Bank Runs and Monetary Arrangements: 
A Computational Examination

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Abstract
What is the role of inter-bank markets and central banks in coping with banking crises? I answer this question by developing a sensitivity analysis to the model in Diamond and Dybvig (1983). I implement an agent-based economic model to analyze different modifications and extensions to the original. In 36 experiments based on three different versions of the one-bank model the frequency of bank runs dropped from 42% to 17%. This was due to changes in the payoffs structure and social network effects whereby depositors go to the bank if at least three of their proximate neighbors went previously. In other experiments with multiple banks interacting in an inter-bank market with banks having the same market share there are no runs escalating to systemic panics. If there is one bank with a market share twice as big as the other ones, a liquidity crisis spreads to more than one bank. On the other hand, when banks cannot interact runs in isolated banks occur with a higher frequency. Finally, adding a central bank unexpectedly increases the occurrence of bank runs. Institutional complexity helps to reduce the frequency of bank runs. Hence, decentralized institutional structures perform better than centralized ones.

JEL Codes: G21, C63
Keywords: Bank runs, Liquidity, Agent-based computational models.

* Thanks to Richard E. Wagner, Rob Axtell, Carl Ramirez, Dan Klein and Axel Leijonhufvud for their invaluable guidance and support. For many important comments to Jeremy Horpedahl, Maciej Latek, Andrew Nelson, Aaron Orsborn and participants of the seminar at the Center for Social Complexity at George Mason University. All errors are mine. Financial support from the Lynde and Harry Bradley Foundation and the National Science Foundation make possible this research.
1. Introduction

The model of Diamond and Dybvig (1983) is perhaps the modern canonical statement of the claim that money won’t manage itself because a regime of free banking is subject to contagious bank runs and failures, wherein insolvency in one bank can spread to other banks that initially were solvent. Deposit insurance and various forms of regulation might serve as means of restraining such runs. Diamond-Dybvig (hereafter) is austere, involving, among other things, a single bank that neither makes loans nor allows checking accounts. The point of this paper is not to challenge Diamond-Dybvig, but rather is to explore how computational modeling might be brought to bear on the relationship between monetary arrangements and bank runs. Here, I implement an agent-based computational model to analyze different modifications and extensions to the original model.

In the literature the relationship between business cycles and banking panics\(^1\) in comparing the National Banking Era and the Great Depression points out a small difference regarding when panics are leading or lagging the cycle.

\(^1\) Here I follow standard definitions to distinguish a ‘bank run’ as a localized liquidity crisis in one bank when the withdrawal rate is so large that it cannot be served; a ‘banking panic’ whereby several banks face a generalized withdrawals that compromise their liquidity; and a ‘bank failure’ in which case a bank or more suspend payments and/or exit the market. See Selgin (1988), Calomiris and Gorton (1991), and Leijonhufvud (1998) for a broader taxonomy of economic crises.
During the former most of the six\(^2\) panic occurrences identified in Calomiris and Gorton (1991) happened during the downturn and near to the peak\(^3\). Whereas the first wave of bank panics during the Great Depression occurred after thirteen months of the turning point in the cycle (Duckenfield et. al. 2006 Vols. 2-3; Friedman and Schwartz 1963). Also, Mishkin and White (2003) study the major stock market crashes in the twentieth century in the U.S. and report 15 episodes in which stock market crashes precede financial instability or distress including banking panics (although without a clear pattern as leading, coincident or lagged indicator of the business cycle in the U.S.). During the post second world war period up to 2001 there had been 10 recessions of smaller magnitude (in terms of lost GDP) compared to the previous period according to the National Bureau of Economic Research (NBER) but only until the 1973 recession did bank failures occur and also later in the 1980s during the savings and loan crisis.

Calomiris and Gorton (1991) test the operational hypotheses stemming from the two competing models that in the early 80s tried to answer the following question, namely: “How can bank debt contracts be optimal if such contracts lead to banking panics?” op. cit. p. 107\(^4\). Those two competing models were: a) the random deposit withdrawals, and b) the asymmetric information models. The seminal paper for the first strand of models is Diamond-Dybvig and for the

\(^2\) Wicker (2000:xii) reports five by excluding the panic in 1896 due to its localized nature to Chicago and Minneapolis-St. Paul without propagating to the whole country.

\(^3\) The panic in 1873 anticipated the peak of the business cycle for one month.

\(^4\) Another way in which this question can be posed that is closer to the current research on ‘emergent’ or bottom-up organizational and institutional processes is: How do banks spontaneously evolve in markets to provide liquidity and related financial services?
second one its origin is more diverse: Chari and Jagannathan (1988), Gorton and Mullineaux (1987), Diamond (1984), and Jacklin and Bhattacharya (1988) are the most relevant ones.

The first modeling strategy focuses on the liability side of the balance sheet of banks, i.e. deposits in which banks’ main role is to provide ‘liquidity’ that contributes to the smoothing pattern of individual consumption. On a pure theoretical basis this model requires two mechanisms to assure the occurrence of a bank run. These are a sequential-service constraint (Wallace 1988) and a lack of a secondary market for trading assets and bank liabilities (Jacklin 1987).

