Simulating an Economy with Smithian Production: Division of Labor as a Computational Object

By Roger A. McCain

The division of labor plays little role in contemporary economics, which is in remarkable contrast to its importance in the ideas of the founder of economics as a distinct study, Adam Smith. This disappearance of division of labor may no doubt be partly a result of Malthus' dismissal of division of labor as an irrelevancy, although Mill and others among later classical economists returned to it. However, another possibility is that the division of labor is an intractable subject for the methods preferred by neoclassical economists: that, indeed, neoclassical economics simply has little or nothing to contribute that would extend Smith's insight. What is intractable for neoclassical methods may nevertheless yield to simulation methods, and there has been some agent-based simulation work done on the division of labor. Nevertheless, our understanding of the division of labor remains much as Smith left it.

As Simon (1995) notes, a computer program can be a theory. More generally, computer programming languages are formal languages, as mathematical analysis is, and in principle any formal language may serve to state a theory. As we will see, object-oriented programming lends itself well to a theory of division of labor.

Smith's ideas have less in common with modern economics than is commonly supposed. For Smith it is the division of labor per se that accounts for increasing productivity. It has little to do with efficient allocation of resources, and the commonsense idea that division of labor enhances efficiency by allowing individuals to specialize in what they are best at is not Smith's sense. Smith denies that there are any such differences in aptitude: "The difference of natural talents in different men is, in reality, much less than we are aware of; and the very different genius which appears to distinguish men of different professions, when grown up to maturity, is not upon many occasions so much the cause, as the effect of the division of labour. The difference between the most dissimilar characters, between a philosopher and a common street porter, for example, seems to arise not so much from nature, as from habit, custom, and education. When they came into the world, and for the first six or eight years of their existence, they were perhaps, very much alike, and neither their parents nor playfellows could perceive any remarkable difference. About that age, or soon after, they come to be employed in very different occupations. The difference of talents comes then to be taken notice of, and widens by degrees, till at last the vanity of the philosopher is willing to acknowledge scarce any resemblance." This provides another reason why Smith's ideas on this point have had so little influence on modern economics. Smith treats production as a complex system phenomenon. It is complexity (in the form of extended division of labor) that leads to increased productivity. But our understanding of complex systems is relatively new, and has had little influence on economics.

To motivate the study reported here, it will be worthwhile to recall Smith's most famous passage on the division of labor. Consider "the trade of the pin-maker; … in the way in which this business is now carried on, not only the whole work is a peculiar trade, but it is divided into a number of branches, of which the greater part are likewise peculiar trades. One man draws out the wire, another straights it, a third cuts it, a fourth points it, a fifth grinds it at the top for receiving, the head; to make the head requires two or three distinct operations; to put it on is a peculiar business, to whiten the pins is another; it is even a trade by itself to put them into the paper; and the important business of making a pin is, in this manner, divided into about eighteen distinct operations, …" In what follows each of these distinct operations will be called a "task." A plan for the division of labor in producing some particular good is then a sequence of complementary tasks[1], and with each person performing just one or a very small number of tasks. In what follows, such a plan of production will be called a "technique."

In Smith's example, each task corresponds to a specific stage in production. With very few exceptions, a particular task cannot occur unless some specific tasks have preceded it. The exception in Smith's description is the first step: "One man draws out the wire, …." In principle, there might be more than one starting task. In the production of muskets, for example, the musket is assembled from a number of parts, a barrel, stock, and trigger assembly, each of which is produced by a separate process beginning with a different startup task. On the other hand, each task contributes something toward the value of the final product. Only one, however, yields a final product. Accordingly, for this discussion, a task is an object T, with the following properties:

(Boolean) T is or is not a startup task

(Boolean) T is or is not a finish task, i.e. a task that gives rise to a finished output good such as a paper of pins or a musket

(double) a scale parameter, indicated by V(T)

---

[1] "Smithian Growth Through Creative Organization" by Patrick Legros, Andrew F. Newman, and Eugenio Proto, presented, American Economic Association annual conference, San Francisco, Cal, Jan. 3, 2009. This paper is a valuable exception to the tendency of current economic research to ignore the division of labor.

(vector of integers) indices of tasks that can immediately follow T; in the case of a finish task this set is null

(integer) index of final output category finished. If T is not a finish task then this is null. Note that a class of tasks might be both startup and finish tasks. These tasks would be techniques of craft production. (Such a task could be very complex, but for the purposes of this discussion of division of labor, one task is the work of one person, so a craft technique would be formally treated as a single task.)

