

Testing the evolution of cooperation through network reciprocity under different imitation strategies

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Abstract

One of the interesting extensions of evolutionary game theory is to limit the possible interactions of each agent to only the subset of population, mainly to those agents belonging to the spatial or social neighborhood of the particular individual. In our article we extend existing literature on the evolutionary games on the networks by examining how both the network topologies and the mechanisms, in our case imitation strategies, through which particular behavior spreads in the population affect the evolutionary dynamics of the game. Our agent-based computational experiments show that both the interaction structure and the way how agents choose to imitate the others strongly influence the dynamics of the game.

1 Introduction

Cooperative behavior is fundamental for the existence of the complex modern societies as we know them today. It is thus interesting for all social scientists to understand how cooperation emerges and persists even when in many situations the rational reasoning tells us that individuals should have more incentives to act in rather selfish than collectively beneficial manner.

Useful tool for such a reasoning is the game theoretical framework ([28]) developed extensively in 1950's and well known metaphor for social dilemmas the prisoner's dilemma game (PD). In the basic version of PD two individuals facing the trade-off between cooperative collectively beneficial behavior and non-cooperative self-interested behavior. Assuming that both players chose their actions simultaneously four possible situations can arise, either both players cooperate or one or both of them defect. Different payoffs are attached to each

of these situations in PD game. For mutual cooperation players receive reward R . If both players defect they are punished by payoff P which is lower than R . If one player cooperates while the other does not the cooperating one leaves with the so-called sucker's payoff S and the non-cooperating one with the highest payoff in the game T usually referred as a temptation to defect. The resulting orderings of payoffs is $T > R > P > S$.

In this setting defection is the best choice irrespective of what opponent does. Hence, in the so-called Nash equilibrium (NE) both players do not cooperate in prisoner's dilemma game. Such a conclusion sounds contradictory to what we observe in the real live. Cooperative behavior is rather abundant than extinct in the modern societies. So, let's say for example that individuals are not like a game theorists and do not deduce the best strategy for the game or the situations that can be represented with it. Rather assume that the behaviors evolve based on a simple evolutionary dynamics in which more successful behavior spreads or is adopted by larger portion of individuals. This is in a very abbreviated version research question of the evolutionary game theory (EGT) ([41]). In the EGT we use concept of evolutionary stable strategy (ESS) which represents the strategy that when adopted by the population of players cannot be invaded by the mutant. But for the prisoner's dilemma game NE and ESS coincide so the assumption of lower cognitive capabilities of the players alone cannot explain why the conclusions given by these undoubtedly useful theoretical tools differ from what we usually observe around us.

Recent research on the evolution of cooperative behavior showed that evolutionary version of the game theoretical models can still be used for explanation of the persistence of cooperation in the population of individuals facing the social dilemma in the form of PD if we limit the interactions of each of them to only subset of population. In the traditional version of EGT models individuals are assumed to interact with every other member of the population with the same probability, the so-called well-mixing assumption. But this assumption is often violated in the real systems and more or less complex topological interaction structures are more abundant. The first studies on spatial and network versions of the PD game showed that more structured population evolve quite differently to the well-mixed ones. In such populations not only cooperation survives but even becomes dominating strategy in the population.

In our paper we continue this research with the motivation to deepen the understanding of how different topological structures influence the evolutionary dynamics. We construct a model of the evolution of strategies in the prisoner's dilemma game played on networks. We use four different interaction topologies: grid, ring, small-world and scale-free networks. To examine their effects on the evolutionary dynamics of the model we introduce four different mechanisms (in our case imitation strategies) through which strategies spread in the population. The aim of our work is to find out if the final outcome i.e. the stationary distribution of strategies in the population is solely result of the topological features of particular interaction structure or result of the combination of both topology and other evolutionary mechanisms. The collection of the results from many Monte-Carlo simulations of our model shows that the latter explanation

is more correct.

