

# On Complexity and Efficiency of Health Insurance Markets

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## Abstract

We propose a computational model of health insurance markets. Two types of agents are present. First, numerous simple agents representing patients use heuristics to select insurance plans or stay out of the market. Second, a small number of insurers behave strategically to design multi-part insurance plans, subject to exogenous regulator-imposed constraints. In our model, insurers compute the best response to the current market situations via recursive simulation logic: each of the insurers has an internal agent-based model of the market, isomorphic with one in which insurers are embedded themselves.

We study effects of different regulatory structures (number of insurers / insurance plans on the market, subsidies, level of coordination / revenue sharing between plans) on computational complexity of decision making of insurers and efficiency of market outcomes. We identify Pareto frontiers for both purely market-based as well as centrally coordinated markets. We find that, despite results of Papadimitriou [1994] and Axtell [2000] for exponential complexity of Nash equilibrium, coordinated (centralized) solutions to health insurance market are computationally tractable.

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## 1 Introduction

Rust [1997] considers the existence of a fundamental limit to knowledge in economics, imposed by the so called problem of economic complexity. A pivotal concern of economic complexity is the following question: does the computational complexity of economic calculations place inherent limits on the ability of economic agents to behave according to existing economic theories of optimization and equilibrium?

Rust argues that a negative answer implies that most existing economic theories are not relevant models of the economy and individual economic behavior. If there is an inherent curse of dimensionality underlying most economic calculations, then actual economic agents with limited computational capabilities would be unable to operate in complex environments with high-dimensional state spaces and choice sets. This would argue against the standard rationality hypothesis that economic agents behave as if they were making the vast number of computations implied by existing economic theories.

This line of thinking was started by H. Simon (Simon [1982]). Simon differentiated between agents capable of achieving rational outcomes *deus ex machina* (*substantial rationality*, without providing a plausible mechanism by which rational results might be achieved) and those which operate according to feasible but potentially less capable heuristics (*procedural rationality*). From a normative point of view, mathematical analysis is a perfect tool to achieve substantial rationality for well-conditioned environments, but it is often overlooked

that constructive proofs (results of which would serve as policy for agents) are prohibitively expensive.

A good example of difference between substantial and procedural rationality is the issue of finding a price vector which clears the market. Existence of such an equilibrium (Walras, Nash) is proved via Brouwer or Kakutani fixed point theorems. Finding the exact values for any set of endowments and agent’s preferences (constructive proof) is done through Sperner’s lemma. Papadimitriou [1994] has shown that Sperner’s lemma is essentially NP-complete and therefore can not be used to compute prices beyond trivial cases. The dual limited rationality formulation (using an agent-based simulation) has been investigated in Axtell [2005] and found to generate comparable welfare outcomes at profoundly lower (polynomial) computational cost.

Exponential cost of fully rational solution to general economic questions should not be seen as a detriment to positive activities such as market design. In this particular field, both Rust [1997] as well Mirowski [2004] argue that one may be able to design a sufficiently precise and consistent solution, one that imposes reasonable computational requirements on both the modeler / market designer as well as future market participants. Such a break of the curse of dimensionality of economic problems necessarily needs to exploit the special structure of the problem at hand<sup>1</sup>.

In this paper, we verify existence of such a structure for voluntary health insurance markets. In particular, we construct a detailed agent-based model (Axtell [2000]) of such a market, where two types of agents are present:

1. Simple health-insurance consumers (with empirically calibrated attributes), using heuristics to select insurance plans or stay out of market;
2. Insurers, who behave strategically to design multi-part insurance plans, subject to exogenous regulator-imposed constraints.

We vary the exogenous regulator-imposed constraints (market designs, ranging from a fully competitive market to a centralized, fully coordinated system of insurance plans, preserving the free choice of consumers) to see what kind of welfare and complexity implications each of them has. Our goal is to establish a frontier of performance-complexity trade-offs, contributing to a fledgling dispute of reform of health care system in the United States.

## 2 Modeling Assumptions

### 2.1 Complexity Measures

In our paper, we adopt the perspective of an insurer facing the problem of setting features of an insurance policy. Following Papadimitriou [1994], Rust [1997], we reconstruct the complexity function,  $X(\epsilon) \rightarrow \mathfrak{R}_+$ . Function  $X$  represents the minimal computation time or cost function of producing an  $\epsilon$ -approximation of the optimal solution for given policy

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<sup>1</sup>For examples, refer to Hanson [2003] for combinatorial spectrum auctions or to Cramton [2003] for electricity markets.

parameters. Many mathematical / decision problems have one or more associated parameters. For simplicity, let's assume that the dimension of the space in which the mathematical problem lives is  $d^2$ . In these cases it is customary to write the problem dimension  $d$  as an extra argument of the complexity function,  $X(\epsilon, d)$ .

Computer scientists say a computational problem is intractable if its complexity function takes the form

$$X(\epsilon, d) = o\left(\left(\frac{1}{\epsilon}\right)^d\right)$$

where the small  $o$  symbol denotes a lower bound on the complexity function. That is, an intractable problem is one for which the lower bound on the computation cost increases exponentially with the problem dimension  $d$ . In economics, problems that have this property are typically referred to as suffering from the curse of dimensionality.

Lastly, the complexity of decision making will depend on the exact regulatory setup used to set rules of market competition. We will denote the set of possible market designs by  $M$  and reconstruct a separate complexity function for each of them, ending up with a set of complexity functions indexed by market designs,  $\{X_m(\epsilon, d)\}_{m \in M}$

As noted in Rust [1997], the logic of exponential growth tells us that regardless of the values of the (typically unspecified) bounding constants in the complexity bounds, as  $d$  increases the number of calculations quickly grows so large that the world's fastest supercomputers would be unable to find an approximate solution to the problem in any reasonable period of time. Since this complexity bounds also constitute arguments for the impossibility of rational or equilibrium behavior, we would like to exclude associated  $m$  from set of possible future market designs.