A technique is an object with at least the following property: a vector of tasks T_1, T_2, … T_n such that 1) task T_1 is a startup task, 2) task T_i is a member of the set of successors of task T_{i-1}, and 3) task T_n is a finish task. Several properties can be derived from this vector. First, the output category produced is the output category indexed as a property of T_n. Second, the employment scale of the technique is n – there is one worker per task. (Managerial labor is abstracted from). Third, the quantity of output is the product of the scale parameters of the tasks, $Q = \prod_{i=1}^{n} V\left(T_i\right)$.

As Kaldor (1934) observes, division of labor as treated by Smith implies increasing returns to employment scale, since a larger work group will allow a more extensive division of labor and so greater productivity. If $\overline{V}$, the mean value of V(T), over all T, is greater than one then Kaldor's observation will follow: the output of a technique T_1, T_2, … T_n will increase as $\overline{V}^{n}$ on the average. This case will be called a Smithian bias. Smith gives a number of reasons for a bias of this sort. The most important one, no doubt, is learning: when a person concentrates on a single task, he learns to perform it more effectively. Yet the Smithian assumption that increased division of labor leads to increased productivity does not require a Smithian bias. After all, it is not the average productivity of a task of size n that is really of interest to us. Rather it is the largest productivity: for a particular output category and employment scale, a rational entrepreneur will choose the most productive technique. But a larger n will imply a larger set of techniques, in principle, and to the extent that this is true the expected value of productivity will increase with employment scale and the division of labor in the absence of any Smithian bias in the sense just indicated.

This tendency was explored in some preliminary research for this paper. Tasks were generated with V(T) determined as a pseudorandom number with a rectangular distribution on the interval (0.8, 1.2). Thus the mean value is 1 and there is no Smithian bias. Again relying on pseudorandom processes, 64,000 techniques were generated from 100,000 tasks. The 64,000 techniques did not exclude duplicates, and in fact duplication was common. The data for distinct techniques are shown in Figure 1, with productivity of labor on the vertical axis and employment scale on the horizontal scale.

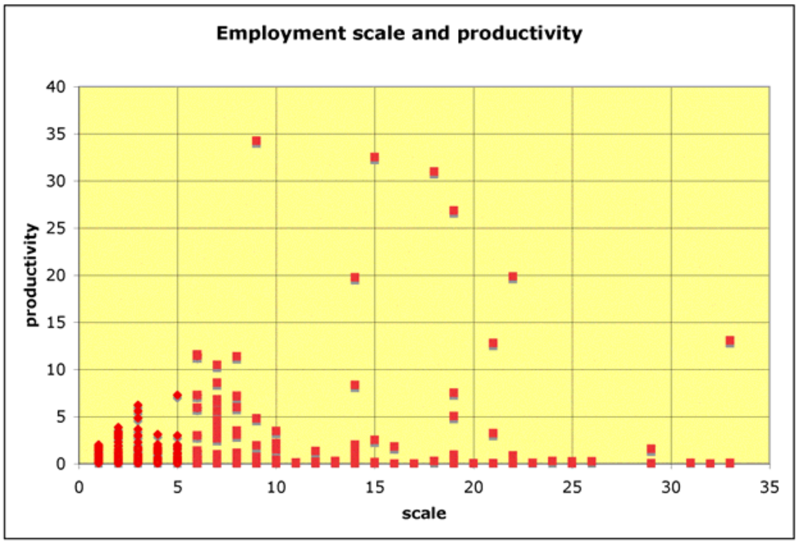

Figure 1. 64,000 Techniques Comprised of 100,000 Tasks

We see that the tendency for productivity to increase with employment is pronounced only in the range of 1-9 employees, and that the maximum productivity of a work group declines from that point on. Examination of the pseudodata in more detail indicates that the techniques for larger groups are highly redundant, suggesting that, beyond a certain point, techniques become fewer rather than more numerous. Recall that each task in a viable technique must be succeeded by one of a successor set that is itself random, must begin with a startup task and end with a finish task. For a given ordered pair of tasks, the probability that the second is not in the successor set of the first is more or less constant. Taking a sequence of length n at random, the probability that one of those tasks is not followed by a member of its successor set increases with n, and eventually this probability overwhelms the increase in the number of sequences so that the number of viable sequences declines. However, it might be more “realistic” not to generate techniques strictly by random processes but to generate families of techniques in which tasks are subdivided. Nevertheless, simulations for this study generate tasks and techniques as described here, and alternative forms for generation of tasks and techniques will be reserved for further research.