In the second modeling case the asset side of the balance sheet of banks is analyzed but without any effect from the liability side. Here bank runs happen as a rational response by depositors that neither have full information about the quality of the loans of the banks nor lower transaction costs to monitor that aspect for every loan. Thus, a bank exists to monitor the quality of the loans of a pool of savers to borrowers. Bank runs occur when those savers or depositors are not sure about which banks are solvent.

In both of those cases ‘outside’ equity is not incorporated (Dewatripont and Tirole 1993) while Dowd (1993) including ‘inside’ equity provided by a bank owner allow him to conclude that with this modification bank runs are less likely. On the other hand, in this literature banks’ liabilities do not play any role as
medium of exchange whether as inside money much less as outside money. Gorton and Pennacchi (1990) elaborate a model that can be deemed as an approximation of the ‘credit theory of money’ spelled out by Schumpeter (1939). Under a setting similar to that of Diamond-Dybvig, they derive how banks overcome the asymmetry of information between informed and uninformed traders by creating or offering a riskless security that can be used as a medium of exchange\(^5\). It remains an open avenue for research the modeling of the both sides of the balance sheet of banks and the effect for asset and liability management.

Calomiris and Gorton (1991) implement their empirical test with data from the National Banking Era in the U.S.. They proceed by distinguishing three opposing predictions yielded by these two models. Firstly, the random withdrawals model differs from the asymmetric information model over the source of shocks triggering the panic. In the former case an idiosyncratic shift in the money-demand is the cause of the panic so unusual increases in withdrawals in the pre-panic periods should be observed. On the contrary, in the latter case the shocks might be falling stock prices, real-state prices downfalls or those occurred in whatever assets mostly held in banks’ portfolio.

Secondly, bank failures or liquidations will come from regionally concentrated demand shocks channeled through the banking network according

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\(^5\) Marimon, Nicolini and Teles (2003) present a model where inside money providers’ competition creates incentives that promote efficiency for the government’s supply of outside money.
to the random withdrawals model. For the second type of models high incidences of bank failures will more likely happen in regions that suffer from negative asset shocks. Lastly, both types of models differ in their predictions regarding the management of the crises. The random withdrawals model predicts that a discount window like the one provided by the Fed during the Great Depression should be a sufficient deterrent for banking panics. While the asymmetric information model predicts that inter-bank transfers or some similar sort of collective action by banks will help to internalize and quickly solve bank runs to avoid they turn into systemic panics.

All in all, Calomiris and Gorton (1991) develop an exhaustive statistical test for the three stages just described without implementing an explicit econometric test in either case (Gorton 1988, develops an econometric model to test the implications of the first set of predictions supporting the asymmetric information model). Their findings reject the predictions of the random withdrawal model during the National Banking Era and the Great Depression in favor of the predictions of the asymmetric information model. The historical work by Wicker (2000: 139-147) also supports these results but he remarks that some of the data used by Calomiris and Gorton may be incomplete and more work needs to be done to fill that gap.

Despite this, in this paper, I will not get rid of Diamond-Dybvig altogether. Rather I will relax some of their assumptions. By doing so, I will show who even
in this tradition of models bank runs are less likely. Next, I proceed in three stages, corresponding to three different types of monetary arrangements. First, I examine a system of independent free banks. Second, I explore inter-bank or clearinghouse types of arrangements among banks. Third, I analyze the interaction between an inter-bank market and a central bank.

1.2. Independent Free Banks

The first version of the model only includes a bank à la Diamond-Dybvig. Once it is understood its details I will introduce a model with several banks. A key feature of this computational model involves the specification of the operating rules for individual banks. Banking is organized through free competition among independent firms. The monetary base is all the wealth deposited by agents in the banks and that can be withdrawn at anytime.

Banks have a multiplicity of depositors, most of whom at any moment will have positive balances on deposit. While banks will want to keep reserves to maintain liquidity against claims for redemption by depositors, they will be able to lend out some of their reserves. By doing this, however, also comes a risk of illiquidity that is not present when banks provide only bailment.

The first thing I seek to model is a simple system of free banking that explores the risk of bank runs and the extent of contagion under alternative parametric specifications. To avoid complexities regarding firms and labor
markets, I assume all firms are sole proprietorships. Individuals populate the model and banks. There is an initial (uniform) distribution of money among the individuals who in turn entrust their money to banks. This model is spatial in character and I will specify the details of each version of the model below.

For each individual, receipts and expenditures are both subject to some random variation. Banks will lend based on some myopic forecast on current experience regarding the behavior of their reserves. From this point of departure there are several experiments that can be performed. The first would be to assume that all banks follow the same rules regarding lending and reserves, and that all individuals are subject to the same random variation. This would be a world of homogeneity and would map relatively directly into closed forms of modeling based on averages and representative agents. The challenge and opportunity for computational modeling would involve the presence of heterogeneity, and along several dimensions⁶. Banks can be heterogeneous but will remain myopic and risk neutral. Individuals would differ in the probabilistic circumstances they face. The task would be twofold: (1) to generate bank insolvency and (2) to generate a process of contagion whereby one bank’s insolvency spreads to an otherwise solvent bank.