## 2.2 Modeling Bounded Rationality in Strategic Contexts

We will model rationality of insurers by an application of the recursive simulation methodology and  $n$ -th order rationality. As defined in Gilmer and Sullivan [2003], recursive simulation requires the simulated decision makers to use simulation to inform their own decision making. In our approach, the structure of internal models used by agents will be isomorphic with the structure of the model in which they themselves are embedded. That kind of modeling is consistent with the economic literature. In fact, so called "Lucas critique" (Lucas [1976]) posits that in order to design a policy intervention, one should model how individuals and institutions account (model) for the change in policy, and then aggregate the individual decisions to calculate the effects of the policy change. This model responds to the critique by allowing agents to respond to novel regulatory environments without being affected by predetermined assumptions about how they 'should' behave.

Recursive simulation technique is part of the answer. In novel situations there may be no analogous games to provide "evidence" for how the agent "should" behave. Instead equilibrium / behavioral guidance must come from sophisticated strategic thinking rather than learning from direct experience. This element is provided by so called  $n$ -th order rationality. A formal definition of  $n$ -th order rationality is presented in Michihiro [1997]. An

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<sup>2</sup>In our case,  $d$  will be used to denote number of competing plans

agent is zero-order rational if it calculates the best response to his beliefs about strategies of other agents and the state of the world. An agent is  $n$ -th order rational if it determines its best response assuming that the other agents are  $(n-1)$ -th order rational. Similar philosophy leading to an iterated process of strategic thinking has been also outlined in [Binmore \[1988\]](#).

The concept of  $n$ -th order rationality differs from concept of Cognitive Hierarchies, see [Camerer et al. \[2004\]](#), in the fact that all agents always believe that others are just a notch less sophisticated than they are themselves, instead of assuming some distribution of rationality types bounded from above with their own rationality level. This makes  $n$ -th order rationality more suitable in games with identifiable, institutional agents, not population games. Similarly, the assumption that agents are able to reconstruct the model they belong to (apart from simple single-stage environments considered by experimental economists mentioned before) implies that they are representations of real-life institutions and corporations rather than individuals. Nevertheless, for application areas like mechanism design and industrial organization, this assumption fits well<sup>3</sup>.

The exact implementation of the recursive bounded rationality scheme used is to be found in Section 3.2. More on algorithm used can be found in [Latek et al. \[2009\]](#).

## 3 Model Description

### 3.1 Reactive Patients

In the United States, health insurance for those under age 65 is typically provided through group plans purchased by employers from commercial insurers, whereas older people rely on government health care programs and individually purchased supplementary coverage. In both cases, health insurance is a complex multi-attribute service, so customers looking to buy insurance face a difficult shopping problem. In the case of pharmaceutical drug coverage (available to retirees under the Medicare Part D program) savvy purchasers must consider which of the many drugs they might use. They must consider whether or not those drugs are in the insurers formularies, and what co-pays and deductibles apply to which pharmaceuticals, providers and services. Comparison shopping is made even more difficult by the fact that many aspects of insurance involve commitments to provide services under hard-to-anticipate contingencies<sup>4</sup>. Behavioral patterns connected to choice of health insurance have been subject of numerous studies, including [Frank and Lamiraud \[2006\]](#), [Liebman and Zeckhauser \[2008\]](#), [Randall et al. \[2008\]](#).

In our paper, patient follow a heuristic accounting for many of aspects of human behavior faced with such a complex decision problem, called  $S(x)$  sampling rationality. This very intuitive scheme, proposed by [Osborne and Rubinstein \[1998\]](#), consists of a consumer generating  $x$  contingencies, evaluating each of them against set of possible decision alternatives and selecting one that performs best according to some pre-specified criteria, presented on

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<sup>3</sup>In fact, it might be the case that the real-life markets become increasingly "recursive" as time flows. This would happen due to spreading of modeling methods amongst the business community and the phenomenon of revolving doors, described in [Heyes \[2000\]](#).

<sup>4</sup>See [Rubinstein \[1993\]](#) and [Spiegler \[2006\]](#) for discussion of relevant models of product obfuscation on general markets.

Algorithm 1.

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**Algorithm 1** Procedure  $S(x)$  used by patient  $i$  to select insurance (or stay out of the market).

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Read patient  $i$  attributes  $\text{costDistribution}_i$ ,  $\text{minimalCoverage}_i$ ,  $\text{tolerableMargin}_i$   
Read in set of available policies  $P$   
Generate set  $E$ ,  $\|E\| = x$  of cost events, by sampling  $x$  times from  $\text{costDistribution}_i$   
**for all** Policies  $p \in P$  **do**  
    Evaluate  $p$  against  $E$ , calculating  $\text{netBenefit}(p, X)$  and  $\text{coverage}(p, X)$   
    **if**  $\frac{\text{netBenefit}(p, X)}{\text{premium}(p)} > \text{tolerableMargin}_i$  or  $\text{coverage}(p, X) < \text{minimalCoverage}_i$  **then**  
         $P \leftarrow P - \{p\}$   
    **end if**  
**end for**  
**if**  $P \neq \emptyset$  **then**  
    Adopt random  $p \in P$   
**else**  
    Stay uninsured  
**end if**

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## 3.2 Recursive Insurers

The description provided here is short version of [Latek et al., 2009, Section 4]. Suppose one is given a multi-insurer simulation  $\Psi$ , populated with  $K$  insurers. The state of the simulation at time  $t$  is defined as all relevant information, excluding policies of agents, and is denoted as  $C_t$ . Each of the agents has an associated action set  $A_t^i$ , which may depend on the current state of simulation  $C_t$ .

Suppose that the behavior of insurer  $i$  at time  $t$  can be described by a policy  $p_t^i$ <sup>5</sup>.