Division of labor, complementary tasks, and techniques constitute a specification of technology, an alternative to neoclassical production functions. It remains to specify an organizational principle, such as a market system. Moreover, since division of labor implies group action, some process of the formation of groups, such as business firms, must be specified. For the purposes of this study the theory of group formation will be highly abstract and derived from cooperative game theory: coalitions will be formed (and sometimes reformed) for production via the division of labor within the coalitions.[2] Yet another aspect of the model that needs to be specified is the process of decision-making. For purposes of this study decision-making will not be “rational” but adaptive in a sense close to that of John Cross[3]. That is, decisions are boundedly rational[4], but the rules and routines that determine the decisions are gradually adjusted on the basis of experience. For this study, learning and decisions take place in coalitions. These two elements – coalitional dynamics and learning – will be discussed in reverse order.

For this simulation, a coalition is an object. Among the properties of a coalition object is a vector of agents, the members of the coalition. Agents also are objects of an agent class; however, in this simulation agents play a very passive role. Among the properties of an agent object is an average (over time) payoff: essentially, the role of an agent is to receive payments from the coalition of which it is a member and remember the payoffs. The average payoff to an agent is computed via a Koyck lag algorithm with a constant 0.5.

Another property of a coalition is a vector of fifteen technique objects. At the initialization of the game, the techniques are created by pseudorandom processes as described above. Yet another property of a coalition object is a vector of real numbers that express the remembered value of each technique. On each iteration of the simulation,

---

[2] See McCain, 2009, for further detail and a discussion of the dynamics of coalition formation in pure theory.

[3] Cross, John G. (1983), A Theory of Adaptive Economic Behavior (Cambridge:Cambridge University Press).

[4] It is not clear that there is any real alternative to bounded rationality in this model, since rational decisions would require an algorithm to determine the best case out of many thousands of tasks and techniques, with a very complex optimization surface. I conjecture that no such algorithm exists, in general. On the other hand, an implication of this approach is that the technology is endogenous.

each coalition chooses a technique at random according to probabilities that increase monotonically with the remembered values of the techniques. The remembered value for that technique is then updated via a Koyck lag process with a constant of 0.5. In favorable circumstances, this process can converge to a best choice; but since the best decision in any case depends on the choices made by others, and other circumstances that may vary from trial to trial, so that the learning problem is complex and may not lead to a stable or optimal result.

In addition, after the fiftieth iteration, the technique with the lowest remembered value is deleted and replaced by a new technique generated as above and with average value. Thus, endogenous technical progress is a possibility. However, new tasks are not created, a subject for further research.

In these simulations, the population of agents is 100 and at initialization, ten coalitions are formed, with their membership determined by pseudorandom processes. After the first fifty iterations, a routine for reorganization by formation of a new coalition is run. The new coalition may or may not be formed; but if it is, the non-members of the new coalition remain grouped as before; that is, the residual partition of the new coalition is formed (see McCain, 2009). In principle, a new coalition is formed if it makes the members better off. In practice, this depends on things that might be difficult even for an ideally rational agent to predict: reorganizations of the nonmembers into still other new coalitions on later iteration (Koczy, 2007, McCain 2009), unanticipated learning by the new coalition or other coalitions, with consequent shifts of the opportunities available to the new coalition.

As a first approximation, the value of the new coalition is computed, averaging over all of the fifteen techniques it knows, at the market prices for the current iteration, and this is compared to the average payoffs for the agents who are members of the coalition, totaled over the members. The comparison of the potential value of the new coalition with the average payoffs of its members is not unbiased, but is biased by a factor of pessimism, $\mathcal{P}$. The new coalition is formed only if the estimated value of the new coalition exceeds the payoffs to its members by a factor of $\mathcal{P}$. If $\mathcal{P}$=1, then a new coalition is formed whenever its value exceeds the total payoffs of the potential members, but in other cases new coalitions will be formed less frequently than that.

The values of outputs and therefore coalitions are determined by a very abstract analog of the market system. For these simulations, there are four distinct categories of output, that is, four goods or services. Individual demands are not modeled (and this will be a topic for further research). Instead, prices are determined by outputs with a system of interrelated demand functions. Where some goods, are strong substitutes, demand elasticity tends to be elastic, while a case in which all goods are complements tends to result in inelastic demands. We first consider a series of simulations with elastic demands, with elasticity around 2.2, and $\mathcal{P}$=1. For this simulation, there are four goods produced, comprising two pairs of strongly substitutable goods, while otherwise the goods are mild complements.

A series of simulations were run with this specification. We will begin by discussing a single, representative simulation, with random number seed 789123. For this simulation, initially coalitions ranged in size from 5 to 13 members. Roughly between rounds 52 and 171, a series of reorganizations first increase the number of coalitions to

11 and then gradually reduce the number of coalitions to 5, after which the coalition structure stabilizes with 7 for the remainder of the simulation. Figure 1 shows the number of coalitions by round in this simulation.