2 Literature review

First spatial models with regular and simple spatial structures have shown that cooperation can be evolutionary viable for a large subset of the PD game parameters ([29], [31], [33]) and drove attention to the impact of spatial structure on the stable state level of cooperation ([13], [37]). Recently, the evolution of cooperation has been examined on networks with heterogeneous degree distribution that better reflect topology of the real world networks of contacts ([6], [44]). Real world networks are very often characterized by high clustering coefficient that enables agents to reciprocate each other to outperform defectors. At the same time, [26] has shown that the path any two individuals are connected with is much shorter than in case of regular lattice (a simple network with high clustering coefficient). Playing PD on Watts and Strogatz ([44]) small-world network derived from ring with random rewiring leads to lower stable state level of cooperation than the ring would have. This result is given by long-range connections that open the possibility of free riding to more players ([47]). Three different dynamics are observed in small-world networks with respect to the level of temptation. For the temptation payoff high (low) system converges towards equilibrium with full (no) cooperation ([1]). The convergence is faster with more rewiring (lower clustering) in the small-world network. Between these two extremes number of cooperators and clustering coefficient are closely related. Cooperation is more successful in the more clustered networks with less long range connections ([25]). [36] shows that the level of cooperation in the small-world networks results from two sources; heterogeneous connectivity distribution leads to the overall enhancement of cooperation for all values of temptation parameter and pure small-world effect that increases survivability of cooperators up to larger values of the temptation parameter. Pure small-world effect is very subtle and in the heterogeneous small-worlds almost indistinguishable. An interesting behavior with sudden breakdowns of cooperation followed by long time recovery occurs in small-world networks if an influential node with directed random links to the other nodes in the network is added to the model ([23]). Scale-free networks generally display high cooperation levels ([47]). The emergence of cooperation is closely related to the degree distribution with highly connected nodes being largely populated by cooperators and defectors occupying nodes with low degrees ([36]). This regularity in the distribution of strategies stems from the following mechanism of hubs' strategy choice observed in the simulations. Let us begin with hub occupied by the defector surrounded by cooperators. Having high degree means having substantially higher payoff from exploitation of many cooperating neighbors. In the next periods most of these neighbors switch their strategy to defection and the payoff of the hub decreases consecutively. Remaining cooperating neighbors are able to profit from cooperation and attain higher payoff than hub surrounded by defectors which in the following periods leads to restoration of cooperation by the hub. Once hubs

are populated by cooperators, the fraction of cooperators in their neighborhood starts to increase which enhances spread of cooperation even further (see [36]). Based on the series of simulation [37] conclude that scale-free topology ensure the prevalence of cooperation in the PD and snowdrift game for the entire range of parameters.

A review of the literature on spatial PD reveals that the imitation rules used by various authors vary significantly ranging from simple rules like “copy the strategy of the neighbor with the highest payoff” (e.g. [29], [47]) to stochastic rules taking into account payoff differences (e.g. [31]). Therefore, the question of robustness and generality of the results, as addressed in our article, against various imitation rules arises. [48] ask similar question in connection with [37] statement that scale-free networks provide a unifying framework for the emergence of cooperation. They test [37] results for three updating rules and conclude that scale-free topology may not be the crucial factor for the emergence of cooperation with updating rules and dynamics governing the game being of the utmost importance. As we will show in the following sections none of the examined topologies can be used solely as an explanation for particular stationary mix of strategies in the population. It is the combination of topological features and other evolutionary mechanisms that matter.

3 The model

In this section we define the basic constituents of our model. Our model is built as an evolutionary prisoner’s dilemma game played on network. The model consists of four parts

- the set of agents,
- the set of strategies and corresponding payoffs,
- the interaction structure, and
- update rules, according to which agents revise the chosen strategy

Contrary to more traditional game-theoretical models we do not assume rational agents able to deduce a particular Nash equilibrium strategy but we allow boundedly rational agents to gradually update their strategies and adapt to the changes in the environment they co-create with other agents. In this sense our model is more closely related to the evolutionary game-theoretic models ([45]) or population games ([35]). Although, additionally to the evolutionary nature of our model we abandon the assumption of well-mixing and instead assume more structured population with more or less complex interaction structure represented as a network. What we are interested in is some steady state or stationary distribution of strategies in the population.

Table 1: **Payoff matrix**

	Agent j cooperates	Agent j defects
Agent i cooperates	R,R	S, T
Agent i defects	T, S	P,P

3.1 Agents

Following [13] note that the real agents neither face the sterilized 2×2 games common in the theory nor do they think like game-theorists we assume only boundedly rational instead of fully rational agents. Although, even if we assume that agents are not able to deduced the optimal strategy for the game we will not leave them absolutely helpless. Additionally to their own payoffs we allow them to observe the payoffs and strategies of their neighbors. This allows to revise their strategies according to these observations.

