firm's average productivity and the extent to which it can grow in the long run. The model considers two components – apart from technological improvements - of an investment decision: capacity expansion and capacity replacement. The latter can be explained either by a thorough physical depreciation or by technological obsolescence, or both. In order to be implemented, such decisions must be financially feasible, *i.e.*, the firm must be capable of paying for the new capital goods either with its own and/or with borrowed resources, subject to a given precautional demand for liquid assets and to an upper indebtedness bound. These financial variables act as a constraint to the firm's desired investment. This Kaleckian (and partly Keynesian) provision is a clear improvement upon the traditional "accelerator" mechanism. The main differences from our model as compared to Kalecki's (1954, ch. 9) are: (i) the introduction of a financial constraint, instead of adding it as a continuous variable, on the investment equation; and (ii) the "accelerator" component itself, adapted to cope with the necessary adjustments of the degree of capacity utilization together with the observed growth projection (Possas, 1987)[37].

At the same time, detailed descriptions of the investment decisions made by behaviorist economists[38] seem to be largely consistent with investment decision routines and with their relation with financial variables, proposed by the present model[39].

5A. Investment decisions

Investment decisions are taken at the end of each investment period (time interval between consecutive investment decisions), which is assumed to comprise *six* production periods each.

Decision making starts with a forecast of average sales for the next production periods [t+6; t+12] when the new capacity, resulting from current investment, will be operative – by definition, its "construction period", also assumed to be equal to the investment period. This forecast is a simple extrapolation of average sales of the corresponding previous investment periods. Expected sales for the next investment period $x^{ve}_{i,T+1}$ [40] are therefore:

$$x^{ve}_{i,T+1} = e_{i,T}\left(1+\gamma\frac{\left(e_{i,T}-e_{i,T-1}\right)}{e_{i,T-1}}\right)$$

(23),

where e_T = average orders in the current investment period[41], by the end of which decision is being made. Assuming that the same absolute change[42] in demand will repeat itself in the following period,

$$x^{e}_{T+2} = x^{e}_{T+1} + e_T\gamma\frac{\left(e_T-e_{T-1}\right)}{e_{T-1}} = e_T\left(1+2\gamma\frac{\left(e_T-e_{T-1}\right)}{e_{T-1}}\right)$$

(24).

In order to determine the desired productive capacity one needs to know the production level that is expected to be necessary, which, as usual, has not only to meet expected sales, but also a given stock level. The latter was established before as a fraction σ of expected sales. As a safety margin for possible forecast errors and unforeseen demand fluctuations, the above result is taken as a fraction α of

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[37] Possas (1987) made a detailed critical discussion on investment determinants in Kalecki's model of 1954, as well as on the accelerator and for the original formulation of the equation used here.

[38] See Cyert *et alii* (1979) and Bromiley (1986), quoted in Possas, Koblitz *et al.* (2001).

[39] For instance, concerning the role of financial variables as constraints to desired investment, see Cyert *et alii* (1979), *op. cit.*, *in* Cyert and De Groot (1987), p.134; *apud* Possas, Koblitz *et al.* (2001).

[40] In this block of equations, subscript T refers to the investment period, where T=t/6.

[41] The average orders over the last 6 production periods.

[42] Note that this is a lower rate of change than the previous one, since $1+2a<(1+a)^2$.

planned capacity. Finally, to obtain the *variation* in planned capacity which will justify investment, one needs only to subtract the existing capacity. Thus, desired increase in the productive capacity $\Delta\overline{x}^*_{i,t}$ is determined by[43]:

$$\Delta\overline{x}^*_{i,T} = \frac{(1+\sigma_i)}{\alpha_i} x^e_{i,T+2} - (1-\delta)\overline{x}_{i,T} \qquad (25),$$

where $x^e_{i,T+2}/\alpha_i$ is the planned capacity, α as defined above (given as an initial condition) and δ the depreciation rate.

Now, the *value* of desired gross investment in fixed capital is obtained by multiplying the desired increase in capacity by b_z, the vector[44] (p x 1) of incremental capital/productive capacity ratio of the firm's sector, and the price of capital goods:

$$I^{*F}_{z,i,t} = \overline{p}'_t b_{z,t} \Delta\overline{x}^*_{z,i,t} \qquad (26),$$

where $\overline{p}'$ is the vector (1 x p) of the sector's average price.

Lastly, desired investment in *technological updating* of the equipment will be determined by a common *payback rule* for each unit of capital equipment, starting with those of lower productivity:

$$\frac{p_{F,t}}{w\left(\frac{1}{\pi_{i,j,t}} - \frac{1}{\pi^F_{i,t}}\right)} \leq b \qquad (27),$$

where $\pi_{i,j,t}$ is the productivity of equipment *j* of firm *i* and *b* the payback period. This technological updating is only possible if the firm has available funds *greater than* desired investment, $I^{*F}_{z,i,t}$.

5B. Financial constraint to investment[45]

Total financial funds available for investment, *F*, are given by:

$$F_{i,t} = F^I_{i,t} + F^{*X}_{i,t} - A^*_{i,t} \qquad (28).$$

F^I is the amount of the internal funds (or cash flow), resulting from deducting taxes and distributed profits from net profits (which defines retained profits P^R), and adding depreciation funds. F^X are the external funds that a firm may borrow[46] up to a given acceptable rate *g* of debt on total capital, exogenously assumed in the simulations as fixed and equal among firms. These funds will only be used when internal ones are insufficient to fund the amount of desired investment[47]. The firm is also supposed to keep a given amount of liquid resources as a means of avoiding short run borrowing due to

[43] This sets the maximum level of required capacity, in order to meet product demand and stock replacement needs.
[44] This vector will contain as many non-zero values as the number of capital goods sectors.
[45] The financial constraint used in this model was largely inspired by Wood (1975) and Minsky (1975), and also employed in Possas (1984). For more details see Possas, Koblitz *et al.* (2001).
[46] In order to invest, firms may need access to credit offered by banks through book-keeping creation of money. This does not warrant an unlimited access to credit, since financial fragility of firms may impose a financial constraint to limit external funds borrowing and a further debt increase.
[47] F^{*X} may be positive – when external funds may be added to internal ones to finance investment without exceeding an acceptable level for the rate of debt on capital; or negative, otherwise – in which case part of the internal funds should be used to reduce the debt. This debt adjustment is made stepwise to reflect some tolerance of the firm to exceed its debt limit so as to avoid sacrificing the whole desired investment.

sales forecasting errors. $A^*_{i,t}$ is the amount of desired additional liquid funds as a proportion ϕ of the already existing liquid capital stock[48].

When the firm's rate of debt on capital exceeds a given risk threshold, which is much above the acceptable level g (in the simulations, 90%), the model assumes it went bankrupt and it will thus be eliminated from the market.

Now, a *financial constraint* on the desired value of total investment can be applied, depending on a number of factors, as shown in the above equation. To simplify matters, we can identify the following main alternatives:

(i) The amount of *total financial funds* available for investment is *negative* ($F_{i,t}$<0).

This situation may result from high losses, high indebtedness, liquidity squeeze or a combination thereof. Generally it will take place if F^I is small or negative; and the firm's reaction will depend on its stock of liquid assets. Should it be insufficient to cover (the negative value of) $F_{i,t}$, the firm will use it up to reduce the debt or to reduce the impact of F^I in case of loss. Otherwise, liquid funds will be reduced in the amount of $F_{i,t}$. In any case the investment in new capacity, as well as that eventually designed to technological updating, will amount to zero.

(ii) The amount of *total financial funds* available for investment is *positive or null* ($F_{i,t}$≥0).

Two situations may happen:

a) available funds are *less than or equal to* desired investment. In this case, the firm will invest the amount available, taking into account the technical indivisibility of investment (the minimum unit of capacity was set as 10 production units in the model). Effective financial flows (external funds plus liquid assets investment) will equal the values that entered in the calculation of $F_{i,t}$, and possible residues due to fixed capital indivisibility will be used to increase liquid assets;

b) available funds are *greater than* desired investment. The firm will be able to invest the desired amount, and the remaining surplus will be destined (when required) to technological updating of the equipment. If these funds are completely used up, effective financial flows will equal their initial values.

If after this stage there still remains some liquid surplus, its destination will depend on its amount. The firm is supposed to use its own funds in the first place, so if the surplus is greater than or equal to the external available funds, the firm will not run into new debt and any remaining surplus will be kept as liquid assets. Conversely, should the external funds be greater the same rule would apply: the firm will only use external funds to the amount strictly necessary to invest.

Block 6: Frontier shift and technological search

Technological search by any firm is accomplished through *process* R&D. The assumption made here is that the industrial sector being modeled introduces technical change basically embodied in the equipment ordered, but at the same time internal R&D is assumed to be crucial for design and technical improvement of the equipment, through learning-by-doing (in K. Pavitt's (1984) taxonomy, it would correspond closer to "scale intensive", with some elements of "science based", sectors). The innovation and diffusion (imitation) processes follow closely those 2 stage processes proposed by Nelson & Winter (1982, ch. 12)[49].

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48 $A^*_{i,t}$ may also be positive or negative, depending on an increase in liquid resources being needed. When it is negative, the firm has additional resources available for investment.

49 In the simulations, among the total of 10 firms three kinds of firms were assumed to exist: firms numbered 1 to 3 are "strong" innovators (higher R&D innovative spending than imitative spending); those numbered 4 and 5 "weak" innovators (the inverse proportion); and firms 6 through 10 are imitators (only imitative R&D).

The equations are as follows:

(a) Productivity associated with an *imitation* draw:

1st. stage – choice of best practice (and corresponding productivity $\pi^M_{i,t}$ *) to imitate*:

$$\pi^M_{i,t} = d_m \max_{i,j}(\pi^0_{i,j,t}) \qquad (29).$$

2nd. stage – probability of imitative success:

$$\Pr(d_m = 1) = 1 - \exp(-\rho_{m,i} p_{i,t} x_{i,t} a_m) \qquad (30),$$

where d_m is a dummy variable representing success (d_m=1) or failure (d_m=0) of the imitative draw; ρ_{mi} is the share of revenue spent in imitative R&D; and a_m is a sector-specific exogenous parameter of "technological opportunity" of imitative success[50].