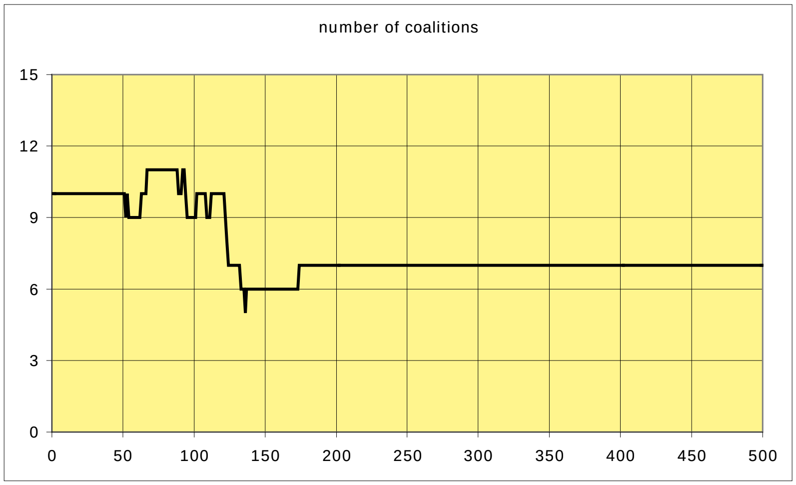

Figure 1. Number of Coalitions in this Simulation

Figure 2 shows the average payoffs to agents in this simulation. The solid (red) line shows the average for all agents and the grayed line shows the average payoff to the members of the coalition that was most recently formed in a deviation. Recorded payoffs for a new coalition are recorded as zero in the iteration at which it is formed. This illustrates two points. 1) The *average* payoffs can fluctuate fairly widely, especially in the period of consecutive reorganizations of the coalition structure. This reflects the inelasticity of demand for the four categories of output. In some iterations, production in some categories may be reduced close to zero, resulting in very high payoffs for the few agents who produce that category. 2) Members of newly formed coalitions often do sometimes than average, even in periods following reorganizations, though they also often experience brief periods of extremely high payoffs. As the simulation stabilizes the deviators tend to do better than the overall average.

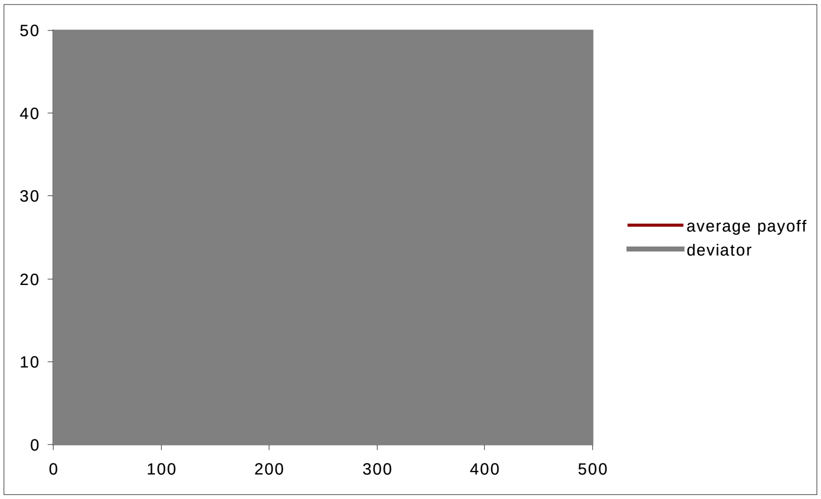

Figure 2. Average Payoffs in a Representative Simulation

Because the demand system is symmetrical, an unweighted sum of the production of the four categories of final product is proportional to real output. The evolution of this aggregate is shown in Figure 3. Examining Figure 3 we see that 1) during the first 50 iterations, there are no reorganizations of the coalition structure nor introduction of new techniques, so that all variation is due to learning. This learning process is productive, yielding growth in aggregate production from less than 90 to above 100, though with some wide fluctuations. 2) Reorganizations may lead to decreases in production, as the reorganization at round 104 is associated with a decline from 100 to 82 – a recession with an 18% drop in production! 3) Once the coalition stucture stabilizes, the learning process again leads to fairly steady growth in production (with occasional volatility) until production stabilizes at about 110 after 150 iterations. The recurrent small fluctuations after round 250 may be cyclical, but should be interpreted with caution because 1) they might be an artifact of the pseudorandom number generator, and 2) the learning algorithm imposes a constant probability of an experimental choice of strategies, which will in general result in a downward fluctuation.