Any agent-based simulation  $\Psi$  can be viewed as a map, which for a given  $C_t$  and a fixed set of policies of agents  $\mathbf{p}_t$  returns both the next state  $C_{t+1}$  as well as a vector of rewards  $\mathbf{r}_t = (r_t^1, \dots, r_t^K)$  for each of the agents:

$$(\mathbf{r}_t, C_{t+1}) = \Psi(\mathbf{p}_t, C_t).$$

Two remarks need to be made about  $\Psi$ . First, in general  $(\mathbf{r}_t, C_{t+1})$  are random variables, either due to nondeterministic decision rules  $\mathbf{p}_t$  or to possible randomness of the agent activation scheme. Therefore, a single run of simulation  $\Psi$  yields only one realization of these variables. Second, we will assume that as long as an agent can construct description of the initial state of  $\Psi$  and assume particular policies for opponents, it can evaluate  $\Psi$ .

The ability to evaluate  $\Psi$  gives agents a powerful forecasting tool<sup>6</sup>. Superimposing  $\Psi$

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<sup>5</sup>Following notation will be adopted:  $p_t^i$  is the policy of insurer  $i$  at time  $t$ . Set of policies for the whole population used at time  $t$  will be denoted as  $\mathbf{p}_t$ , a  $K$ -dimensional vector of policies. Lastly, by  $\mathbf{P}$  we will denote a scenario of policy trajectories for each of the agents for a set number of periods:  $\mathbf{P} \equiv \{\mathbf{p}_t, \dots, \mathbf{p}_{t+h}\}$ , parameter  $h$  will be defined later.

<sup>6</sup>We need to underline here that our focus is on domains, like industrial organization, with players already using advanced analytics to guide their decision making.

produces forecasts about future rewards  $\mathbf{r}_t, \dots, \mathbf{r}_{t+h}$  and future states of the core simulation  $C_{t+1}, \dots, C_{t+h}$  for any arbitrary horizon  $h$  and scenario of policy trajectories  $\mathbf{P}_{t,h} = (\mathbf{p}_t, \dots, \mathbf{p}_{t+h})$ .

We assume that insurer  $i$  wants to maximize expected discounted stream of rewards for a certain planning horizon  $h$  by controlling policies  $(p_t^i, \dots, p_{t+h}^i)$ :

$$\max_{(p_t^i, \dots, p_{t+h}^i)} \sum_{j=0}^h \gamma^j E(r_{t+j}^i)$$

where  $\gamma$  is the discount rate<sup>7</sup>.

For non-trivial  $\Psi$ ,  $i$ 's payoff and trajectory through state space will depend not only on policy agent  $i$  sets, but also on policies of the other agents.

We assume that agents in the simulation behave according to  $n$ -th order rationality scheme. Let us denote by  $\Xi^i(d, h) = (\Xi_0^i(d, h), \dots, \Xi_h^i(d, h))$  the optimal policy trajectory  $(p_t^i, \dots, p_{t+h}^i)$  of  $i$ -th player in planning horizon  $h$  assuming that his order of rationality is equal to  $d$ . A 0-order rational agent replicates his last action. Assuming that the initial state of the simulation  $C_t$  and last actions of all agents  $\mathbf{p}_{t-1}$  are public we get:

$$\Xi^i(0, h) = \underbrace{(p_{t-1}^i, \dots, p_{t-1}^i)}_{h+1}$$

The values of  $\Xi$  for agents having rationality order  $d > 0$  are defined recursively. In each period  $d$ -th order rational player  $i$  assumes that other players ( $k \neq i$ ) will be playing  $(d-1)$ -th order rational strategy  $\Xi^k(d-1, h)$ . Therefore  $i$ -th player will optimize:

$$\begin{aligned} \Xi^i(d, h) \equiv \operatorname{argmax}_{(p_t^i, \dots, p_{t+h}^i)} \sum_{j=0}^h \gamma^j E(r_{t+j}^i) \\ \text{subject to:} \\ \forall j \in \{0, \dots, h\} : (\mathbf{r}_{t+j}, C_{t+j+1}) = \Psi(\mathbf{p}_{t+j}, C_{t+j}) \\ \mathbf{p}_{t+j} = p_{t+j}^i \cup \{\Xi_j^k(d-1, h)\}_{k \neq i} \end{aligned}$$

In short, the optimization process for agent  $i$  would involve following steps:

1. Clone the current state of the simulation  $C_t$ <sup>8</sup>.
2. Assume a particular policy trajectory for other agents  $\mathbf{P}_{t,h}^{-i} \equiv \mathbf{P}_{t,h} - \{p_t^i, \dots, p_{t+h}^i\}$ . If  $d > 0$ ,  $\mathbf{P}_{t,h}^{-i}$  is obtained by solving problems  $\Xi(d-1, \bullet)$  for competitors. If  $d = 0$ , by assuming that they will continue their last policies for the next  $h$  periods.

<sup>7</sup>Note that in order to calculate expected values of  $r_{t+j}^i$  it is necessary to run simulation  $\Psi$  multiple times.

<sup>8</sup>In this paper, we assume that  $C_t$  is all public information. Private information includes current policies as well as individual  $d$  and  $h$ . There is neither need for parameters  $d$  and  $h$  to be homogeneous in the population nor need they be public. In the case of heterogeneity, agent  $i$  will use his private values to instantiate  $\Xi^i(d_i, h_i)$ . Please see Section ?? for discussion of alternatives.