(b) Productivity associated with an *innovation* draw:

1st. stage – probability of innovative success:

$$\Pr(d_n = 1) = 1 - \exp(-\rho_{n,i} p_{i,t} x_{i,t} a_n) \qquad (31),$$

where d_n is a dummy variable representing success (d_n=1) or failure (d_n=0) of the innovative draw; ρ_{ni} is the share of revenue spent in innovative R&D; and a_n is a sector-specific exogenous parameter of "technological opportunity" of innovative success.

2nd. stage – productivity obtained by innovation, $\pi^N_{i,t}$ *(only if* d_n*=1)*:

$$\log(\pi^N_{i,t}) \sim N(\mu; \sigma^2) \qquad (32),$$

where μ and σ are given exogenously.

The final choice, that will define the productivity of the firm's "internal" best practice $\pi^F_{i,t}$, will be the technology with the highest productivity among available alternatives:

$$\pi^F_{i,t} = \max(\pi^F_{i,t-1}, \pi^N_{i,t}, \pi^M_{i,t}) \qquad (33).$$

Block 7: Income generation

The unit surplus, s_t, can be obtained subtracting from price the indirect taxes and unit costs; however, its total amount can only be defined after sales:

$$s_t = (I - \hat{\tau}^i)p_t - \varphi_t \qquad (34),$$

where $\hat{\tau}^i$ is the diagonal matrix of indirect tax rates, charged over the sector's revenue. Total surplus is defined *ex post*, multiplying its unit value by total sales. The aggregate amount of surplus in the economy, TS_t, will be the sum of all sectors' surpluses:

$$TS_t = s_t^T x_t^v \qquad (35).$$

The total wage, TW_t, analogously, is defined aggregating wages through sectors, including public sector wages W^g. To simplify, a flexible labor contract is implicitly assumed, that is, the amount of

[50] To be precise, this is not the only variable reflecting the degree of technological opportunity of a given technology: the exogenous productivity growth of the best practice may be interpreted in a similar way, even more so since Nelson & Winter. To avoid ambiguity we decided to call the latter effect simply as "productivity gains" of the technological frontier.

labor employed is determined by the level of production and there is no labor supply constraint. Wage unit is subject to change every four periods according to changes in each sector's average productivity.

$$w_{jt} = w_{jt-1}\left(1+\gamma^{w}\left(\frac{\overline{\pi}_{jt-1}-\overline{\pi}_{jt-5}}{\overline{\pi}_{jt-5}}\right)\right) \quad (36),$$

$$WT_t = w_t x_t + W^g \quad (37).$$

The GDP in each production period is given by the sum of total wage and surplus with the indirect taxes:

$$Y_t = TS_t + TW_t + T_t^i \quad (38).$$

Finally, the two functional income classes must be converted into *m* personal income classes. This is done by a matrix (*m* x 2) of personal income appropriation (personal income class x functional income class), as in the equation below:

$$y_t^d = (I - \hat{\tau}^d)R\begin{bmatrix} DP_t \\ WT_t \end{bmatrix} \quad (39),$$

where $\hat{\tau}^d$ is the (*m* x *m*) diagonal matrix of the income tax rate and DP_t is total distributed profits.

The real income of each class, $y_{h,t}^r$, is determined by deflating the corresponding money income by a class-specific consumer price index ($CPI_{h,t}$):

$$y_{h,t}^r = \frac{y_{h,t}^d}{CPI_{h,t}} \quad (40),$$

where the index is a Paasche one whose weights are given by the marginal propensity to consume domestic and imported goods by each class.

Block 8: Public and foreign sectors

8A. Public sector

Government, as mentioned above, is introduced in a partially exogenous and very simplified way. The main components of this block are government expenses and tax revenues (income and indirect). These expenses are determined with a surplus target[51], by the difference between the expected taxes (corrected by the expected growth rate) and the target surplus, subject to a minimum level determined by the amount of public sector real wages. The former is set in the initial conditions as some fixed proportion of the government expenses and it then increases at an exogenous rate. The excess of government expenses over public sector wages, g_t^g, is equally divided into consumption and investment:

$$G_t = g_t^{T*}T_{t-1} - \xi Y_{t-1}$$

[51] A rule for changing endogenously this target may be subject to simulations.

$$C_t^g = \left(G_t - W_t^g\right)c^g \qquad I_t^g = \left(G_t - W_t^g\right)\boldsymbol{\kappa}^g \qquad (41);$$

where C_t^g e I_t^g are (p x 1) vectors; c^g *and* κ^g the proportions.

Government revenue is obtained from indirect and income (direct) taxes. The indirect taxes are paid by sectors according to their sales proceeds. Income taxes are applied over the total amount of personal income of each income class, with class-specific rates.

8B. Foreign sector

The second partially exogenous block is the foreign sector, composed, in this version, exclusively by the trade balance, supposed to be identical to foreign balance of payments[52][53].

Exports are determined by a fixed coefficient, χ_i, over the "rest of the world" income, Y_t^x (measured in domestic currency) and the corresponding income elasticity on the world market, η_i. This simple form captures both the general international situation, expressed by the world income, and the sector-specific conditions expressed by the export coefficient and elasticities. In order to define the exports in terms of units of output, this value is divided by each sector's average price:

$$e_{z,t}^x = \chi_z \left(\frac{er_t p_{z,t}^*}{p_{z,t}} \right)^{\varepsilon x} \left(Y_t^x\right)^{\eta_i} \qquad (42).$$

Aggregate exports are:

$$X_t = p_t^x e_t^x \qquad (43),$$

where p_t^x is the (1 x p) vector of export prices (in domestic currency) and e_t^x the (p x 1) vector of exports from each sector.

Imports are determined in the same way as the domestic demand. The intermediate goods imports are defined by technical coefficients and planned production; the consumer goods imports are defined by a linear function with increasing lags[54] and decreasing marginal propensity; capital goods imports were already explained.

Aggregate imports value in domestic currency is given by the sum total imports by each sector multiplied by the respective international prices and the exchange rate:

$$M_t = er_t\left(p_t^m e_t^m\right) \qquad (44),$$

where p_t^m is the (1 x p) vector of international prices[55] and e_t^m the (p x 1) vector of imports of products corresponding to each sector. Finally, it is possible to determine the trade balance in domestic currency, which in this preliminary version of the model will be equal to the balance of payments.

[52] It does not include compensatory capital flows. Contrary to many models of balance of payments constraint, no assumption is made about the trade balance.

[53] The other components of foreign balance of payments will be introduced in a later version of the model.

[54] The same lags as in domestic consumption.

[55] International prices are subject to exogenous rates of change, higher for intermediate goods.

Block 9: Entry and Exit

In this model, the total number of firms is not constant, but entry only occurs in case of exit. This is not so realistic, but the sectors are considered to be in their "mature" state and only its main firms are modeled. Exit of incumbent firms is completely endogenous and may result from two causes: (i) market share, or productive capacity, falls below a critical level; and (ii) the firm's rate of debt on capital exceeds a given risk threshold.

The possibility of entry of a new firm is limited to an exit caused by financial problems, which can be seen as a proxy to a take over or a merger. Market conditions, however, are also taken into account: an entry may only happen when there is steady growth, defined by a minimum of periods when orders have been increasing in the sector. Besides, even if there is market turbulence (defined by the exit of more than one firm), the number of entering firms is not allowed to exceed one by each investment period. The initial conditions of new entrants are set arbitrarily: its market share is made equal to the average of exiting firms, and price and productivity equal to sector average.

4. Preliminary simulation results

The simulation results presented in this section only give a very brief idea of the trajectories generated by the model. Further work will provide a more systematic analysis of the parameters and initial conditions, as well as of the time series generated by the simulations. Also, given the size of the model, a deeper analysis will be made on each block separately. Nevertheless, since the stochastic part of the model is very limited, the results of each simulation run based on the standard or "benchmark" conditions are analytically relevant[56]. Models like this are basically deterministic, although highly path-dependent, being more sensitive to initial conditions than to the random seed of the stochastic components.

In the benchmark setup we assume p=4 and m=4. The sectors are: consumption (one), intermediate (two) and capital (one), with different number of firms, 10 for the last three sectors and 20 for the consumption goods sector. All firms are identical in each sector except for technological and price strategies, according to which they may be divided in three groups: (i) strong innovators – which allocate a larger part of R&D expenses to innovative search and put a higher weight on desired price; (ii) weak innovators – a smaller part of R&D to innovative search, but also a higher weight on desired price; and (iii) pure imitators – all R&D to imitative search and higher weight on average price.

As in the original multisectoral model of Possas (1984) and in the tradition of Kalecki (1954), the main macrodynamic result of the simulations using the benchmark setup is the cyclical behavior of the GDP (fig. 1) over a positive trend. Fluctuations are relatively stable, although their pattern, as expected, is more complex than in aggregate analytical models. These results can be observed in many different simulations, as shown below. To capture the main results, trend and cycle will be discussed separately.

Before turning to the aggregate time series analysis, some results at the micro level deserve being reported. The first three sector variables shown in fig. 6 - average sector price, average sector productivity[57] and unit wage - are closely related. Capital goods sector, one with the greatest technological opportunity parameter, followed by the two intermediate good sectors, after a few

[56] In fact, a simple sensitivity exercise was made. Each initial condition was replicated 10 times and the results of the main aggregate time series were basically the same, of which some results will be shown. The ones chosen are considered to be representative runs, in the sense that they exhibit intermediate values for the main series. When possible, results from different runs will be compared.

[57] Since there are different "vintages" of capital goods, this is calculated as the average of each firm's average productivity.

periods exhibits the highest average productivity, lowest price and highest unit wage. As shown above, the increase in wage follows the increase of productivity with some lag.