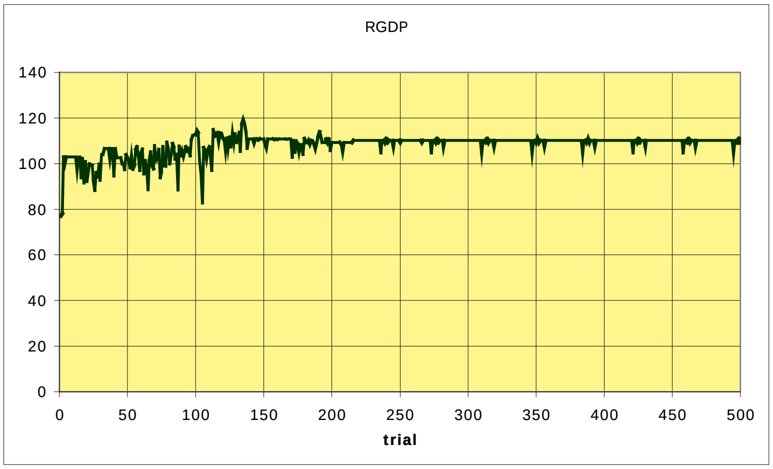

Figure 3. Production in this Simulation

While this simulation with random number seed 789123 was representative in many ways, it was less volatile than some. Consider for contrast the simulation with random number seed 876345. Production in this simulation is shown in Figure 4, and the number of coalitions in Figure 5. We see, that roughly between rounds 70 and 210, the simulation is fairly stable with a quite small number of coalitions, usually as few as two, and production in the range of 120. Between rounds 210 and 220, a series of reorganizations leads to a coalition structure with 5 coalitions and production that then stabilizes at 109-111, never recovering the production in the intermediate period. The following is conjecture, to be verified by a rerun with more extensive reporting: the formation of the new coalition resulted in reduction of the size of the residual coalitions, with the result that they could no longer man highly productive techniques that had previously been used, perhaps producing only one of each pair of substitutable goods.