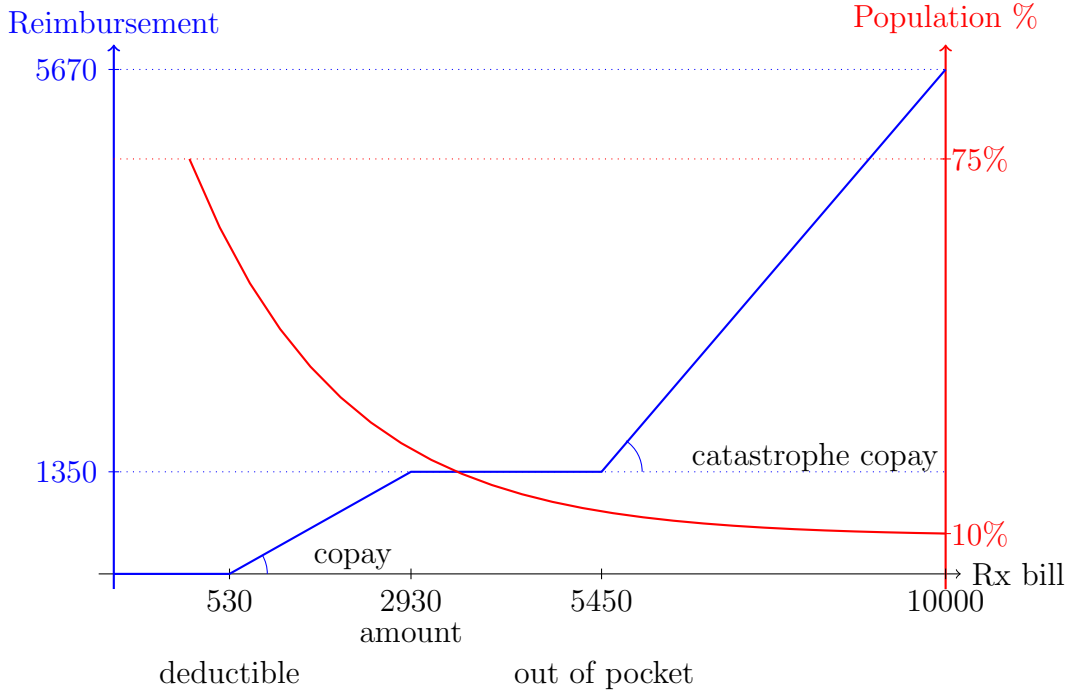


Figure 1: Definition of multi-part health insurance policy. Each policy is a point in  $\mathfrak{R}^6$ , composed of following dimensions: {premium, deductible, copay, catastropheCopay, amount, OOP}. Bounds for each of the dimensions used in the experiments are listed in Table 1.

- Adjust  $(p_t^i, \dots, p_{t+h}^i)$  such that the expected discounted reward stream is maximized. The objective function is evaluated by running the core simulation  $\Psi$  for  $h$  periods forward, keeping  $\mathbf{P}_{t,h}^{-i}$  fixed.

Solving  $\Xi(\bullet, \bullet)$  generates an extended best-response dynamic. For  $d = 1$ , the best response is calculated, while  $d = 2$  yields the best response to one's expectations of the others best responses. Parameter  $h$  controls how myopic the insurers are. Definitions of dimensions contained in each  $p_t^i$  are presented in Figure 1.

## 4 Results

### 4.1 Experimental Design

In Table 2 we define experimental design for our study. For each of the experimental treatments, we will gather following statistics:

Performance Market-level and plan-level data on performance includes: profits, enrollment levels, benefits paid;

Parameter	Range	Meaning
numPatients		
premium		
deductible		
amount		
copay		
catastropheCopay		
OOP		
costDistribution		
tolerableMargin		
minimalCoverage		

Table 1: Simulation parameters used during experiments from Section 4. Differences between different treatments are summarized in Table 2.

Complexity Again, data will be recorded on both plan as well market (decision maker where appropriate) levels. Performance measures include wall clock time necessary to reach convergence of appropriate  $\Xi(\bullet, \bullet)$  scheme as well as number of calls to  $S(1, \bullet)$  patients decision making procedure.

All experiments will be run on the same reference patient population of 500 patients. The artificial distribution of average costs in the population correspond to red curve from Figure 1.

The analysis of data will proceed as follows. In Section 4.2 market performance will investigated and later correlated with market complexity in Section 4.3, showing unexpectedly low complexity. In Section 4.4, we will search for factors causing such a low complexity and introduce an analytical framework that is conducive to quantifying influence of micro structure on market complexity.

## 4.2 Analysis of Performance

This Section intends to answer the question: How does the market behave as a function of the market design and the number of insurers, leaving out the market complexity of the picture for now. Figures 2 presents how the number of plans on the market and design treatment influence different performance metrics. Most of the results are intuitive. Two items deserve mention. First, please note that in the pure market-driven treatment, introducing new insurers does not collapse total market profits. Second, note that even under most benevolent central planner, no 100 % enrollment is achieved. Both results are driven by patient heterogeneity which in the first case enables market segmentation and in the second causes least risky patients to stay out of the market.

	Market design set $M$			
	Market	Multi-plan monopolist	Coordinated plans	Social planner
Objective	Profit	Profit	Enrollment & Profit	Enrollment
Number of plans $d$	$\{1 \dots 8\}$	$\{1 \dots 8\}$	$\{1 \dots 8\}$	$\{1 \dots 8\}$
Number of decision makers	$\{1 \dots 8\}$	1	$\{1 \dots 8\}$	1

Table 2: Setups for all four market designs  $M$ . Market designs set is composed of four treatments  $M = \{\text{market, monopolist, coordinated, planner}\}$ . Each treatment was composed of smaller experiments, where number of plans on the markets (denoted  $d$ ) was varied. For each pair  $(m, d) \in M \times Z_+$  100 runs were performed.

Different aspect of increasing number of plans is addressed by Figure 3. For treatments where decision makers aim at maximizing profit, one can scatterplot net benefits paid to patients versus total market profit as a function of number of plans on the market, visualizing how welfare in the system is shared. First, patients benefit little from additional plans under monopolist conditions, even though enrollment levels rise. Second, the downward sloping benefit-profit exchange plot compares favorably with result obtained experimentally by Johnston [2007] for a identical patient population where human subjects played role of plan managers.

### 4.3 Complexity of Health Insurance Markets

Figure 4 presents average computational cost  $X_\bullet(0.01, \bullet)$ , recorded per decision-maker and per market. To account for the fact that on larger markets customers are forced to take samples over larger number of plans, we express the computational cost in cost of executing  $S(d)$  procedure (for example, sampling each of  $d$  plans once), the most primitive procedure called by recursive insurers during the plan design phase.

The striking fact is that, for most treatments, the marginal cost (in  $S(d)$  units) of adding a new plan to the market is essentially 0. The only exception is the market based treatment, where this cost is linear in number of plans. Please note that under general conditions, we would expect exponential rise of cost in number of plans both for of market competition as well as social planner’s problem (per general results on finding Nash equilibrium, Walras allocations, etc.). This observation constitutes the main result of this paper.