In fig. 7, some indicators of industrial structure and their change over time are shown: a concentration index (inverse HHI), number of firms, degree of asymmetry in firms' performance and market turbulence. The last two were suggested by Dosi *et al*. (1995), the former being a measure of dispersion of the level of competitiveness of firms[58] and the latter defined in terms of changes in market share involving incumbents, exiting and entrant firms[59]. The concentration index and the number of firms taken together show that the sector starting with the larger number firms - consumer goods - increases concentration at a much faster rate. Market turbulence indicator shows the highest frequency of peaks in capital goods, due to the exit of firms highly indebted possessing significant market share as compared to exiting firms in other sectors. The absence of such peaks in consumer goods results from the fact that in this sector exit is mainly related to the lack of technological success. Another interesting result is that, while sudden reductions in the degree of asymmetry in most sectors are associated with exit, in capital goods sector, where technological opportunity is the highest, it is more volatile and basically independent of entry and exit of firms.

4.1 Cycle

In order to analyze the variables at the business cycle frequencies and compare their co-movements we applied a bandpass filter[60]. Two general theoretical points deserve attention. Firstly, just like the traditional neo-Keynesian and Kaleckian models, the regularity of the main observed fluctuations may be explained, in very general lines, by the lagged dual effect of investment, stimulating demand in the short term through multiplier effects and adding productive capacity in a longer term, whose eventual utilization may exceed or lag the desired level, propagating the original impulse (fig. 2). Secondly, a comparative analysis of simulations under different assumptions has shown that the relative stability of the fluctuations, unlike traditional aggregate neo-Keynesian models, is due to a much more complex investment function, where the usually explosive accelerator effect is balanced by the influence of the degree of capacity utilization and by a very effective financial constraint. It is important to notice that the financial credit constraint applies at the *micro* level, as explained above in block 5B, and not at the *macro* level, due to some supposed shortage of "savings" and resulting rise of interest rates[61], as usual in mainstream theory and models.

As can be seen in the results ahead, the cyclical movements seem to replicate some important stylized facts. As Stock and Watson (1998) do for their empirical data, it is possible to analyze the co-movements of the main macroeconomic variables generated by the model by means of the cross correlation of their trajectories at business cycle frequencies to a benchmark variable, representing the business cycle at various leads and lags. As usual, we applied filtered GDP as the benchmark variable and computed cross-correlations at all leads and lags from t-5 to t+5. Based on cross-correlograms a variable may be classified: (i) depending on their sign and shape as "procyclical", "countercyclical" or "acyclical"; (ii) acording to their peak value as "leading", "lagging" or "coincident". As in Stock and

[58] The one used here is the standard deviation of the ratio between the competitiveness of each firm and the sector average.
[59] Calculated as the sum of the absolute value of the change in market share plus the market share of the exiting firms.
[60] Baxter, King (1995). The parameters of the filter were the usual ones (6, 32, 12). Other values were tested with similar results.
[61] As explained by Keynes (1937, p. 211): "the novelty in my treatment of saving and investment consists, not in maintaining their necessary aggregate equality, but in the proposition that it is, not the rate of interest, but the level of incomes which (in conjunction with certain other factors) ensures this equality". Given the causal relation mentioned in the first section of this paper, it is the decision to invest, combined with consumption and other sources of demand that set the level of income, and not the other way around. Therefore, no trade-off between real consumption and investment (and other aggregate demand components) can arise; they jointly set the level of (money and real, given the price level) income, which is never given in advance, while investment (together with public deficit and trade surplus) determines savings as a mere *ex post* residue.

Watson, we calculated the correlation between x_t and y_{t+k}, where x_t is the bandpass filtered series listed in the figures below and y_{t+k} is the k-period lead of the filtered log of real GDP. As to the sign and shape, a large positive correlation at k=0 indicates procyclical behavior of the series, and a large negative correlation indicates countercyclical behavior. As to peak values, a maximum correlation at a negative k indicates that the cyclical component of the series tend to lag the aggregate business cycle by k periods, and vice versa.

The first important result is the co-movements of real GDP, consumption and investment. Consistent with well-known stylized facts, the latter fluctuates more and the former less than GDP, *i.e.* investment is much more volatile than consumption (fig. 3). As can be seen in the cross-correlograms below (fig 4), although both consumption and investment are procyclical, investment is a leading while consumption is a lagging variable, as should be expected in a Keynesian model.

The analysis of the remaining variables is also important to give a wider picture of the model. Employment is a procyclical and coincident variable, which is a consequence of the characteristics of the labor market described before: the hiring process is completely flexible and there is no excess supply of labor; at any period firms can freely hire and fire workers. Government expenses are a procyclical and lagging variable, as a result of the fiscal policy assumed in this first version of the model, according to which any countercyclical role of government spending is ruled out. Finally, given the way exports were introduced, they are acyclical: at leads the correlation is negative and at lags positive, but in neither case is its absolute value greater than 0.5. The main dynamic impact of exports is on the trend component. Although not reported here, these results are independent of the parameters of the filter.

In a first attempt to explore the expectations formation, different values of the expectations parameter γ were tested, and the results are found in fig. 8-11. As commented before, this parameter influences the amplitude of the cycle, but it does not affect the trend. This is shown in fig. 8, where the log series of GDP, investment and consumption are plotted, giving contrasting results for different values of γ. Fluctuations are the most volatile for the highest value of γ, which is confirmed by the standard deviations of the cyclical components in fig 9, while the average rate of growth[62] seems to remain unaffected by this parameter, as seen in fig. 11. Finally, note that besides this volatility effect, changes in γ do not affect the cross correlograms (fig. 10).

Another important parameter of the model which influences the cycle is the lag of the investment period, that is, of how many production periods it is consisted. The same figures shown above are reported here (figures 12-15) comparing two other values for this lag, 4 and 8, with the standard value of 6. As the expectation parameter, the change of the lag does not alter the trend as can be seen in fig. 12 and confirmed by fig. 15 of the average growth rates. On the other hand, as can be seen in fig. 13, the standard deviation of the cyclical component of each series the change from 6 to 4 influences the volatility if the cycle of all major aggregate series, while the change from 6 to 8 has a major influence on the productive capacity utilization, but not so much on the others. However, contrary to the change on the expectation parameter, the cross-correlogram seems also to be affected by the change in the investment period, not in the sign or peak, but in its values.

4.2 Trend

Amongst the main possible determinants of long term trend[63] in the neo-Keynesian and Kaleckian traditions, one should consider as the main ones the aggregate demand effects of structural changes, expressed as autonomous changes in (i) investment, particularly related to innovations; (ii) exports;

[62] Average growth rate (AGR) is calculated as the difference between the average of log values of GDP in the last ten production periods and the average in the first ten periods divided by the total number of periods.

[63] For a more robust analysis a low pass filter should have been applied to isolate the trend component. However, even a rough visual description is sufficient to point out some results that will be briefly commented.

(iii) government expenditures, and (iv) consumption. Although a more detailed analysis is still required, at least a brief account of each one in our simulations can be provided.

In this version of the model[64] these autonomous expenditures, mainly exports, are playing the major role in the long run growth, as shown in fig 16 and 19. In the case of exports, a simple relation between income elasticities of exports and imports allows for such behavior: as domestic income increases pulled by export growth, imports increase cannot offset the export drive. It should also be noted that the growth rate of exports seems to leave unchanged the cyclical component, as can be seen in both fig. 17 and fig. 18. The only partial exception is the volatility of the degree of productive capacity utilization, which seems to reduce when foreign demand is stable.

As to government expenditures, although the surplus target changes in a pro-cyclical way, potentially producing counter-cyclical effects, these are not enough to level off the cyclical movement, as shown before. Some simulations, not reported here, showed that when the surplus target is fixed, the effect of government spending is just to intensify the amplitude of the cycle. An autonomous consumption growth was not simulated, although it would be logically expected to increase to some extent the long run growth trend.

Finally, as is well known in theory, investment related to innovations is considered to be one of the most important autonomous demand components capable of generating an increasing long run trend[65]. However, fig. 16 shows that when the rate of growth of exports is zero, the long run trend disappears, suggesting the limited impact of this component on long run growth. In current neo-Schumpeterian models, which focus mainly on the *supply* side, the main element of the long run trend is technological innovations, but in a different sense than in Kalecki's and Keynesian models, as well as in the present model, where the *demand* side is the main dynamic drive of the economy. In the presence of innovations, demand could seem not to play a role of its own neither in the trend nor in the cycle (as in RBC models). But in order to generate long run growth, from a Keynesian macro perspective, innovations must necessarily influence one of the demand components - in this case mainly investment, but possibly also consumption, provided product innovation is taken into account. (which is not the case in this version of the model).

When such demand side effects are not explicitly considered, models based on technological process innovations tend to generate deflation, which could have a positive impact on consumption through a steady increase of real income. However, general price deflation is not usually observed in real economies and therefore any results strictly based on that should be taken with caution. What is most likely to happen as a general effect of process innovations is a change in relative prices, which may influence consumption in a positive way if and only if consumption goods tend to become relatively cheaper – which is far from an obvious result - according to relative productivity and cost changes. This relative price effect, if it happens, would also imply an increase on real income and would have the same effect on the growth trend as an increase in autonomous consumption. By the way, fig. 6 shows the opposite effect in the present model, where relative prices of consumption goods are increasing.

[64] We are now working on a different version incorporating product innovations as well, an important additional source of autonomous demand components.

[65] See Kalecki (1954), ch. 15.