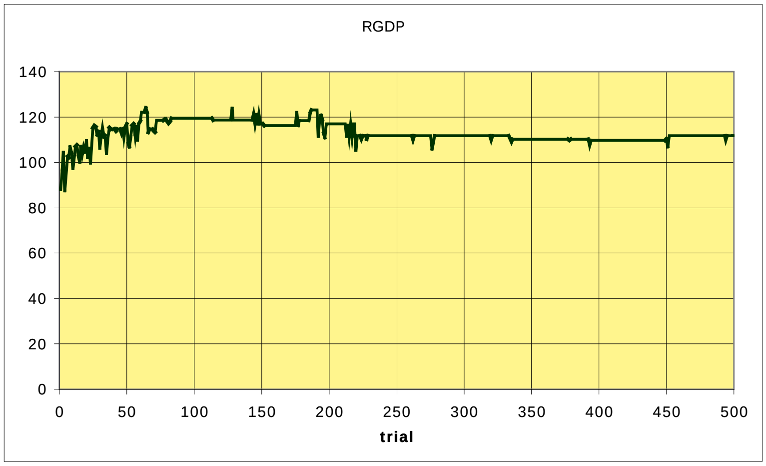

Figure 4. Production in a Contrasting Simulation

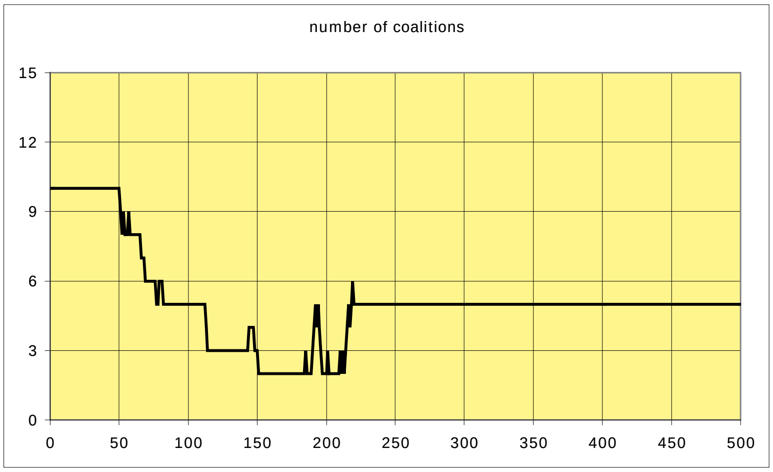

Figure 5. Number of Coalitions in The Contrasting Simulation

For this stage of the study, eight simulations were run with distinct random number seeds, as a rough indication of the range of possible outcomes. Figure 8 shows the number of coalitions over 500 iterations in each of the eight simulations. We see that six of the eight stabilize with seven to eleven coalitions, one with 5, and one does not stabilize within 500 iterations. There seems some tendency toward consolidation with fewer, larger coalitions, perhaps evidence of some degree of economies of scale. Unweighted average production is shown for the eight simulations in Figure 7. We see that the initial period of learning, for about the first 100 iterations, is characterized by increasing production, while all simulations stabilize or become almost stable with production between 100 and just over 120 units. The outlier in production, which nearly stabilizes with production at about 130, continues with production at that level while the number of coalitions varies between 10 and 12. Outliers in the number of coalitions and outliers in production are different simulations.

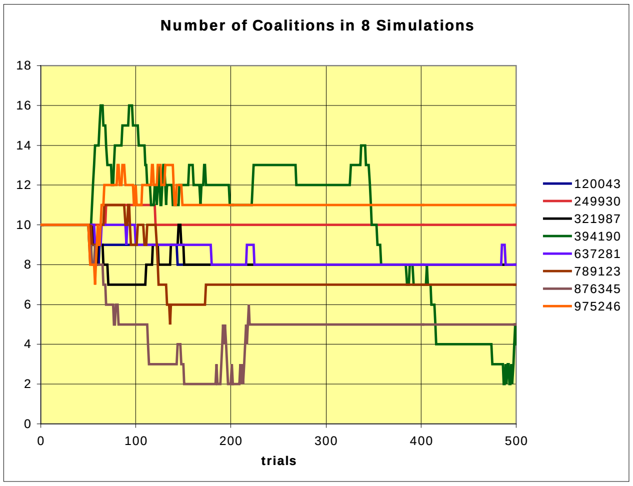

Figure 6. Number of Coalitions

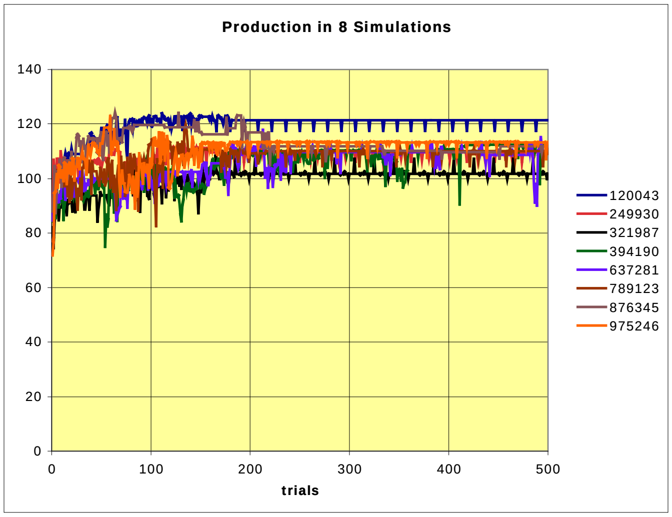

Figure 7. Production

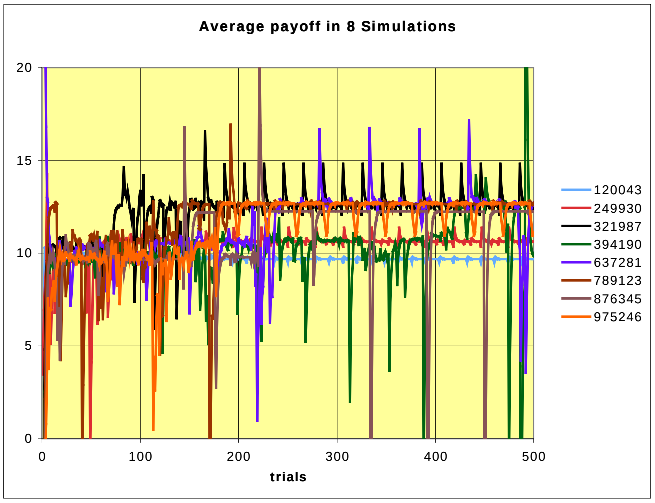

Figure 8. Payoffs

Figure 8 shows average payoffs for the eight simulations. As noted, the average payoff for this model can fluctuate widely, but the payoffs tend to stabilize later in the simulation as coalition structures do.

These simulations are, in some sense, well behaved. Their stability is in part a result of the assumed elasticity of demand. In an alternative valuation model, with all four goods complementary and consequently with elasticity of demand at about 0.86, with pessimism at 1 (i.e. no pessimism) stablization was not observed, as the constant reorganization of the coalition structure frustrates the learning process and leads to relatively many new coalitions that, after further reorganizations, prove to be unprofitable. With pessimism $\mathcal{P}$=2, however, this tendency was largely offset.