Valid question is how different is the behavior of worst-case compared to the average cost and how much of this result is influence by choice of  $\epsilon$ . A related question is: how much (in terms of computational cost) does achieving desired levels of market performance costs. Figure 5 presents scatter plot in performance  $\times$  cost space, where lower right edge of each of the points clouds constitutes a Pareto front. When better optimization algorithm is used and all of patients randomness eliminated, it should be possible to reduce all the clutter and move all data points to respective Pareto frontiers. As the scatter clouds are rather narrow, we can say that at least on market level, our result is robust with respect to changes in  $\epsilon$  and worst-case / average-case perspective.

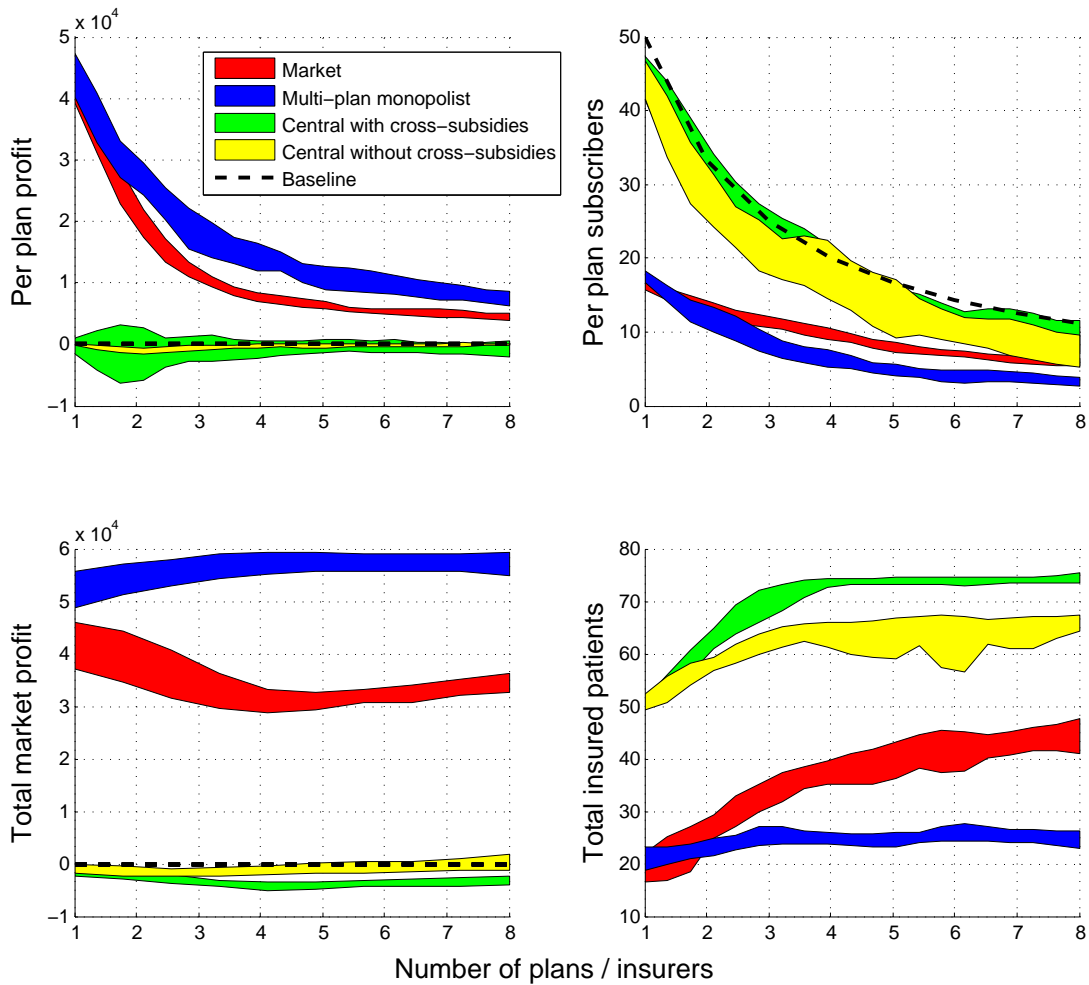


Figure 2: Measures of market- and plan-level performance as a function of market treatment and number of plans.

## 4.4 Market Microstructure

### 4.4.1 Framework for measuring coupling

In this Section, we will attempt to find and quantify the micro structure causing the optimization to exhibit diminishing costs in added plans. For this, we will use so called  $N - K$  model as a reference framework for describing complexity of multi-component systems.

In [Kauffman and Levin \[1987\]](#)  $N - K$  model,  $N$  refers to the number of parts of the system and each part makes a contribution to the overall fitness that depends upon that part and upon  $K$  other parts among  $N$ . In simplest setting, it is assumed that each part has two possible states and represent the state of the system by  $E \in \{0, 1\}^N$ . The fitness of  $E$  is defined as:

$$\Phi(E) = \sum_{i=0}^N \phi_i(\eta_{i_1}, \dots, \eta_{i_K})$$

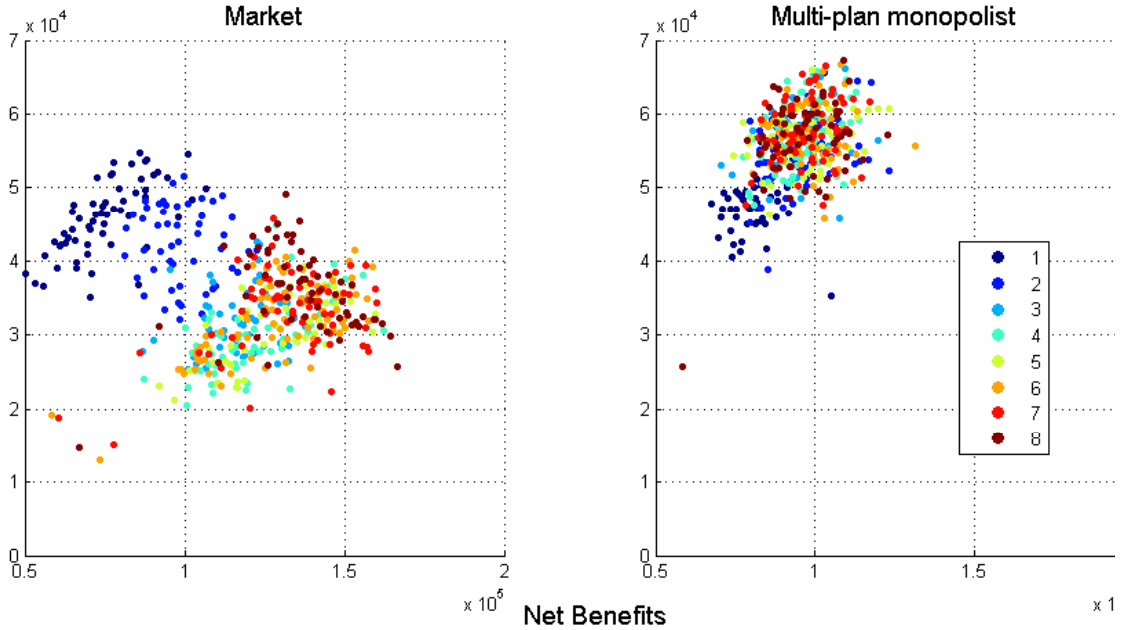


Figure 3: For treatments where decision makers aim at maximizing profit, one can scatterplot net benefits paid to patients versus total market profit as a function of number of plans on the market, visualizing how welfare in the system is shared. Please note that patients benefit very little from additional plans under monopolist conditions, even though enrollment levels rise.

where the  $\phi_i : \{0, 1\}^K \rightarrow \{0, 1\}$  are i.i.d draws from a space of lookup tables of size  $K$ . As noted in [Durrett and Limic \[2001\]](#), when  $K = 0$ , finding optimal solution  $\eta^*$  is trivial. The parts do not interact, so there is only one maximum  $\eta^* = (\eta_1^*, \dots, \eta_N^*)$ , which can be obtained by choosing  $(\eta_i^*)$  to maximize  $\eta_i \rightarrow \phi_i(\eta_i)$  for each  $i$ . The other extreme case  $K = N - 1$  is also trivial. Each  $\phi_i$  is a function of all  $N$  coordinates, so the fitness of each  $\eta$  is a sum of  $N$  independent uniforms and as values of  $\Phi(\eta)$  are independent for different  $\eta$ , exhaustive search of  $E$  space is the only optimization method available. The non-trivial cases of  $0 < K < N - 1$ , when coupling is not global, may yield to different optimization methods at a cost that is increasing function of  $K$ .

Therefore, if we were to find the coupling parameter  $K$  for different market treatments and number of plans, we could use it as a measure of strength of microstructure on health insurance market. Let's start the analysis by comparing structure of our model and  $N - K$  mode. As outlined in Figure ??, we need to match three components: design space  $E$ , intermediary fitness functions space  $\phi$  and final fitness  $\Phi$ :

$E$  Design space  $E$  is the space of  $N$  insurance plans, which means that  $E \subset \mathfrak{R}^{6N}$ . For purpose of the complexity discussion, we will treat each plan as a point in  $E$  (bundling dimensions into groups of 6).

$\phi$  Each patient can be viewed as a separate  $\phi_i$  function that assortatively links to plans in the feasibility subspace (term defined below) and produces individual welfare outcomes.

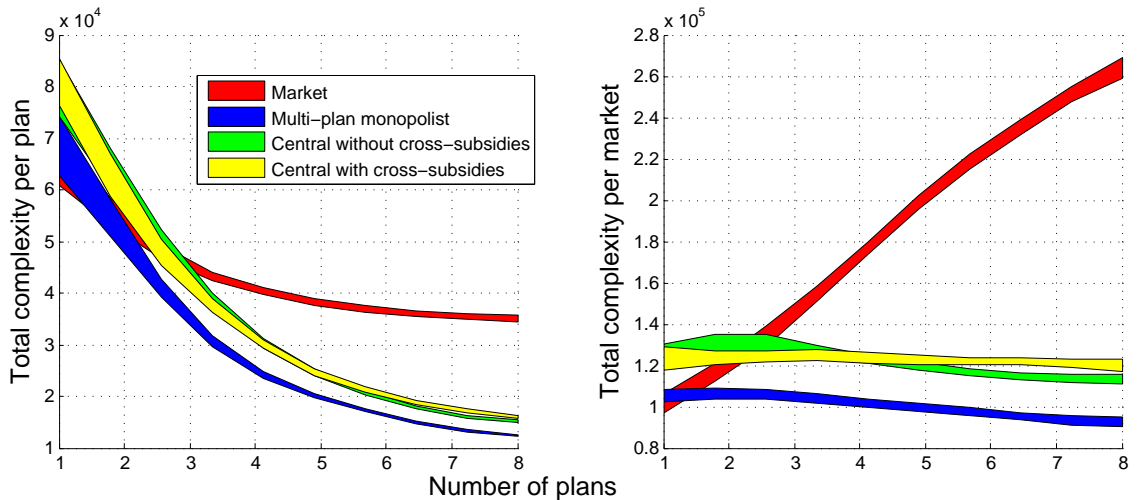


Figure 4: Average computational complexity (with associated 95 % bootstrapped CI) of different treatments, expressed in  $S(d)$  units.

$\Phi$  Aggregation function that takes decisions of individuals and produces social welfare outcome (market profits, net benefits paid, number of insured, depending on treatment).

Please note that each patient defines a region in  $\mathbb{R}^6$ , subspace of potential polices that would be deemed feasible by this patient (in understanding of Algorithm 1), which means that they can chosen with non-zero probability under any treatment and market scenario. We will call this subspace feasibility subspace.