APPENDIX I.- *Parameters and initial conditions*

Macro

Quarterly GDP	295
Quarterly Government Expenses	70
Government Wage Rate	0.8
Superavit Rate Target	0.025
Intersts Rates Debts	0.01
Interest Rates Liquid Assets	0.0075
Exchange rate	1

Income Classes

	Class A	Class B	Class C	Class D
Income	82	51	32	18
Profits share	0.6	0.25	0.1	0.05
Wage share	0.4	0.3	0.2	0.1
Income Tax	0.2	0.15	0.1	0

Sectors

	Consumption Good	Capital Good	Intermediate Good 1	Intermediate Good 2
Input Output Coefficients	Matrix	Matrix	Matrix	Matrix
Capital/Output relation	1	1	1	1
Labor Productivity (Vintages)	1	1	1	1
Desired Capacity Utilization	0.8	0.71	0.75	0.8
Desired proportion of stocks	0.1	0.1	0.1	0.1
Price Parameters	Matrix	Matrix	Matrix	Matrix
Desired Debt Rate	0.6	0.6	0.6	0.6
Desired liquidity rate	0.05	0.05	0.05	0.05
Profits distribution rate	0.5	0.5	0.5	0.5
Learning by doing Parameter:				
Bonus	0.05	0.05	0.05	0.05
Skill increase	0.05	0.05	0.05	0.05
Expectations Parameter	0.3	0.3	0.3	0.3
Mark up	1.5	1.6	1.6	1.6
Technological Parametes:				
Technological Opportunities	0.004	0.007	0.005	0.005
an	0.263	0.731	0.6	0.6
am	0.131	0.366	0.3	0.3
Sector Wage	0.22	0.18	0.18	0.18
Physical Depretiation Period	60	60	60	60
Payback Period	42	42	42	42

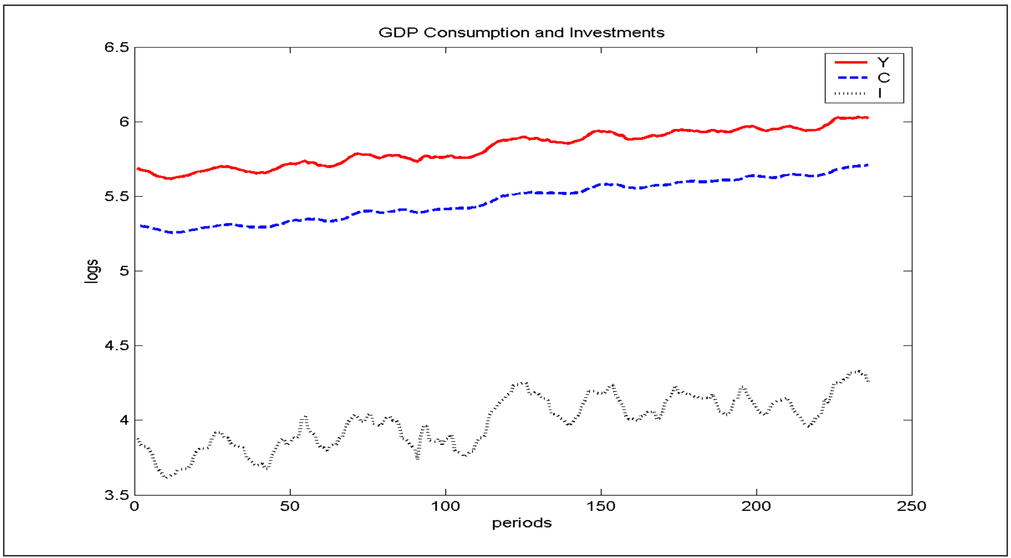
Firms

	Consumption Good			Capital Good			Interm. Good 1			Interm. Good 2		
R&D Effort	S	W	I	S	W	I	S	W	I	S	W	I
Price Equation Param	0.3	0.3	0.7	0.3	0.3	0.7	0.3	0.3	0.7	0.3	0.3	0.7
R&D Imitation	0.005	0.01	0.01	0.0175	0.035	0.035	0.015	0.03	0.03	0.015	0.03	0.03
R&D Innovation	0.01	0.005	0	0.035	0.0175	0	0.03	0.015	0	0.03	0.015	0

S – strong innovator, W – weak innovator, I – pure imitator.

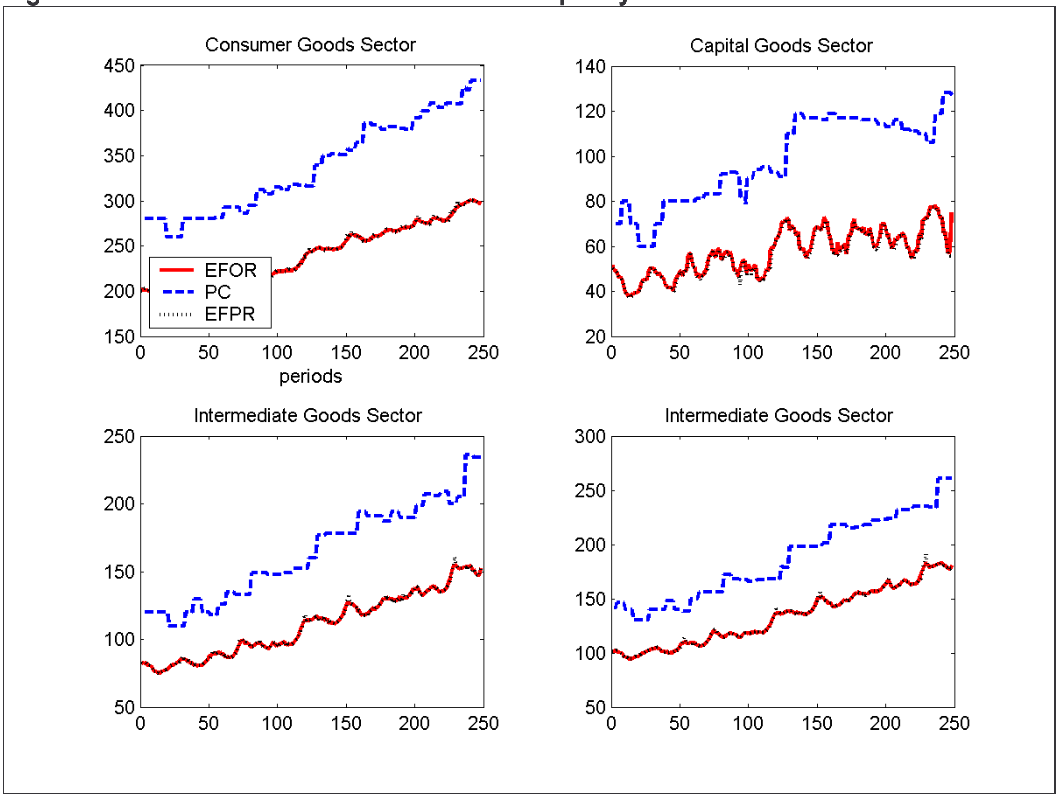
APPENDIX II. Figures

Figure 1 – GDP, Consumption and Investment



Variables measured in log terms

Figure 2 – Effective Orders and Productive Capacity



EFOR – effective orders; PC – productive capacity; and EFPR – effective production.