3.2 Strategies and payoffs

Each player can take an action from the set $A_i \in \{C, D\}$, where C denotes cooperation and D defection. We assume that agents use only pure strategies so each agent either cooperates or defects with probability 1. If we denote a_i the action of agent i and a_j the action of agent j , the i 's payoff $\pi_i(a_i, a_j)$ from this interaction is given by the payoff matrix (table 1). Based on the different orderings of the values in the matrix one can define social dilemma games with different tensions between the individually and collectively beneficial actions. In our paper we use the following ordering of the payoffs $T > R > P > S$ which results in the prisoner's dilemma game.

The overall payoff from the interactions with all agents in the i 's neighborhood is then given by the sum

$$\Pi_i = \sum_{j=1}^{N_i} \pi(a_i, a_j)$$

We are aware that the choice of the accumulated payoffs as a measure of the agents performance strongly influence the results of our analysis. Comparing [37] and [48] one could find out that using for example averaged payoffs instead could lead to fundamentally different conclusions. We will add this as an additional argument for the careful interpretation of the results from similar experiments.

3.3 Network

The agents are arranged in a network. The network consists of

- a finite set of nodes $N = \{1, \dots, n\}$, and
- a set of links $L = \{ij \mid \{i, j\} \in N \times N\}$.

The set of nodes in the network represents the set of agents and the set of links the interactions between them. The network (N, L) can be represented by the $n \times n$ adjacency matrix g , where

- $g_{ij} = 1$ if $ij \in L$ and $g_{ij} = 0$ otherwise,
- $g_{ij} = g_{ji}$, and
- $g_{ii} = 1 \forall i$

The 1's in the matrix means that agents interact with each other. We also assume that the matrix g is symmetric so if agent i is affected by the action of agent j it is also true that agent j is affected by the action of agent i . By setting g_{ii} equal to 1 we allow for a self-play. Let $N_i = \{j \in N : g_{ij} = 1\}$ denotes the set of neighbors of agent i and $k_i = |N_i|$ the number of agent i 's neighbors. In the network theory the k_i is called the degree of node i . Note that because we allow for a self-play N_i also includes i .

In our model we use four networks with different connectivity structure: *grid*, *ring*, *small-world* and *scale-free*. Each network can be characterized by number of topological properties. The *degree distribution* of the network defines the probability of finding a node with particular number of neighbors. According to [3] we can divide networks into *single-scale networks* with a characteristic degree and exponential or Gaussian degree distribution and *scale-free networks* without any characteristic degree and power-law degree distribution. In our case, grid, ring and small-world networks belong to the former group while the scale-free network to the latter.

Another characteristics that differentiate the networks used in our article are *average distance* and *clustering coefficient*. First, denote

- *path* of length l between node i and j as a sequence of nodes $(i, i_1, \dots, i_{k-1}, j)$ such that $g_{ii_1} = g_{i_1i_2} = \dots = g_{i_{k-1}j} = 1$, and
- *distance* between node i and j as length of a shortest possible path between them.

Then the *average distance* is the average of *distances* between all pairs of nodes of a network. The average distance is an important measure of how far the nodes are from each other and how fast the information or the particular behavior can spread through the network. In our case, we can observe rather long average distances for the grid and ring networks and small distances for the small-world and scale-free networks.

The *clustering coefficient* can be defined as the average fraction of the pairs of nodes from the node's neighborhood that are also neighbors to each other. Clusters play important role because the actions of the members of the clusters usually have larger effects on each other the non-members. In our case the grid has a clustering coefficient equal to 0 which means that none of the two neighbors of particular node are also neighbors to each other. As we will see this fact has important implication for the dynamics of our model. On the other

side, we usually find a large number of clusters in ring, small-world and scale-free networks.