1.3. Cooperative Arrangements among Banks

⁶ See Axtell (2000) where he points out three different reasons to use agent-based instead of equation-based models. My claim is that this model falls in his third category wherein writing down the equations does not shed light on the problem.
Free banks were not independent but rather operated within webs of associations and cooperative relationships, as well as creating multi-branch structures. Since branch banking and cooperative associations accomplish much the same task regarding the maintenance of liquidity, I shall work with the association from within an environment of otherwise independent banks.

The rules of association generally map into risk-sharing insurance arrangements. This computational model should generate less insolvency in the presence of such clearinghouse arrangements.

1.4. Central Banking

How would things differ when central banking is introduced, especially within a computational framework? The central bank must be described by a different rule of operation than what pertained to clearinghouses. It will also be necessary to pay attention to the central bank’s budget constraint.

2. An Overview of Diamond and Dybvig and related models

In the Diamond-Dybvig model they assume a continuum of depositors with two types of them: impatient and patient ones. The model has three periods and the agents’ types are ‘discovered’—actually, randomly selected from a uniform distribution—in the second period. The agents interact in a coordination game.

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7 One feature of such arrangements was usually controls placed on individual bank portfolios as a condition for belonging to the association. This was necessary to relieve some of the moral hazard that would have otherwise resulted.
with no mixed strategies. In the first period each depositor makes a deposit in the
bank—playing the role of nature more or less in this game—that has a linear
production function with constant returns to scale. Also, the bank neither lends
money nor owns equity. On the third period the bank’s investment matures
providing positive returns. There is no uncertainty either.

The sequence of actions proceeds as follows: if only impatient agents
withdraw during the second period then a bank run does not occur and the
population coordinates on the Pareto superior equilibrium. Conversely, if patient
agents imitate the behavior of impatient agents on the second period instead of
waiting to withdraw in the third and last period, a bank run will occur and the
population will coordinate on the second Nash equilibrium that is not Pareto
optimal.

The following are some relevant extensions to this model to analyze the
robustness of its conclusions. Making agents heterogeneous regarding their
preferences and discounting rates instead of having only two types of agents’
populations that is tantamount to having only two agents. Increasing the number
of banks (see below Temzelides (1997) for such an extension). Adding owner’s
capital or equity to the bank’s balance sheet (Dowd, 1993). Also including the
lending activity of the banks (see Diamond, 1984).
In the Temzelides (1997) model the original setup of Diamond-Dybvig is extended to a repeated game environment. Thus, the author is able to analyze the evolution of agents' learning during the game and it is claimed that this reinforces the reduction in the likelihood of bank runs. This model also incorporates a case of multiple isolated banks, randomizes strategies of patient agents; bank size becomes a control variable, random matching between depositors and banks, banks are subject to demand shocks, there is uncertainty in payoffs due to the random matching process, and furthermore, introduces a small world network for agent(s)-bank(s) interaction for an alternative matching process.

Agents' learning in the simple repeated game version allows them to coordinate longer on the Pareto superior equilibrium than when the game is played only for one-shot. Moreover, if the bank’s size increases, then the population of agents coordinates mostly on the inferior Pareto equilibrium. On the other hand, under the local interaction rule, i.e. small world network, financial contagion is more feasible among banks.

In my first approximation to model bank runs within an agent-based computational model I add heterogeneity across depositors regarding their preferences and discount rates. Initially, there is only one bank that is investing part of its idle funds in bonds that can be turned into cash by selling them in the secondary market. I analyze how network topology can affect the feasibility of
bank runs incorporating neighborhoods. In a second version of the model I incorporate more banks and model an inter-bank market for lending among banks and study the contagion process. I also control bank size to see how this affect to the frequency of bank runs. Finally, I add a central bank type of agent.

3. Implementation in an Agent-Based Computational Framework

Here (implemented in Netlogo 4.0.2) I present a computational replication and introductory modifications to the Diamond-Dybvig model. In this initial version there is only one bank located in the center of the grid and 441 depositors. There are two types of depositors: impatient and patient ones.

During the initialization of the model the depositors make their unitary deposits in the bank then their types are randomly assigned out of an equal probability of being an impatient agent. The parameterization of the computational model is summarized in Table 1, which is a base scenario that I will explore.