Figure 3 –Co-movements – variables at business cycle frequencies

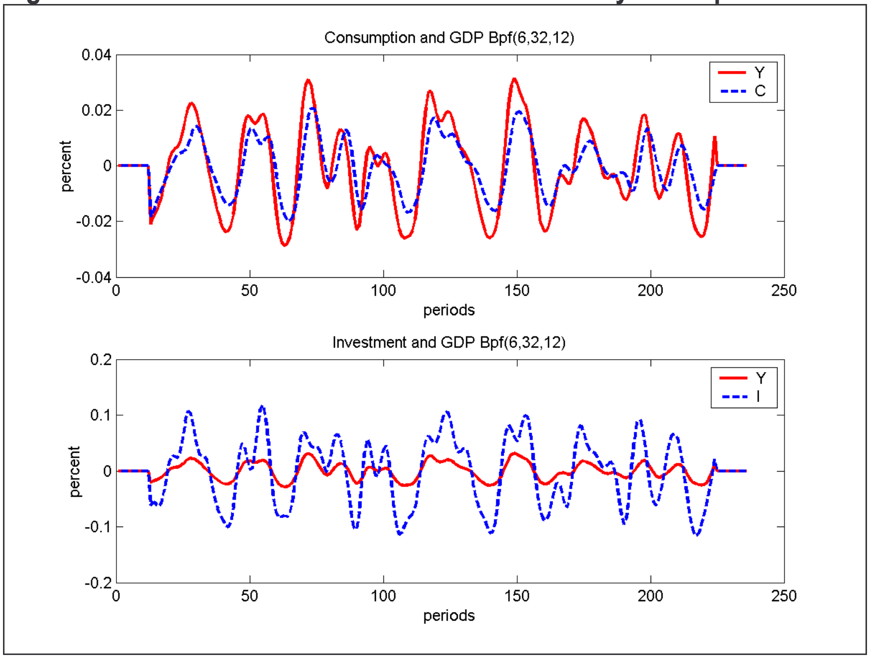
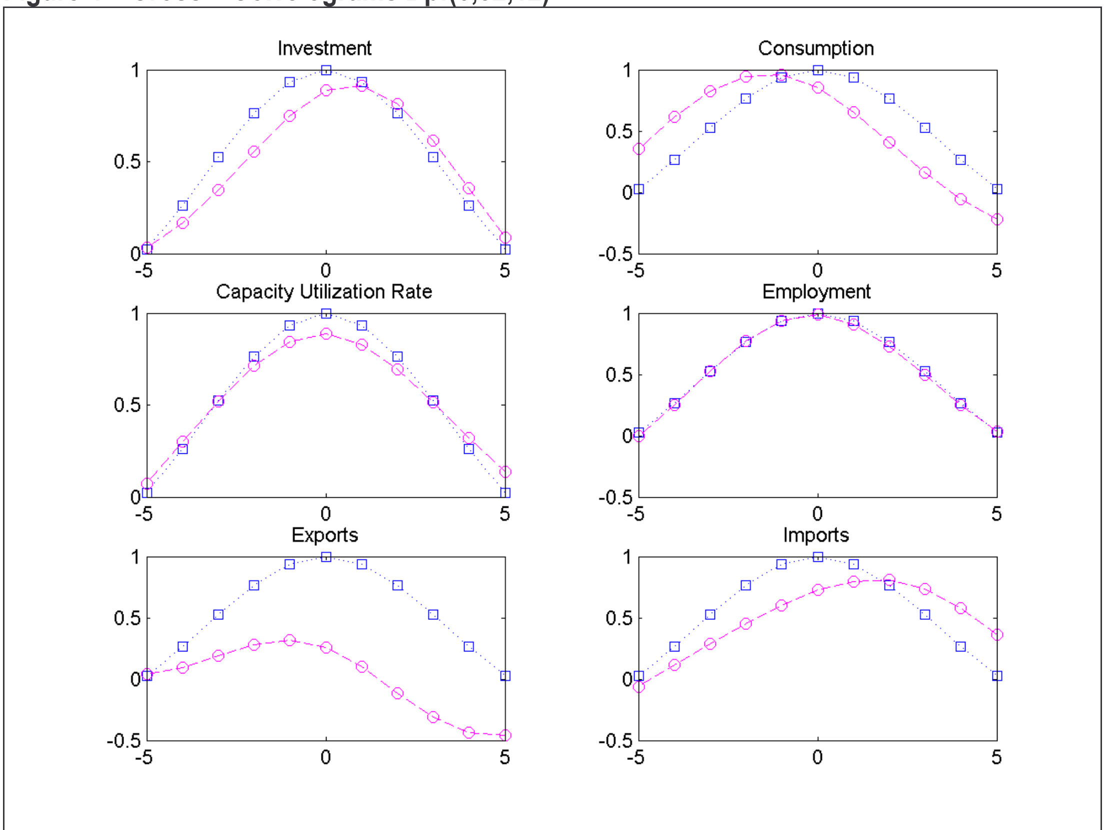
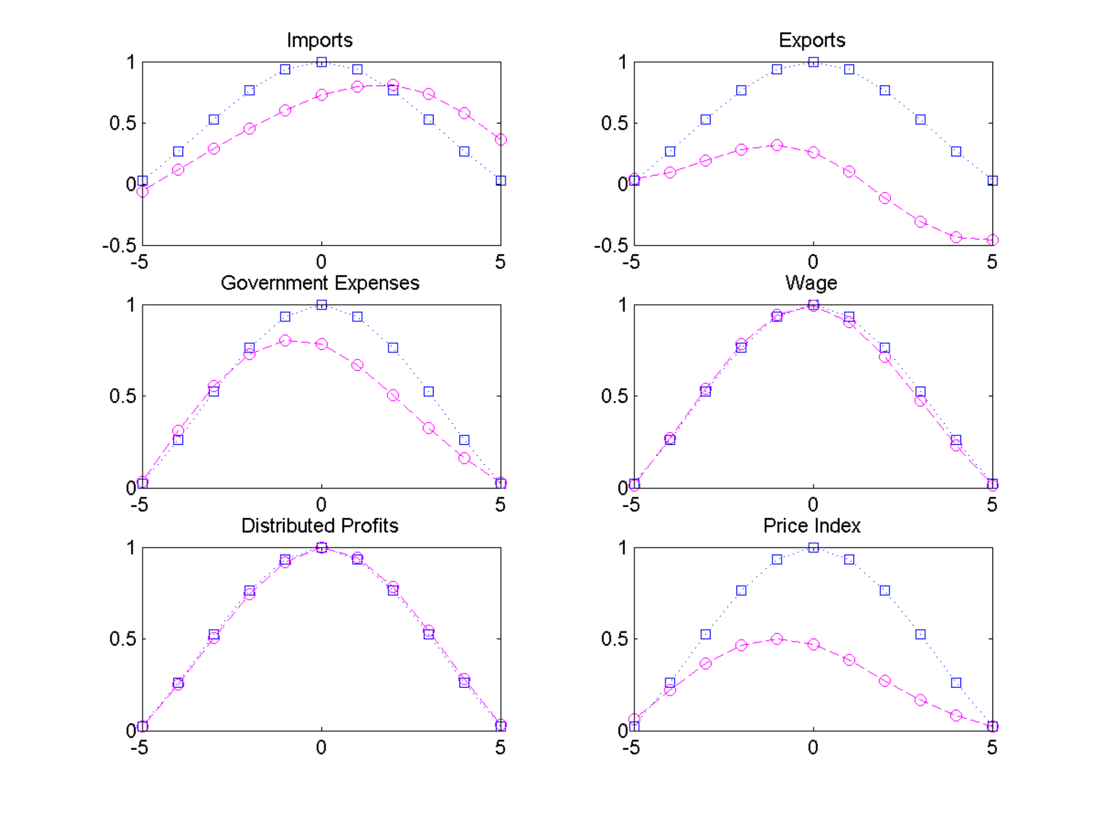


Figure 4 – Cross – Correlograms Bpf(6,32,12)



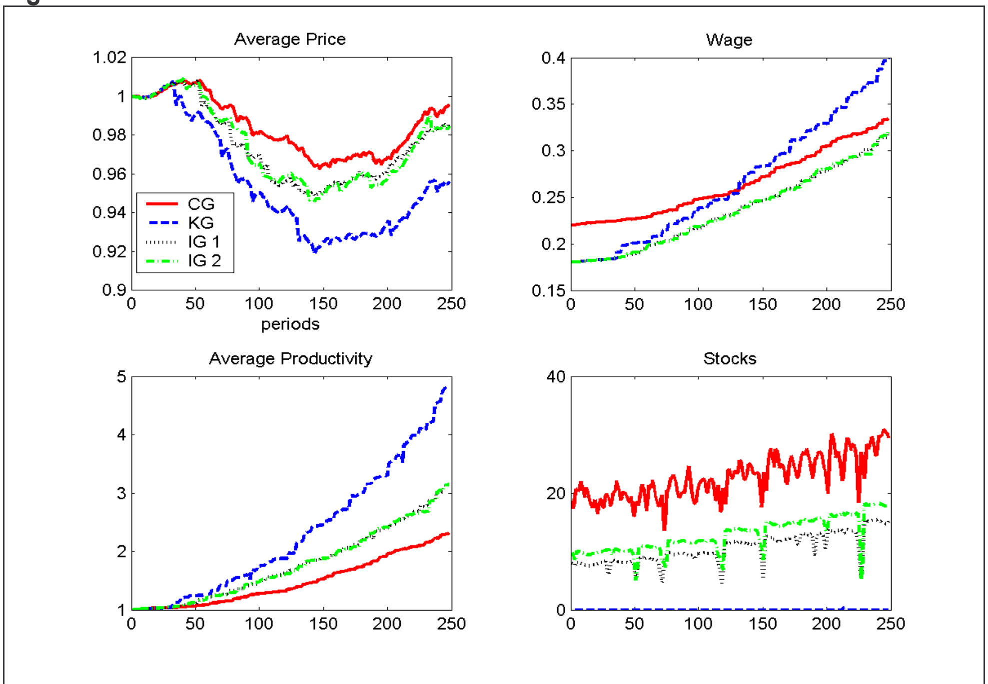
Squares - GDP auto-correlation; circles - Cross-correlations. These results are an average of the cross correlogram of 10 different simulation runs.

Figure 5 - Cross – Correlograms Bpf(6,32,12)



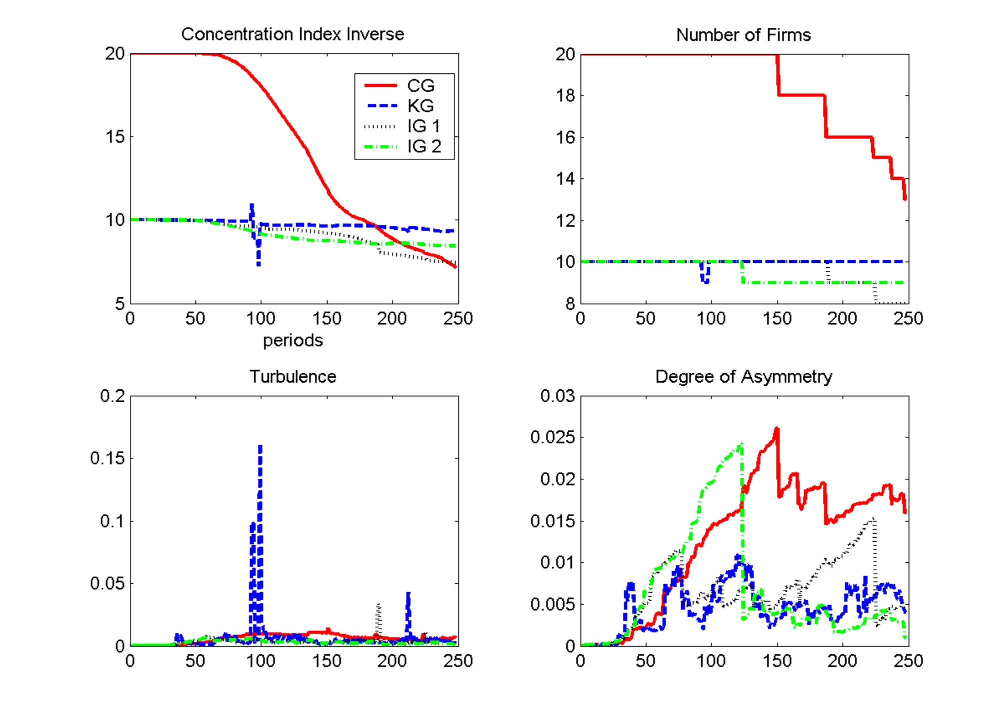
Squares - GDP auto-correlation; circles - Cross-correlations. These results are an average of the cross correlogram of 10 different simulation runs.