Our choice of the examined network topologies was not completely arbitrary. The grid network is often used representation of the local interaction structures both in natural and social sciences. The first spatial evolutionary models of Nowak and May ([29], [30]) used the grid as a simple representation of spatial space. The ring network is important for two reasons. First, since [34] introduced circular space into the economic literature of spatial competition ([19]) the ring has been widely used as representation of space in economics and other social sciences. Second, ring network serves as a starting point for construction of another important kind of network, the small-world network. So, it would be interesting to examine how the slight modifications needed for the transition from one type of network to another influence the dynamics of our model. In small-world networks two important features of both natural and social real-world networks i.e. the high clustering and small average distance appear simultaneously. While the first observations of small-world phenomena in social networks came with the experiments of American sociologist Stanley Milgram ([26]) later examination of other networks both from natural and social systems proved that the small-world characteristics can be found for example in the neural network of the nematode worm *C. elegans*, the electric power grid of Western part of USA or the network of film actors ([44]). The large body of literature focusing on the dynamic processes on small-world networks reflects both the great interest in this kind of network topology and its importance. The last but not least the scale-free networks capture another interesting feature of many real world networks. In many cases, from the cellular metabolism and protein regulatory networks, through the network of Hollywood movie actors or the web of human sexual contacts to the world-wide-web network ([8]), there exist nodes that have huge number of connections to other nodes while at the same time most of nodes in the network have only few such connections. The distribution of number of connections for individual nodes is then skewed and some times best represented by the power-law distribution. The “extreme” topology of scale-free networks has large impact both on the dynamics of processes that happen on this kind of networks ([38]) and on the robustness against errors and attacks ([2]).

3.4 Update rules

While we hold the set of available strategies and the interaction structure fixed individual agents could revise the choice of the strategies. If the updating of rules happens after each time period for all agents in the model we call it a *synchronous updating*. Synchronous updating means that after each period (i.e. after all agents played the game with all their neighbors) all agents simultaneously revise their strategies according to the updating rules defined below. We also decided to use an *asynchronous updating* in which each agent has some probability to revise her choice of strategy after each period. As originally reported by [20] synchronous and asynchronous yield fundamentally different dynamics

of the model. Each researcher should carefully choose the appropriate triggering mechanism for the problem under study.

As noted earlier while we do not allow agents to deduce the best strategy for the game we allow them to observe the actions and results of agents they interact with. In our model we employ four different updating rules in which information about actions and results from the previous period are used:

- *The max rule*
- *The mean rule*
- *The min rule*
- *The fitness based rule*

We did not choose the above rules completely ad-hoc. The first three rules - max, mean and min - were identified in the experiments of [27]. Authors experimentally tested what kind of strategies people use when they have to choose between two actions with uncertain outcome and the only information they have is the experience of other people. The last rule - fitness - was chosen to connect our work with inspiring works of Nowak and May ([29], [30], [31]) and make our results comparable to theirs. What follows is the detail description of the above rules.

For the *max rule* we can write that

$$a_i(t+1) = \begin{cases} a_i(t) & \text{if } \Pi_i > \max\{\Pi \in N_i\} \\ a_{j \in N_i}(t) & \text{if } \Pi_j = \max\{\Pi \in N_i\} \end{cases}$$

Imitation of the strategy of the best performing individual within the neighborhood proved to be very efficient algorithm ([14]). However, in some situations it can be a consequence of a cognitive distortion reported e.g. by [21] known as “illusion of control”. People believe they are able to influence outcomes of their actions despite the fact that they do not have any factual influence over it. The expectation of their success is much higher than available information suggests. The max rule has been widely used within the spatial PD game literature ([49], [1], [23] etc.).

For the definition of the following rules first denote

- N^C , resp. N^D the set of cooperators, res. defectors, and
- Π^C , resp. Π^D the set of payoffs of cooperators, resp. defectors.

The *mean rule* can be than written as

$$a_i(t+1) = \begin{cases} C & \text{if } \bar{\Pi}_{N_i}^C = \max\{\bar{\Pi}_{N_i}^C, \bar{\Pi}_{N_i}^D\} \\ D & \text{if } \bar{\Pi}_{N_i}^D = \max\{\bar{\Pi}_{N_i}^C, \bar{\Pi}_{N_i}^D\} \end{cases}$$

This is the only heuristics compatible with Bayesian optimization. Individuals deterministically choose their strategy by comparing average payoff of the cooperating neighbors with the average payoff of the defecting neighbors.