Eight simulations were run with this specification and the same random number seeds. Figure 9 shows the number of coalitions over 500 iterations in each of the eight simulations. We see that seven of the eight stabilize with eight to eleven coalitions, and the tenth with 15 coalitions. Unweighted average production is shown for the eight simulations in Figure 10. We see that the initial period of learning, for about the first 100 iterations, is characterized by increasing production, while six simulations stabilize or become almost stable with production between 90 and 120 units. The outlier in production, which nearly stabilizes with production at about 130, continues with production at that level while the number of coalitions varies between 10 and 12. Outliers in the number of coalitions and outliers in production are again different simulations.

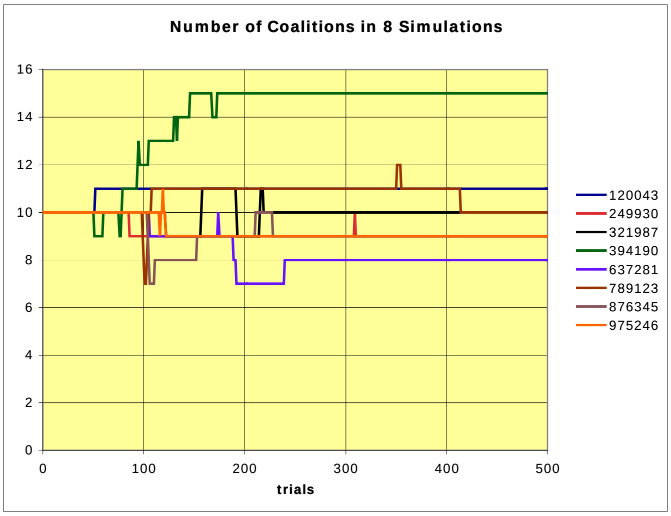

Figure 9. Number of Coalitions

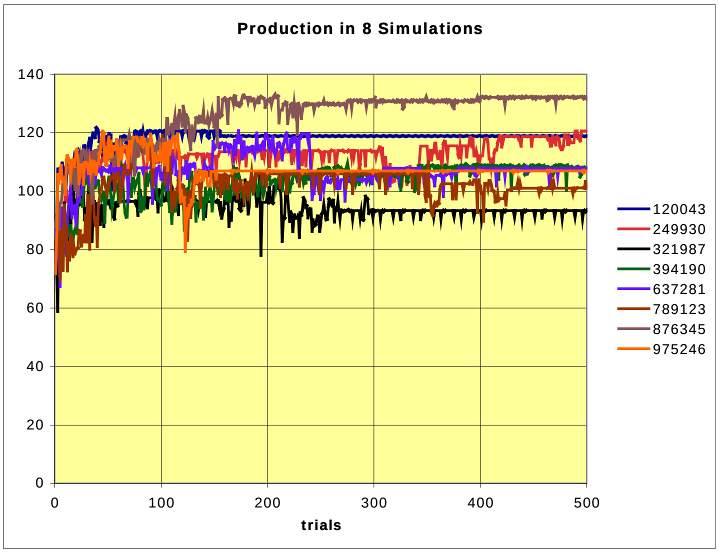

Figure 10. Production

To explore the role of the pessimism variable, a series of simulations were run with the random number seed 394190, with the pessimism constant varying from 1 (neutral) to 5 and 50. The number of coalitions formed by iteration is shown for these eight simulations in Figure 12. For simulations with pessimism at 1 or 1.5, the number of coalitions fluctuates unpredictably throughout the 500 iterations, with many occurrences of very small numbers of coalitions, in the range of 2 to 5. For all simulations with pessimism at 2 or greater, the coalition structure stabilizes, and in all cases but one it stabilizes with the number of coalitions between 9 and 12. With pessimism at just 2, the number of coalitions stabilizes at 15, already noted as an outlier among the simulations with pessimism at 2. With pessimism at 50 there are, unsurprisingly, no reorganizations at all.