Figure 6 - Sector Variables



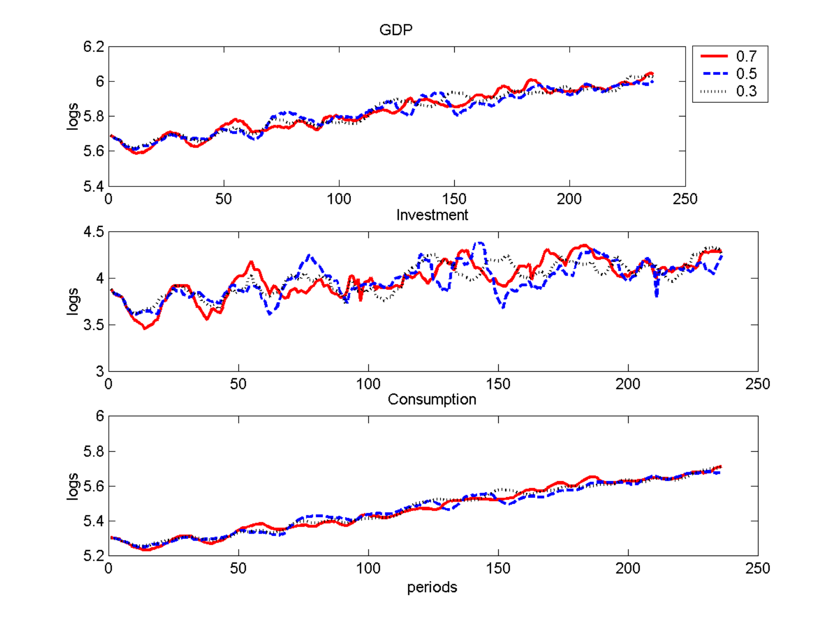
CG – consumption goods sector; KG – capital goods sector; IG 1 – intermediate goods sector 1; and IG 2 – intermediate goods sector 2

Figure 7 - Sector Variables II



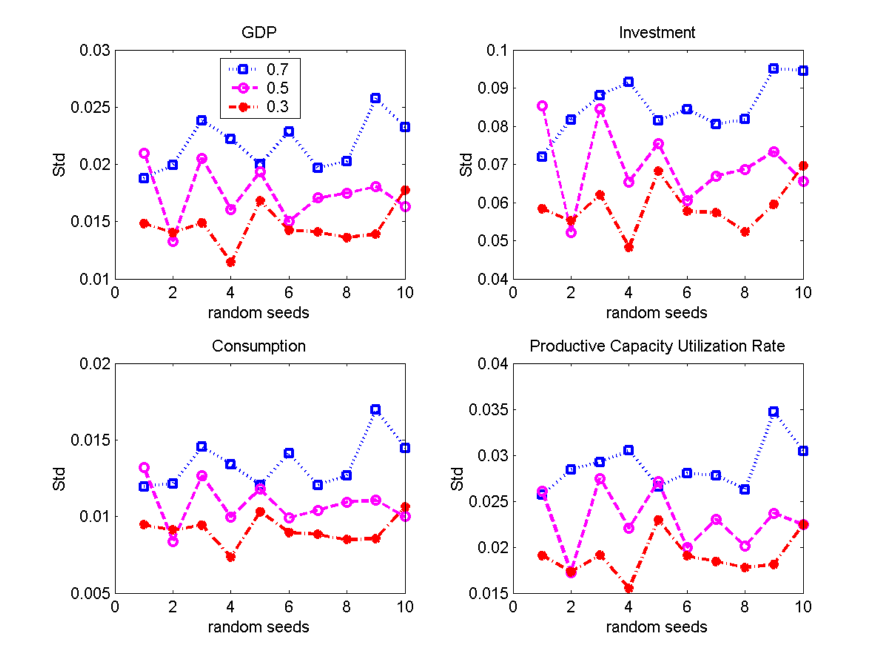
BC – Consumption Good sector; BK – capital good sector; BI_1 – Intermediate good sector 1; and BI_2 – Intermediate good sector 2

Figure 8 – Expectation Parameter



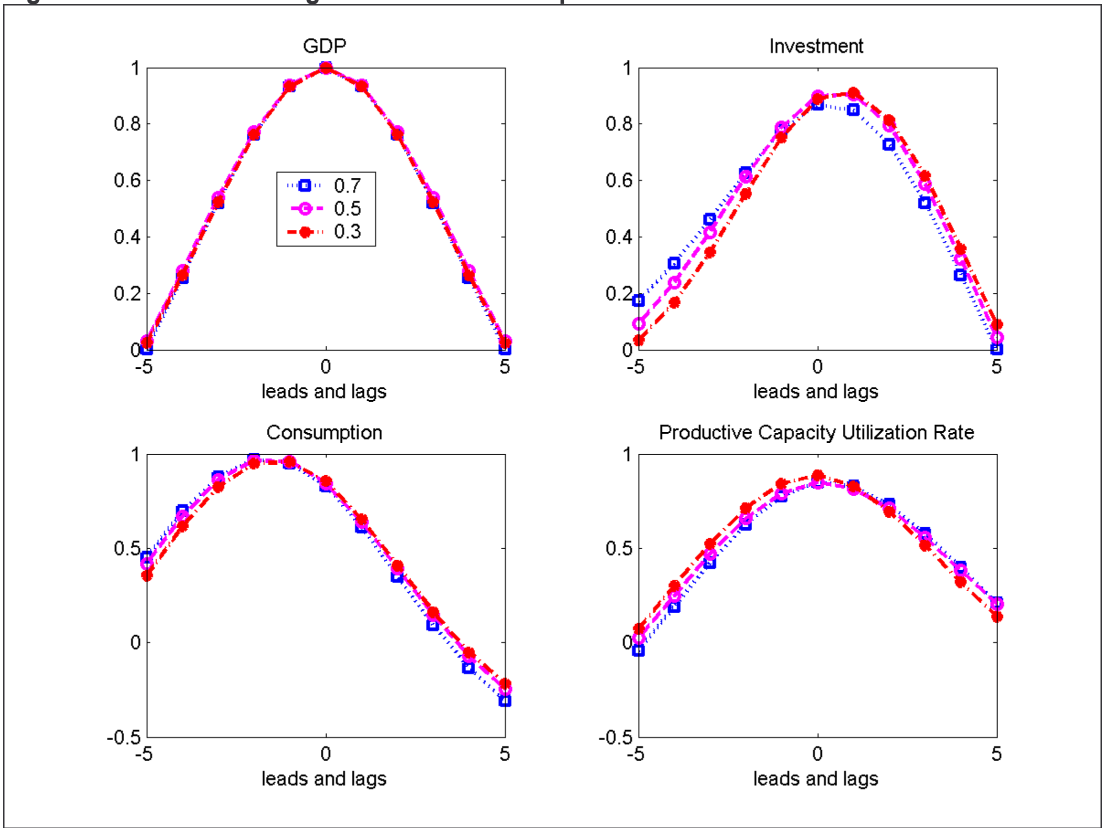
GDP, investment, and consumption with different expectation parameters γ (0.3, 0.5, 0.7).

Figure 9 - Standard Deviation for different expectation parameter



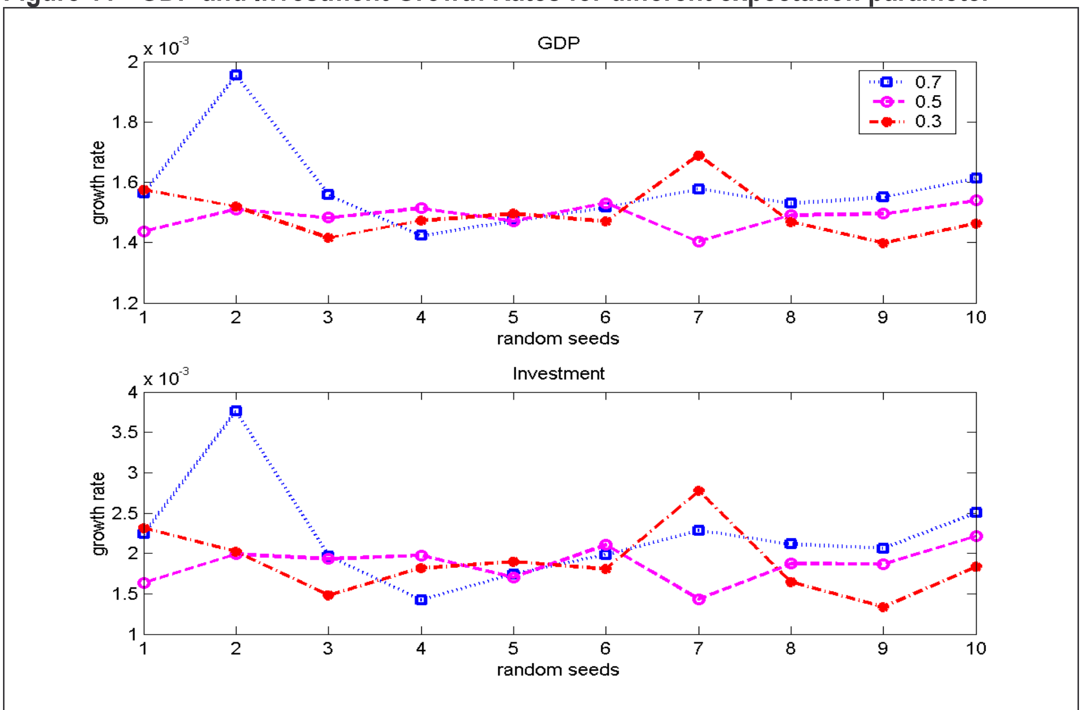
Standard deviation of detrended log GDP, log investment, log consumption and productive capacity utilization rate bpf (6,32,12), for different expectation parameters γ (0.3, 0.5, 0.7) and 10 different simulation runs.