Intuitively, topology of feasibility subspaces would determine the  $K$  (coupling parameter) of our health insurance market. Should each subspace have just a few neighbors, the patient population can be decomposed such that plans designed to maximize profits / social outcomes for one of segments of patients population would be irrelevant to behavior and choices of rest of the population. This would allow for monopolist / social planner to conduct dimension-wise search of design space  $E$ , consecutively targeting different segments of patient populations.

In case of pure market-based solutions, this argument no longer holds. Competitors are unable to bootstrap the decomposability of the policy design problem and must rely on their short-term projections of behavior of other competitor, potentially overcrowding some of the feasibility subspaces leaving others unoccupied. The magnitude of this phenomena can be best measured by regressing number of plans that can be found in each of the feasibility subspaces on characteristics on it's owner. Under coordinated solution, we expect that regression would yield no significant coefficients. Under market-based treatments, we should observe strong relations, pointing to overcrowding of some of market segments.

#### 4.4.2 Visualizing decomposability

First, notice that the decomposability of the problem translates directly into distribution of plan sizes, measuring number of active dimensions in design space (we shall call a plan

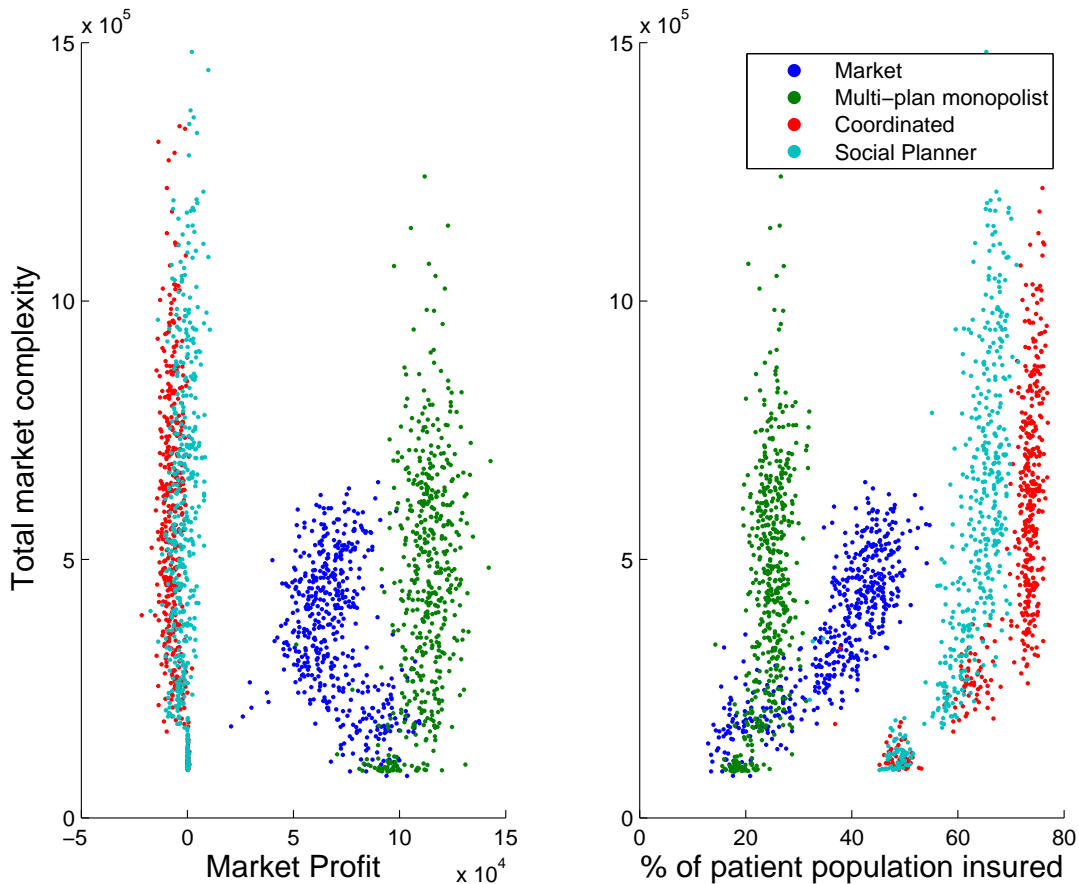


Figure 5: Pareto frontier of performance / complexity tradeoffs for all treatments. Panel separates the treatments such that performance measures are directly comparable.

active if it attracts patients, if number of subscriber is 0, plan will be inactive). For pure market-based treatments, Figure 7 indicated that all plans are always active. For coordinated / centralized treatments, planners tend to use up to 4 active plans, discarding the rest such that they do not attract any subscribers. Given the heterogeneity of patients population, 4 plans seem enough to ensure maximum coverage attainable without subsidies or, in monopolist treatments, extract maximal possible profit from patient's population.

Under market-based treatments, maintaining so many active plans comes at a cost. On Figure 6, we show the average number of plans to be found in feasibility subspaces of patients, conditioned on treatment and average cost. For coordinated treatments, each patient has (even for large number of plans), a 1 or 2 feasible choices. This means that the coupling parameter  $K$  in our system is independent of  $N$ , making up for decreased computational cost in those treatment.

## 5 Discussion

The connection between our simulation and  $N - K$  model can be strengthened by investigating following items:

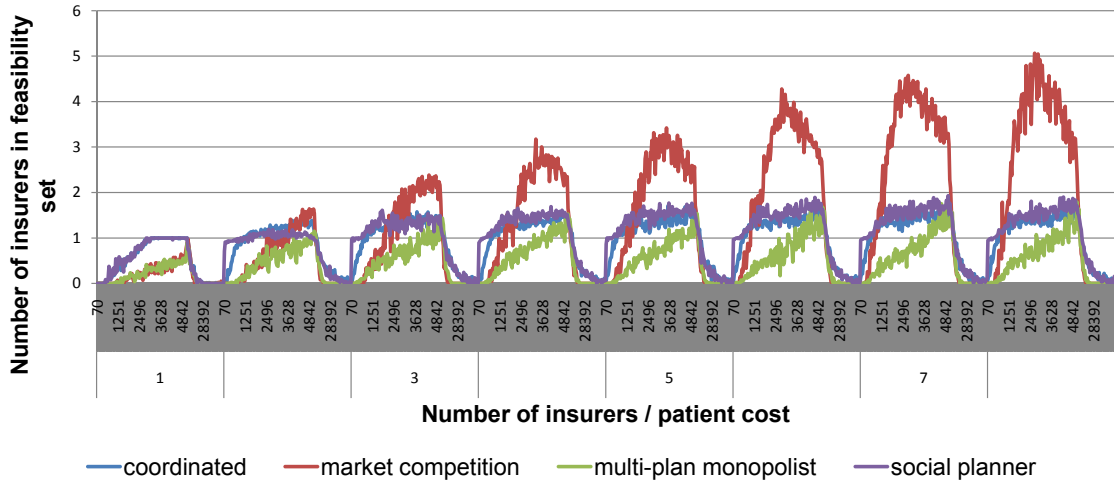


Figure 6: Average number of plans that can be found in feasibility sets of patients under different treatments, conditioned on patients `averageCost`. Two other sources of patient heterogeneity, `tolerableMargin` and `minimalCoverage`, are unaccounted for, but at the same time uncorrelated with `averageCost`.

1. Performing cluster analysis of policies and later re-projecting the policy clusters onto patient populations, to see what exactly makes some policies attractive to which segments of patients and how insurers deal with adverse selection problem.
2. Random sampling of design space to establish topology of feasibility subspaces.
3. Bringing in analytical results about  $N - K$  model to see if the decrease if they are informative with respect to decrease of average computational cost.

We noticed that the results on computational complexity depend on the feasibility subspaces having narrow support. This is equivalent to ability of patients to identify irrelevant alternatives. This leads to following more focused research questions / hypothesis to be verified:

- If the heterogeneity of patient population increases, the coupling parameter as well as computational cost should decrease. This result is conditioned on patients being allowed enough samples to discern between plans in a reliable way.
- Informed patients increase segmentation and decrease computational cost, even though insurers need to do more complex representations in recursive modeling.

Thesis 1 can be verified by manipulating the patient characteristics. Thesis 2 states, that intelligent enough patients should self-select smaller feasibility subspaces. It has clear policy implications, that fostering automated plan comparing would decrease competition in the traditional sense, but might improve system outcomes. In our setting, the control factor that needs to be changed to confirm or refute Thesis 2 is the number of samples  $d$  patients are allowed to take.

We have also identified a technical challenge, one that can lead to developments in theory of computational complexity itself:

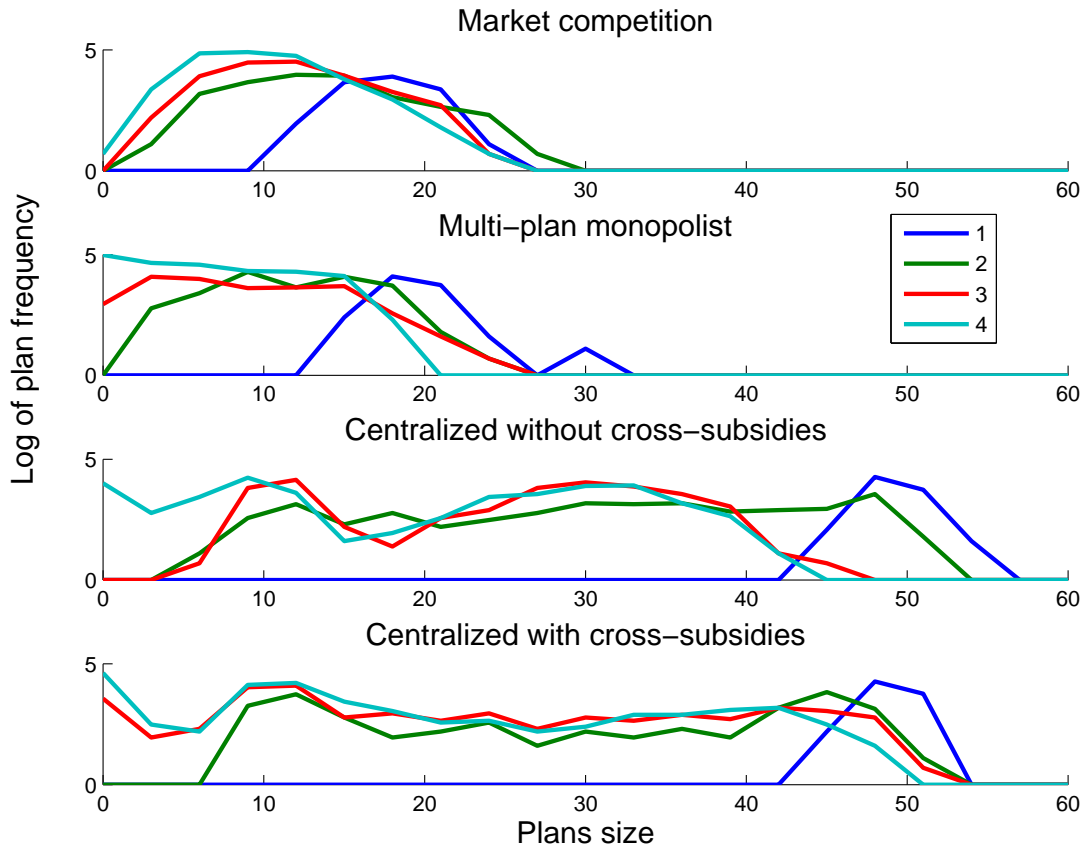


Figure 7: Distributions of plan sizes under different treatments and  $d \in \{1, 2, 3, 4\}$ . Plan size is expressed in percentages of the patient population enrolled.

- Given parameterized  $N - K$  with some reasonable assumption on  $\phi$ , what is expected outcome (in % of optimal solution) that can be expected with probability of  $\delta$  after investing  $T$  time into optimization?

This is reverse question (maybe a dual formulation) of Probably Approximately Correct learning problem that currently is not covered by any theoretical or practical results.

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