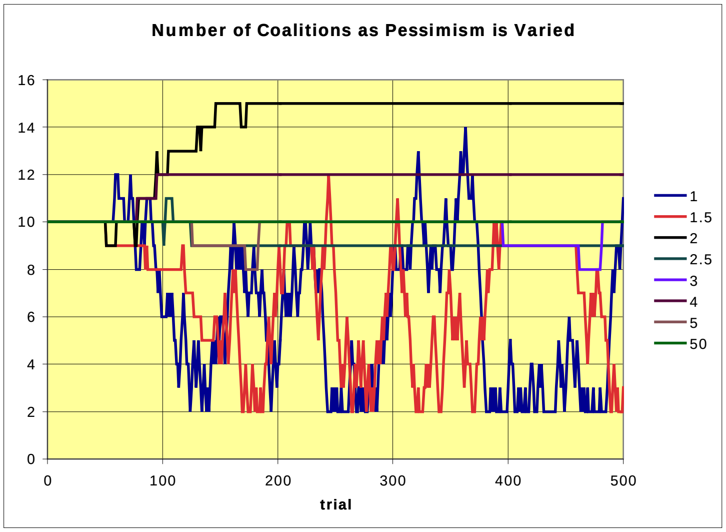

Figure 12. Number of Coalitions as Pessimism Varies

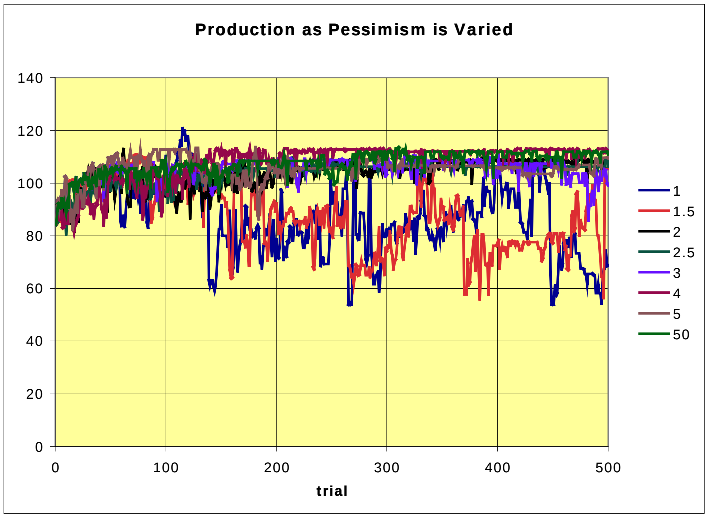

Figure 13. Production as Pessimism Varies

Figure 13 shows the production for simulations with different degrees of pessimism. Here, again, for simulations with pessimism at or above 2, production increases with learning over the first 150 iterations, and then tends to stabilize. Where pessimism is less than 2, however, production becomes quite volatile and, on the average, low. It is clear that, with pessimism at low to neutral levels, the constant reorganization of the coalition structure frustrates the learning process and leads to relatively many new coalitions that, after further reorganizations, prove to be unprofitable. For this series, however, there are only slight improvements with greater pessimism.

These summaries should be taken with caution, as they reflect a fairly small number of distinct simulations. The study is, however, preliminary, and in addition to larger numbers of trials the research will be extended in a number of ways:

1) The demand system will be revised to allow for unsymmetrical demands.
2) Markets will be modeled explicitly in agent terms, with purchases by individual agents and adaptive learning of utility-maximizing demands on their parts.

3) Explore other kinds of coalitional dynamics, including entrepreneurial dynamics and dynamics with individual hiring.

4) Make pessimism endogenous, as a property of individual agents and a product of their experience.

5) Allow the creation of new tasks, as well as new techniques.

6) Enlarge the numbers of agents, tasks and techniques.

Concluding Summary

Multi-agent simulation seems a natural way to study division of labor. However, division of labor requires groups or organizations within which the division takes place, and implies interactive decisions and learning processes, all of which can be modeled or represented in different ways. In this study, learning and coalition formation have been simulated as processes in the nature of evolutionary computation. The preliminary simulations reported here suggest that a coherent pattern of production and valuation can emerge in such a simulation, provided that coalition formation is somewhat pessimistic.

REFERENCES

[1] Cross, John G. (1983), A Theory of Adaptive Economic Behavior (Cambridge:Cambridge University Press).

[2] Kaldor, Nicholas (1934), “The Equilibrium of the Firm,” Economic Journal v. 44, no. 173 (March) pp. 60-76.

[3] Koczy, Laszlo (2007), “A Recursive Core for Partition Function Form Games,” Theory and Decision v. 63, pp. 41-51.

[4] Patrick Legros, Andrew F. Newman, and Eugenio Proto, (2009) “Smithian Growth Through Creative Organization” presented, American Economic Association annual conference, San Francisco, Cal, Jan. 3.

[5] Roger A. McCain (2009) Game Theory and Public Policy (Elgar, in press).

[6] Simon, H. A. (1995), “Artificial Intelligence: An Empirical Science,” Artificial Intelligence v. 77, pp. 95-127.

[7] Smith, Adam (1994), The Wealth of Nations (New York: The Modern Library).