Figure 10 - Cross correlogram for Different Expectation Parameters



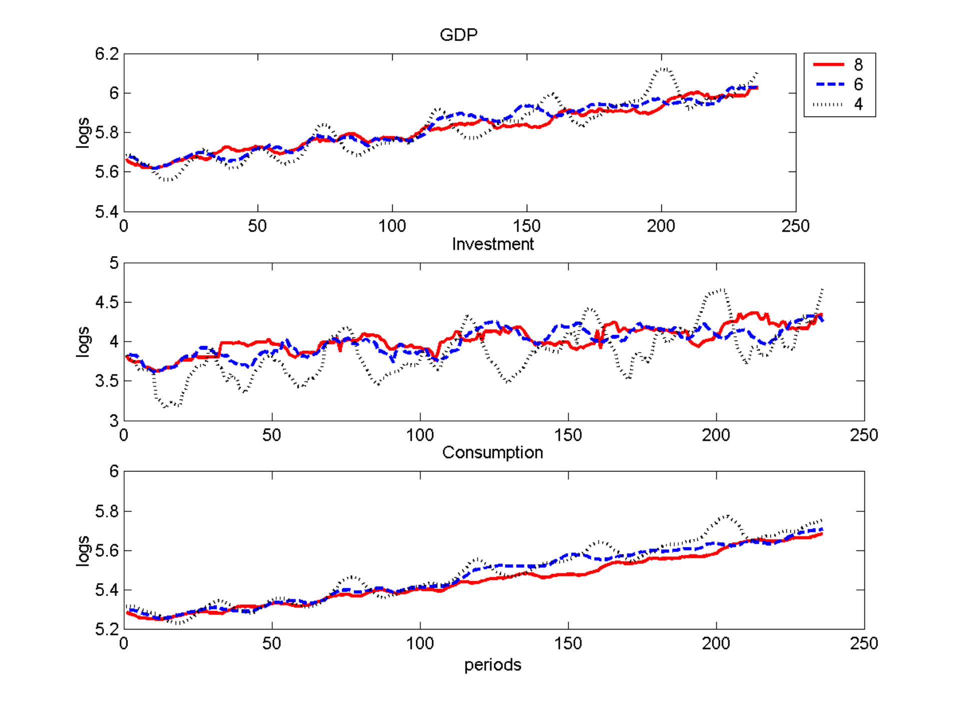
Detrended Variables in log terms, except for Capacity Utilization Rate, average over ten simulation runs

Figure 11 - GDP and Investment Growth Rates for different expectation parameter



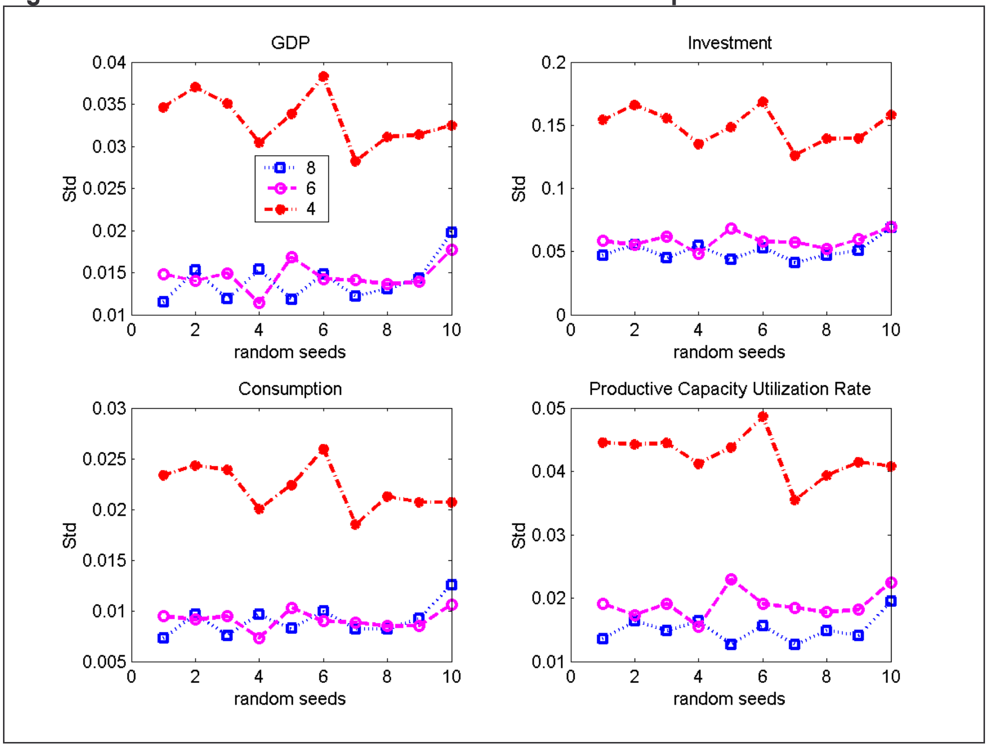
Average Growth rate of GDP and Investment for different expectation parameters γ (0.3, 0.5, 0.7) and 10 different simulation runs.

Figure 12 – Investment Period Lags



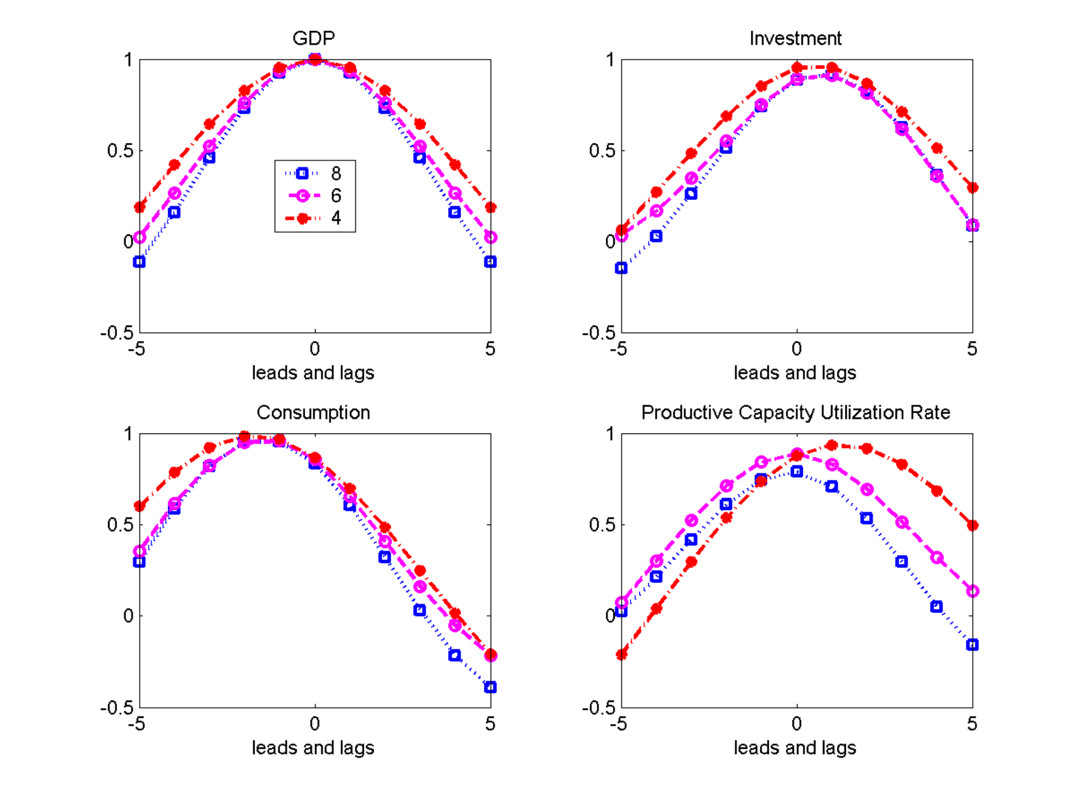
GDP, investment, and consumption for different investment period; each investment period consist of either 4, 6, or 8 production periods.

Figure 13 - Standard Deviation for different investment periods



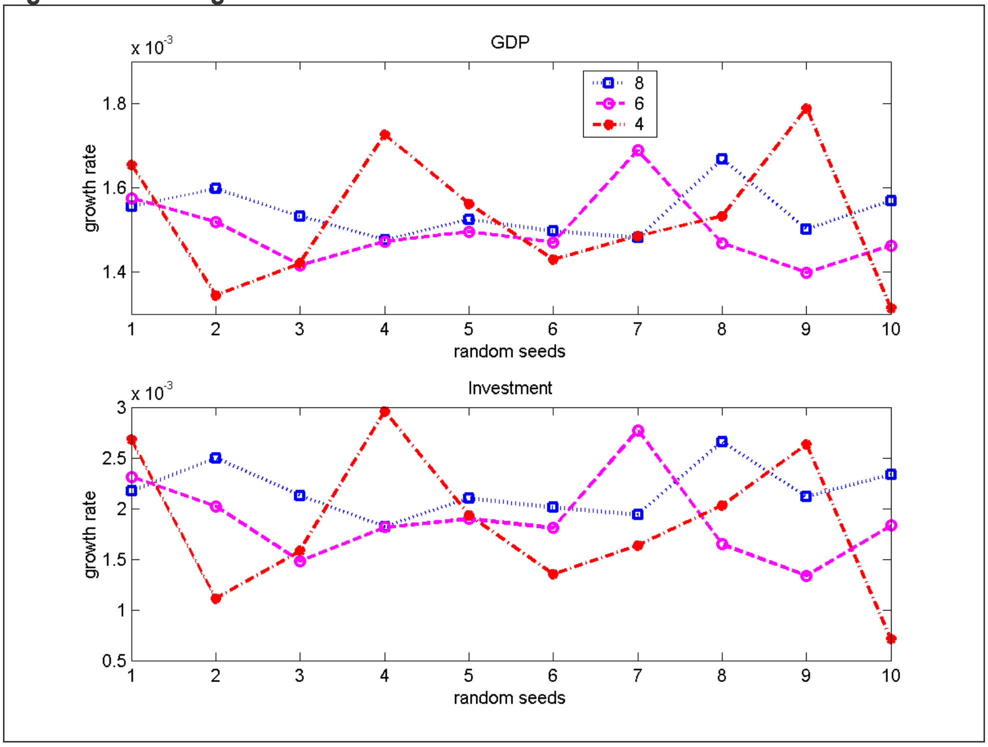
Standard deviation of detrended log GDP, log investment, log consumption and productive capacity utilization rate bpf (6,32,12), for different investment period lag (4, 6, 8) and 10 different simulation runs..