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8 Technically, the grid is a torus with 21 by 21 patches that does wrap either vertical or horizontally. Every patch is an agent (depositor), so that this is where the number 441 comes from. Because I am not incorporating any rule for agents’ movement or mutation this is enough for my analysis.
### Table 1. Model Set-up

<table>
<thead>
<tr>
<th>Model Attribute</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depositors, ( D )</td>
<td>441</td>
</tr>
<tr>
<td>Banks, ( B )</td>
<td>1</td>
</tr>
<tr>
<td>Initial Deposit per agent</td>
<td>1</td>
</tr>
<tr>
<td>Agent Type</td>
<td>( p ) (impatient/deposited) = 0.5</td>
</tr>
<tr>
<td></td>
<td>( p ) (patient/deposited) = 0.5</td>
</tr>
<tr>
<td>Withdrawals</td>
<td>Impatient-type = 0</td>
</tr>
<tr>
<td></td>
<td>Patient-type = 0</td>
</tr>
<tr>
<td>( r_1 )</td>
<td>1.2</td>
</tr>
<tr>
<td>( R )</td>
<td>2</td>
</tr>
<tr>
<td>Initial Bank’s Deposits</td>
<td>Sum initial deposits by all agents</td>
</tr>
<tr>
<td># Agents Withdrawing</td>
<td>n-served 0</td>
</tr>
</tbody>
</table>

In the running stage (go procedure) impatient agents will start to withdraw a random proportion of the sum of their initial deposit plus a return. But this will be carried out sequentially in order, agent after agent\(^9\). This allows me to introduce the ‘sequential service constraint’ of the original model.

Let the payoff for impatient agents withdrawing before those who are patient be:

\[
V_1(f_j, r_1) = \begin{cases} 
1.2 & \text{if } f_j < \frac{1}{r_1} \\
0 & \text{if } f_j \geq \frac{1}{r_1} 
\end{cases} 
\]  

(1)

and the return for patient agents be:

\[
V_2(f_j, r_1) = R(1 - r_1 f_j)/(1 - f) 
\]

(2)

where \( f_j \) is the number of bank’s customers being served at time \( t \) as a fraction of the total number of initial deposits and \( r_1 \) is the gross return for those agents withdrawing before the bank’s investment has matured otherwise the return is \( R \).

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\(^9\) Since Netlogo 4.0 exists the possibility of asking agents in an orderly fashion throughout the grid one by one.
The following relationships are established: \( 1 \leq r_1 < R \). Finally, \( f \) is the fraction of impatient agents in relation to the total number of agents. Equations (1) and (2) are slightly different from equations (2) and (3) in Diamond-Dybvig model.

There are also two regimes for the rates of return, which are: 'fixed' and 'random'. The first follows the Diamond-Dybvig assumptions regarding homogeneity across agents. The latter allows me to analyze heterogeneity across agents. In Table 1, the case for \( r_1 \) fixed (1.2) and equal across agents is presented. Also, \( R \) here is set equal to 2. For the second case the rates of returns are randomly drawn from a uniform distribution: \( r_1(1, 1.5) \) and \( R(1, 2) \).

Patient agents have a higher fitness or payoff from which they can consume (withdraw). That is \( V_2 > V_1 \), given the sequential service constraint, the availability of funds in the bank, and the rate of withdrawals.

Below I describe the model’s agents and its features.

**Depositors**: each depositor has information about its deposits, amount withdrew, payoff or fitness, returns for withdrawing at an early or later date, and an account to register how much is left in the bank.

**Bank(s)**: register their initial deposits, the amounts withdrew by depositors at every time-step during the simulation, how many agents have been served, and their final balance. The bank invests according to the following condition:

\[
I_t(b_{t-1}, R) = \begin{cases} 
R & \text{if } n_j < n_1 \\
0 & \text{if } n_j > n_1
\end{cases}
\]  
(3)
where $I_t$ is the bank’s investment per period; that takes its positive balance $b_{t-1}$ to be invested at the rate of return $R$—which is the same gross rate of return that agents will receive for being patients. This will happen so long as the number of agents withdrawing before the investment matures $n_j$ is less than or equal to the total number of impatient agents $n_1$. Finally, if the bank goes bankrupt the simulation stops.

*Main interaction rule*: There are two rates of return to determine depositors’ payoffs and accounts. Two different regimes for agents’ consumption can also be chosen. Firstly, agents consume altogether their respective payoffs, $V_1$ and $V_2$, every time they withdraw. In the second case each agent withdraws a (random) proportion $w_j (0, 1)$.

Once all the agents are initialized the depositors or bank’s customers have to decide whether to withdraw at every period in the simulation. Impatient agents withdraw first, and then patient agents have to decide whether to withdraw. The decision of withdrawing now or later also depends upon the following relationship taken from Diamond-Dybvig, that is, that the proportion of customers being served with respect to the total number of customers may or not be less than the inverse of the return for withdrawing earlier, as explained in equation (1).

The bank balances its account and keeps serving its customers until it has run out of money. Every customer can withdraw from the bank only after it has served the previous customer. This is not a concurrent procedure.
3.1 Results:

Figure 1

In Figure 1 I present a computational model based on the conditions exposed above. In this figure there are four panels. In panel a) after two time-steps the bank ran out of savings or liquidity to serve its clients. The blue line tracks the change in final balances or net deposits in the bank. The black line records total withdrawals from both types of agents. The green line depicts only the total withdrawals from agents (impatient or patient) withdrawing in earlier periods. Finally the brown line depicts those withdrawals from those agents (only patient ones) who wait. There were only 336 agents who could be served during this experiment. The remaining 105 could not even get their initial deposits back.

In panel b) the bank does not run out of assets. The variables achieve a stationary equilibrium whereby total withdrawals hover over 300 value units. It is important to observe that the only modification in this experiment from the previous one is that the consumption schedule per agent is variable or heterogeneous across population. In panel c) again with constant consumption schedule per agent but with heterogeneous rates of returns after five time-steps the bank ran out of assets and only could serve to 347 agent depositors. This is 127 more than the total number of impatient agents, i.e. 58% greater. Thus, patient agents withdrawing earlier than they were supposed to do it bring about
the bank run. There were 94 depositors who were unable to withdraw after the bank went bankrupt. Lastly, in panel d) with variable consumption schedule and heterogeneous rates of returns across agents a bank run does not occur. The bank’s balance and total withdrawals hover over 200. Another stationary equilibrium is again achieved.

[About Here]

Figure 2

In Figure 2, I present a modified version of the previous computational model whereby the depositor-bank contract is modified to have a different payoff structure. In this case, the payoffs for each period are given by:

\[
V_1(f_j, r_1) = \begin{cases} 
1 & \text{if } f_j > f \\
 c_1 & \text{if } f_j \leq f 
\end{cases}
\]  

\[
V_2(f_j, R) = \begin{cases} 
R & \text{if } f_j > f \\
R(1 - c_1 f)/(1 - f) & \text{if } f_j \leq f 
\end{cases}
\]

where \( f_j \) is the number of bank’s customers being served at time \( t \) and \( c_1 \) is the optimal consumption for those agents withdrawing at period one, otherwise they consume \( c_2 \) in period two. The latter is equal to the second expression in the payoff function for \( V_2 \). The following relationships hold: \( c_1 < c_2, c_1 \geq 1, \) and \( R > 1 \). Finally, \( f \) is the total number of impatient agents. Besides these changes the other characteristics of the agents and the rest of the simulation environment remains the same as before. These payoffs are simpler than those in equations (1) and (2) and yield different results as I will report on Table 2.
The plots in Figure 2 show the results of the same four experiments I implemented previously. In panel a) after three time-steps the bank did run out of assets. The bank served 365 clients, that is 76 of them were unable to get any funds back. In panel b) there is not a bank run, after fifty time-steps. In this case the consumption schedule per agent was heterogeneous across agents. In panel c) with heterogeneous rates of return and the same consumption pattern for all agents a bank run does not occur after fifty time-steps. The bank’s balance declined to 5 value units after three time-steps, but then recovered to fluctuate around 45 value units. Note that in this case earlier withdrawals are always higher than later ones. Finally, in panel d) with rates of return and consumption patterns heterogeneous across agents a bank run does not occur after fifty time-steps. The bank remains liquid with about 200 units in available funds.

[About Here]

Figure 3

In Figure 3 another modification was added to the previous setting. This time I inserted a social network component to the model. Impatient agents make their decision to withdraw first and then patient agents ask to three of their eight neighbors—whether patient or impatient—if they have already withdrawn any funds from the bank in order for them to start to withdraw. Again, I experimented with these four variations as in both previous cases. In this model only two bank runs occur. The one depicted in Figure 3 is the first case with fixed

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10 I implemented a Moore neighborhood with a radius of one that has, at most, eight neighbors for the agent at the center.
interest rates and constant consumption across agents (see panel a). Also, total withdrawals are higher than the bank’s final balance in panel c), in which consumption patterns are constant for the agents. The reverse is true when this is changed to a heterogeneous regime across agents, panels b) and d).

<table>
<thead>
<tr>
<th>p(impatient/deposited)</th>
<th>Scenario a</th>
<th>Scenario b</th>
<th>Scenario c</th>
<th>Scenario d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>Run</td>
<td>no run</td>
<td>Run</td>
<td>no run</td>
</tr>
<tr>
<td>0.5</td>
<td>Run</td>
<td>no run</td>
<td>Run</td>
<td>no run</td>
</tr>
<tr>
<td>0.75</td>
<td>Run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
<tr>
<td>Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>Run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
<tr>
<td>0.5</td>
<td>Run</td>
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<td>no run</td>
<td>no run</td>
</tr>
<tr>
<td>0.75</td>
<td>no run</td>
<td>no run</td>
<td>Run</td>
<td>no run</td>
</tr>
<tr>
<td>Model 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>Run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
<tr>
<td>0.5</td>
<td>Run</td>
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<tr>
<td>0.75</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
<td>no run</td>
</tr>
</tbody>
</table>

Table 2: Experimental results. [Average after 50 simulations]

In Table 2 I show the overall results not just for the four cases or scenarios that I already described for each of the three different models, but also for two additional variations when the probability of being an impatient agent is 0.2 and 0.75. Thus, I ran 12 experiments per each model or 36 experiments overall. In model 1 there were five occurrences of bank runs (42%); in model 2 only three occurrences (25%); and in model 3 only two of these occurrences (17%). Hence, the simple modifications that I have added to the first model reduced the frequency of bank runs. Especially when I modified the topology of the ‘artificial’ world by adding Moore neighborhoods this frequency dropped by 60% compared with the first model.
I have implemented in these simulations changes that allow me to explore the dynamics of the Diamond-Dybvig model within not just a repeated version but also within a population of heterogeneous agents. The aggregate dynamics that I analyzed was the result of the individual or agent-based interacting behavior subject to the rules of the simulated environment.