Figure 14 - Cross correlogram for Different Investment Period lag



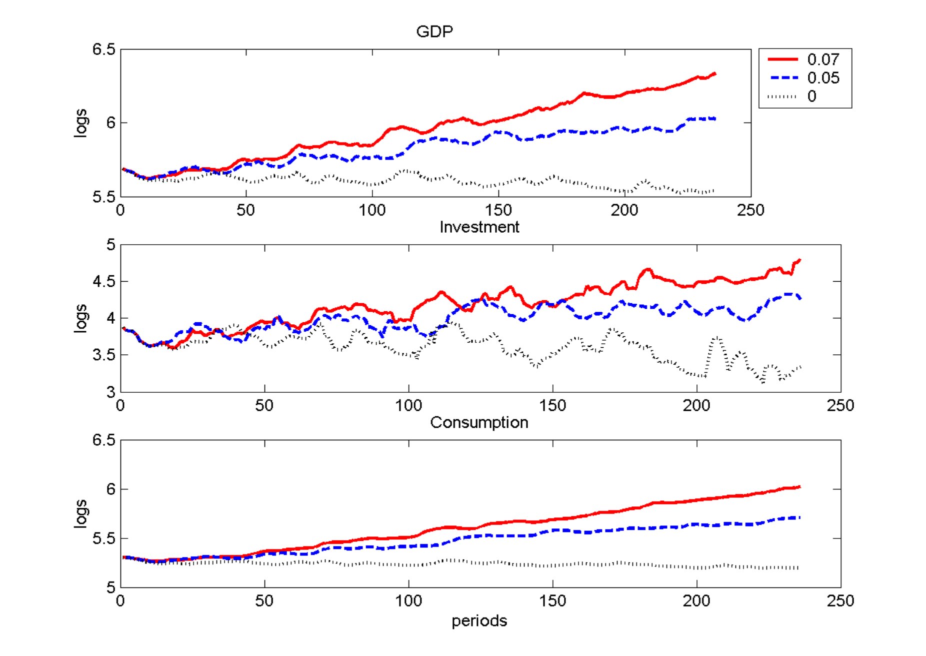
Detrended Variables in log terms, except for Capacity Utilization Rate, average over ten simulation runs

Figure 15 - Average Growth Rates for different Investment Period



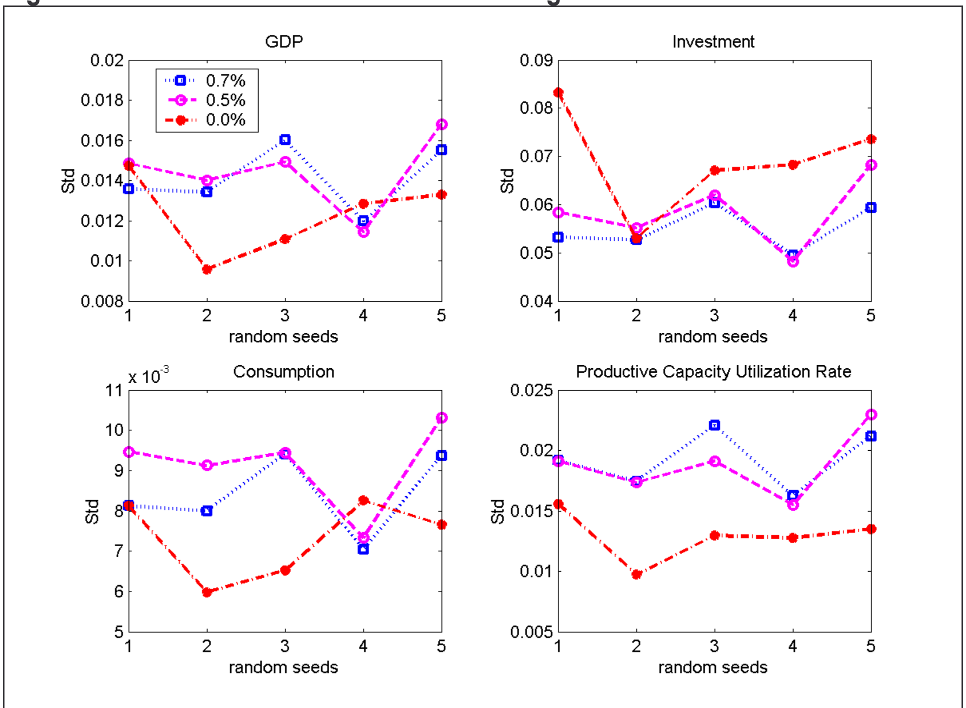
Average Growth rate of GDP and Investment for different investment period lag (4, 6, 8) and 10 different simulation runs.

Figure 16 – GDP, Investment and Consumptuion with different growth rates of external market



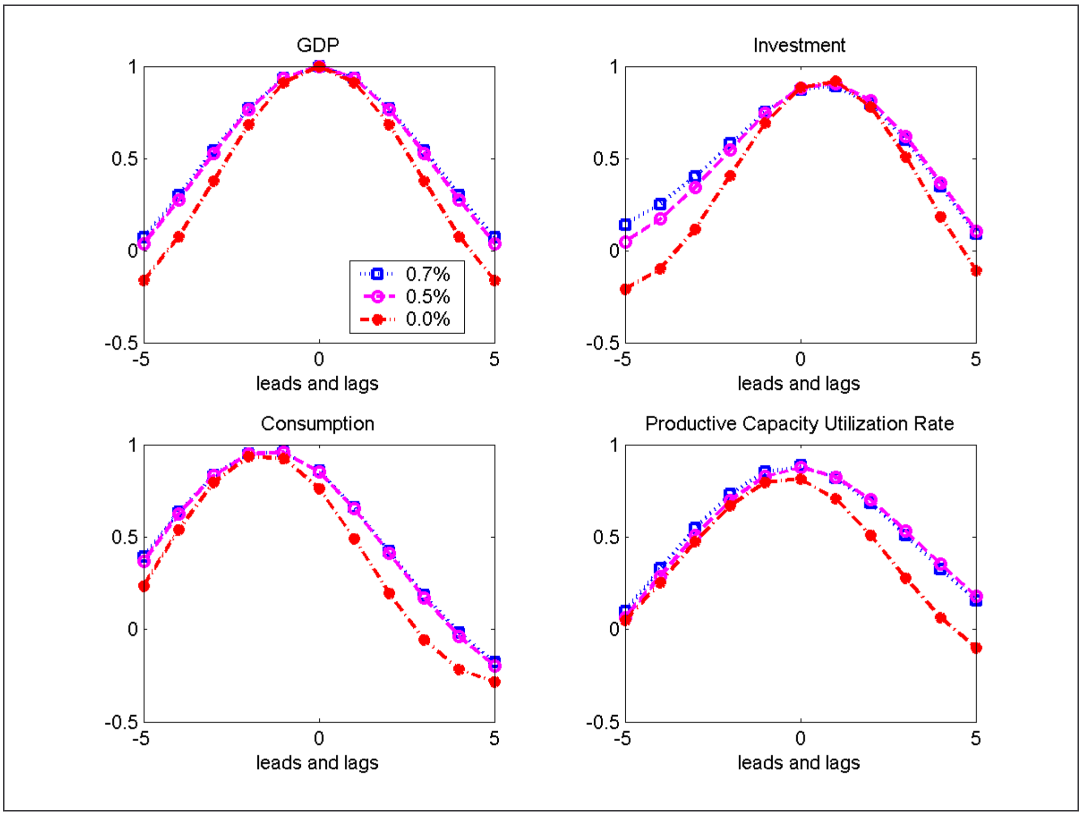
Variables measured in log terms

Figure 17 - Standard Deviation with different growth rates of external market



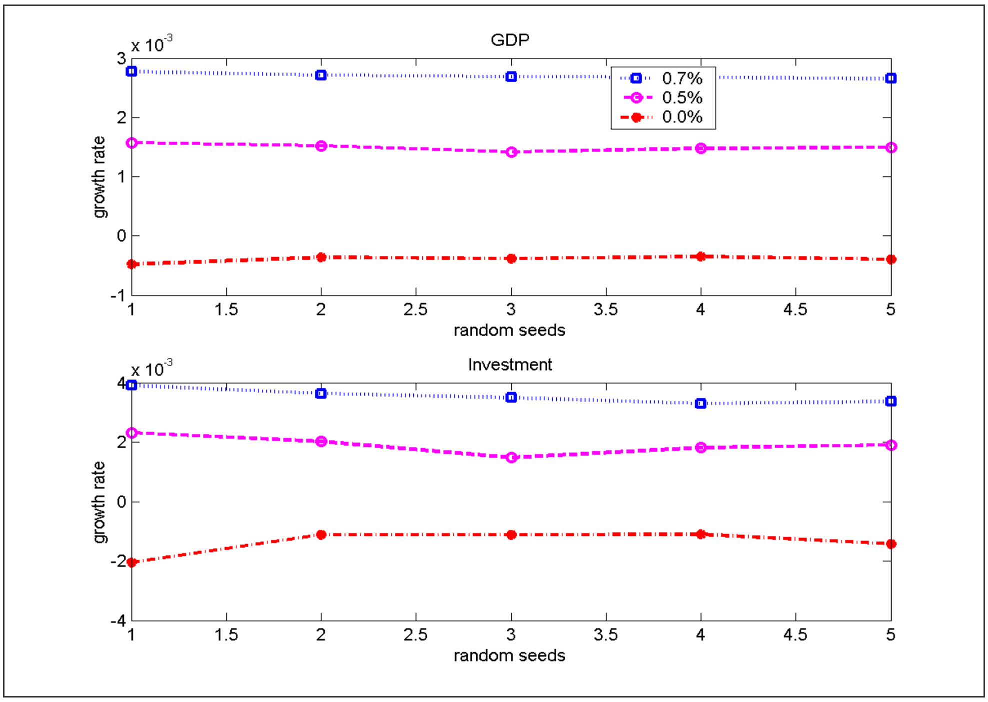
Standard deviation of detrended log GDP, log investment, log consumption and productive capacity utilization rate bpf (6,32,12), for different rate of growth of external market (0.7%, 0.5%, 0%) and 5 different simulation runs.

Figure 18 - Cross Correlogram



Detrended Variables in log terms, except for Capacity Utilization Rate, average over 5 simulation runs

Figure 19 - Average Growth Rate



Average growth rate of GDP and Investment for different rates of growth of external market (0.7%, 0.5%, 0%) in 5 different simulation runs.

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