3.2 Multiple Banks:
Here I introduce a substantial change to the model by increasing the number of banks. There will be four banks. Each bank will have no more than ten customers. Thus, there will be a banking market with four banks and forty depositors. I have chosen the second specification from this model in the previous section with a slight extension. In this version the payoff structure was modified according to eqs. (4) and (5) where the decisions whether to withdraw depend simply on the size of the queue and the payoff for consuming earlier is always lower than postponing consumption. The extension is just a modification of the rule under which patient agents make their decision whether to withdraw not just based on the queue size but also considering how much higher is the interest rate paid by the bank on deposits given each customer ‘subjective’ interest rate.

My aim here is to answer the following: under what conditions a liquidity crisis in a given bank can spread or be contagious to others and how fast does this occur? To make this operational I implement an inter-bank market whereby
each bank with a temporal lack of funds but with customers in the queue can borrow money to any other bank that has a positive balance. After serving its customers the bank will be required to repay the loan with interest. If the bank is unable to repay its debt and/or to serve its customers it will go bankrupt. Regarding customers they, also, stop withdrawing from the bank if they have consumed all of their savings from it.

<table>
<thead>
<tr>
<th>Model 4</th>
<th>No Interbank Market</th>
<th>Interbank Market</th>
<th>One Big Bank</th>
</tr>
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<tbody>
<tr>
<td>#Patient</td>
<td>16</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>#Impatient</td>
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<td>24</td>
<td>11</td>
</tr>
<tr>
<td>Run</td>
<td>Yes</td>
<td>None</td>
<td>Only big one</td>
</tr>
<tr>
<td>Time-step</td>
<td>6</td>
<td>25</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3: Constant consumption and heterogeneous interest rates across customers and banks and with p(impatient/deposited) = 0.5. [Average after 50 simulations]

In Table 3 the simulation results show three different experiments run within this version of the model. In the baseline scenario each bank is isolated from the other banks and, in turn, its customers. At the second period two of the four banks cannot keep serving their clients. In the next period another bank ‘fails’ and by the sixth period the last bank also stop serving its clients. Thus, there is an overall bankrupt of; i.e. a banking panic; the system that takes place gradually. This banking panic, though, is neither due to a contagion effect brought about by customers sharing information nor one localized bank run being spreading out to the whole system.
In the second case (second column in Table 3) a basic ‘inter-bank market’ is modeled to explore how such an institutional environment can facilitate or not financial contagion. As a matter of fact, bank runs did not occur in any of the banks and each of them could serve all of its customers. This result was a bit surprising since what I expected compared with the previous case was actually to observe the reverse. Each bank determines its own interest rate policy and when it will require making a loan from another more liquid bank. The latter depends on how many impatient versus patient agents each bank has in its queue and what are the agents’ particular ‘subjective’ interest rates expected from trading with the bank.

In the last case I present an extension of the previous one. That is I started with the inter-bank market model and reduced the number of customers to twenty-five from the original forty. Then I allocate the customers arbitrarily to make sure that only one of them will get ten customers and the rest only five per bank. By doing so, I get an inter-bank market with one of them twice as big in customers and liabilities (deposits). Here I observe another unexpected result, which is a bank run at period 4 of the bigger bank and the smaller banks being able to serve all of their customers. One interesting aspect of the extension is that before running out of liquidity the bigger bank lent money to another smaller bank that could serve its customers.

3.3. A new agent as a Central Bank:
In this extension of the model I add a new agent to the previous multi-bank model with its inter-bank feature. The characteristics of this ‘artificial’ central bank are the following: a) it controls the monetary base of this economy, b) it collects the reserves from the banks, c) it establishes the legal reserve ratio, d) it determines its policy for a discount rate, and e) it can lend money to any of the other banks. Its balance is the sum of the monetary base plus the total reserves from the banks.

This model will allow me to analyze the interaction between these two institutional arrangements; that is, a central bank and an inter-bank market; which to a great extent is found currently in most of the countries. The central bank can implement its policies by means of three instruments, namely: 1) altering the quantity of monetary base, 2) shifting the legal reserve ratio for the banking system, and 3) changing its discount rate below or above the fixed inter-bank market rate of 1% as in the previous model. I develop experiments based on the different policy alternatives for the central bank and the probability of being impatient for depositors. The results for the frequency of bank runs are reported in Table 4.
The monetary base, the discount rate, and the legal reserve ratio are each changed twice, times the three choices provided by the value of the probability of being impatient amounts to 24 experiments. These are showed in gray boxes in Table 4. When the probability of being impatient is 0.5 it does not matter for the results if the legal reserve ratio is either 2% or 30%. I arbitrarily selected those two reserve ratios, although they may be interpreted as the typical levels set by developed and developing countries, respectively [source]. The results are that with a monetary base of 5 units (the total money supply is monetary base + deposits = 5 + 40) only 25% of the banks suffer runs or liquidity crisis. However, when the monetary base increases up to 8 units there are no runs at all. These results hold even when the central bank discount rate changes from 0.08% (below the inter-bank market rate) to 0.12% (above the inter-bank market rate).
An interesting dynamics appears when I change the probability of being impatient to 0.2 and 0.75. In the case of $p = 0.2$ I observe that changing the legal reserve ratio does matter. When the reserve ratio is 2% and with a monetary base of 5 units 50% of the banks is unable to satisfy its demand for withdrawals. This does not occur when the monetary base is increased to 8 units. But when the legal reserve ratio is increased to 30% there are no liquidity crises at all. The central bank utilizes the money collected through the reserves to lend money to banks that cannot meet its customers’ demand even if this bank has also got loans from the inter-bank market [what if this is not allowed]. On the other hand, when the probability of being impatient increases to 0.75 liquidity problems occur in 25% of the banks only when the reserve ratio is 2%.

Some more general observations are that the level of the discount rate does not matter for the different set of policy changes I analyze. When the reserve ratio is 2% there is a higher occurrence of bank runs. When the probability of being impatient increases to 0.75 I observe the fewer number of bank runs compared either to 0.5 or 0.2. For the last two cases the difference is that when $p = 0.2$ bank runs occur only when the reserve ratio is 2%, while when $p = 0.5$ the level of the reserve ratio does not matter. But the intensity of the bank runs is greater when $p$ is 0.2 than when is 0.5 in which case the crises are of similar ‘magnitude’ for the two reserve ratio levels. It seems natural to ask, why are there more bank runs when $p$ is 0.2 than when $p$ is 0.75? Ex-ante it does not make sense. But the answer rest on the observation that in this model fewer
impatient agents imply a smaller magnitude of anticipated withdrawals compared with total withdrawals from patient agents that are turn out to be larger.

Last but not least, why do bank runs still occur when I have a central bank and an inter-bank market working together? Firstly, each bank balances its accounts by deducting reserves deposited in the central bank. Secondly, each of them can borrow no more than 10% of the outstanding balance of the central bank at every period. Loans from the central bank and the inter-bank market are scheduled to pay in the next period plus and interest out of any remainder in banks’ balances. The main difference with the previous model, which has only an inter-bank market where no runs did occur, is that in this version the reserves are centralized in the central bank and are not anymore at the disposal of each of the banks competing in the inter-bank market for funds. Banks will get indebted first by requiring funds from the inter-bank market, and then they will proceed to ask to the central bank for any loan. However, if any bank that has no required funds yet from any other bank and does not have money to keep serving its customers it can get the funds from the central bank, too.

4. Discussion

I have implemented agents within a microeconomic environment and study their statistical aggregates. To some extent these statistical patterns are ‘emergent’ in the sense of Epstein and Axtell (1996) because they were not
imposed upon the agents' behavior. These aggregate patterns ‘grow up’ from the microeconomic structure in which the agents are embedded. Because the models also include interaction between depositors and banks (and in the third model among the depositors, too) they can also be an example of self-organized complex systems.

In another sense I can try to go beyond of my own framework to incorporate a broader view of ‘emergent’ social patterns. In each of the models agents’ interaction took place within a set of rules based on economic behavior. These were part of the design of the environments that I specify for each of the models. But, how can those rules being also the result of an emergent process? On one hand, this can be a question answered by evolutionary computation. On the other hand, I can provide a rationale for that process from an evolutionary economic point of view. I take the latter approach here.

In the model of multiple banks I experimented with a version in which there was neither an inter-bank market nor a central bank. In this case I was dealing in a stylized manner with isolated banks that did not pool reserves when liquidity was scarce. Its behavior to a great extent is similar to a unit-banking system. A clearinghouse association is an organization that purports to overcome the lack of pooled reserves for a banking system. This together with the appearance of an inter-bank market for loans explains the evolution towards a
more integrated system that allocates reserves throughout all banks by portfolio adjustments.

How could these institutional solutions emerge? Every time that there was a high increase in demand for withdrawals individual banks suffered important reserve losses that led to banking runs. Some banks failed while others don’t. That is, banks with excess reserves could not increase profits by lending to other banks with lack of liquidity. It is as if an opportunity for increasing business was not been exploited. Here lies the economic origin of the inter-bank market. The development of more institutionalized forms to cope with liquidity risks is rather the result of a trial and error process. Several times the banking industry should have suffered massive losses or panics. Until a group of bankers may decide to establish organizations such as a clearinghouse association to reduce the transaction costs of checks clearing, transfer of net balances and more importantly to pool reserves to accommodate liquidity across the banking industry.

This gives place to the distinction between members and non-members of these types of associations or private clubs that provide public goods to members. This is important to naturally test under what scheme banks may reduce the overall risk of panics. Due to a unitary banking industry all the network externalities that a branch-banking industry may offer under clearinghouses will be absent. At a localized level member banks will be covered even in a unitary
system by the pooling of reserves with all the other local banks also participating of this type of associations.

In the models I have not yet incorporated relevant industry characteristics such as branch banking. Calomiris (1992) and Ramírez (2003) present evidence for the pre-Great Depression period comparing branching regulations across the U.S. and Virginia versus West Virginia. Their results show that banks under states that allowed branching were more resilient to agricultural or seasonal crises than banks under states that did not. An evolutionary account of banking institutions should make room for an explanation of the different industrial architectures that may flourish within different rules, and other set of institutions belonging to property rights and monetary arrangements. I leave such extensions for future work.

But even if I take into account the emergence of more resilient industrial architectures this should not be interpreted as a claim that certain banking rules and institutions eliminate altogether the risk of failure. Tussing (1967) presents a compelling rationale for the reduction of wasted resources if banks were treated as any other commercial firms regarding bankruptcy. His claim is another way to argue that if bankers know that they will be bailed out during economic crises, then there will be incentives for them to mis-allocate their resources.
The emergence of institutions such as the central bank has been varied in developed countries. The Bank of England was explicitly founded for purely fiscal reasons (White 1999: 81-3). On the other hand, the Federal Reserve System was the result of a prolonged public discussion resembling what the discussion on social security reform currently is. At least between 1894 up to its foundation in 1913 there debates in which bankers from New York, Chicago, the American Bank Association, several Chambers of Commerce throughout the states, academicians, and politicians, were engaged in (Wicker 2005). The main argument for its foundation albeit was not the frequent banking panics of the National Banking Era but what was considered its ultimate cause namely the inelastic money supply (Wicker 2005: 22-41).

For some a monetary system reigned by a central bank is a sub-optimal solution (Hayek 1978, Mundell 1999) compared either to a classical gold standard system or a competitive private system of money (Klein 1974). In this vein, it is interesting how recent historical research on the origins of the Fed (White op. cit.) notes how the original proposals for monetary and banking reform in the U.S did not include at all the existence of a central bank. It was during the travels of the members of the Monetary Commission; organized by senator Nelson Aldrich between 1908 and 1910; that the idea of establishing a central bank was adopted. Since the leading economic countries at that time like England and France had central banks it seems that imitative behavior can also lock us in a standard not necessarily Pareto optimal.
5. Concluding Remarks

I have modified and extended the original Diamond-Dybvig model. Firstly, I developed a discrete framework for agents and time rather than keeping the unrealistic continuity assumption. Secondly, I introduced the important effects of social networks and provide a better rationale for social interaction. Next, I increased the number of banks and gradually add institutional complexity to the baseline model.

The agents are very simple in that they do not have sophisticated cognitive capabilities or full information. But they do interact within a microeconomic environment in a dynamic fashion. Yet I was able to study the ‘emergent’ aggregate results à la Epstein – Axtell stemming from the agents’ interaction. In most of the models introduced here; except in the inter-bank market case; I still found the occurrence of bank runs. I will include a discussion of this issue in the next section. The models are still very stylized yielding mostly qualitative results. An important step forward is to empirically validate their main implications.
References


Schumpeter J. (1939) Business Cycles.


a) Fixed rates of return to $r_1 = 1.2$ and $R = 2$; constant consumption $w_j = 1$; 221 patient and 220 impatient agents.

b) Fixed rates of return to $r_1 = 1.2$ and $R = 2$; variable consumption $w_j = (0, 1)$; 221 patient and 220 impatient agents.

c) Random heterogeneous rates of returns; constant consumption $w_j = 1$; 221 patient and 220 impatient agents.

d) Random heterogeneous rates of returns; variable consumption $w_j = (0, 1)$; 221 patient and 220 impatient agents.

Figure 1
a) Fixed rates of return to $r_1 = 1.2$ and $R = 2$; constant consumption $w_j = 1$; 221 patient and 220 impatient agents.

b) Fixed rates of return to $r_1 = 1.2$ and $R = 2$; variable consumption $w_j = (0, 1)$; 221 patient and 220 impatient agents.

c) Random heterogeneous rates of returns; constant consumption $w_j = 1$; 221 patient and 220 impatient agents.

d) Random heterogeneous rates of returns; variable consumption $w_j = (0, 1)$; 221 patient and 220 impatient agents.

Figure 2
a) Fixed rates of return to $r_1 = 1.2$ and $R = 2$; constant consumption $w_j = 1$; 221 patient and 220 impatient agents.

b) Fixed rates of return to $r_1 = 1.2$ and $R = 2$; variable consumption $w_j = (0, 1)$; 221 patient and 220 impatient agents.

c) Random heterogeneous rates of returns; constant consumption $w_j = 1$; 221 patient and 220 impatient agents.

d) Random heterogeneous rates of returns; variable consumption $w_j = (0, 1)$; 221 patient and 220 impatient agents.

Figure 3