We define the *min rule* in the following way

$$a_i(t+1) = \begin{cases} C & \text{if } \min \Pi_{N_i}^C = \max \{ \min \Pi_{N_i}^C, \min \Pi_{N_i}^D \} \\ D & \text{if } \min \Pi_{N_i}^D = \max \{ \min \Pi_{N_i}^C, \min \Pi_{N_i}^D \} \end{cases}$$

The min rule reflects strong loss aversion (the tendency to avoid losses). This behavior was first reported by [22] and individuals following the min rule heuristics choose the strategy with the highest minimum in the neighborhood.

The last rule we use in our model is *fitness based updating* which can be defined as follows

$$P(a_i(t+1) = a_{j \in N_i}(t)) = \frac{\Pi_j}{\sum_{j=1}^{N_i} \Pi_j}$$

Fitness based updating mechanism (or roulette-wheel selection) is used in many models of spatial PD (e.g. [31]). The probability that node i will become cooperator in the next round is proportional to the total payoff received by i 's cooperating neighbors (plus i 's payoff if i cooperated in the last round).

We simplified the above definitions by omitting the cases in which there is now clear “winner” of the revision process. For the max rule it can for example happen that several agents reached the maximum observed payoff. Similar situation can arise for other three rules if either both strategies have the same observed average payoff, or have the same observed minimum or are used by the same number of agents. In those cases additional rule has to be applied to let the agents choose among them. We decided that agents choose randomly among strategies in those cases. This is of course arbitrary choice and we would like to note that other mechanisms could change the results.

While there exist some approximate analytical results for similar evolutionary models with simple interaction structures (see e. g. [42]) the complex topological features of networks of interactions and use of boundedly rational agents in our model make the use of computer simulation the simplest available tool to analyze the behavior of the model. In the next section we give the details of computer simulations used and present the results of our experiments.

4 Simulation results

In all our simulations we let 2500 agents play the prisoner’s dilemma game on the four different types of networks and using four different imitation strategies described in the previous section. For each combination of network type, imitation strategy and type of synchronization we repeated the experiments 50 times which results in the $4 \times 4 \times 2 \times 50 = 4000$ simulation runs. Each run consisted of 3500 periods for the synchronous updating and 14000 periods for the asynchronous case. During asynchronous updating each agent had a probability 0.25 to revise her action. Average connectivity of networks was set to 4

Table 2: **Simulations results**

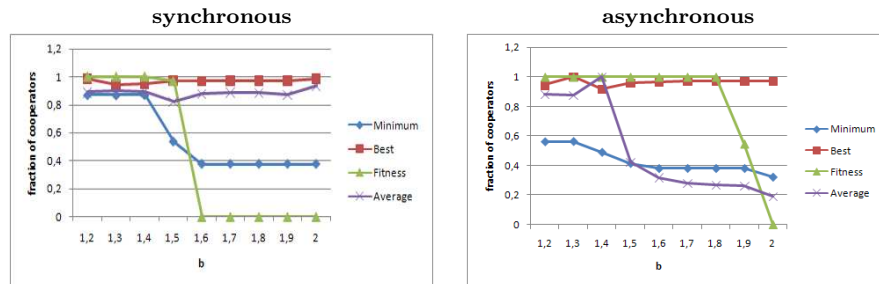
	grid		ring		small-world		scale-free	
	sync	async	sync	async	sync	async	sync	async
max rule	0.62	0.67	0.97	0.97	0.78	0.75	0.91	0.95
mean rule	0.53	0.60	0.88	0.49	0.64	0.57	0.44	0.57
minimum rule	0.47	0.58	0.56	0.43	0.58	0.46	0.56	0.57
fitness rule	0.51	0.83	0.44	0.84	0.48	0.84	0.41	0.84
average	0.54	0.67	0.71	0.68	0.62	0.66	0.58	0.73

The numbers in the table were calculated as averages over last 1000 periods in the case of synchronous updating and over last 4000 periods in the case of asynchronous updating and then averaged over the all 50 experiments. The standard deviations of for the measurements in the table ranged from 0.00 to 0.15.

for all types of networks i.e. each agent was connected on average to four other agents. According to [29] we used an one-parameter version of the PD payoff matrix with $2 \geq T = b > R = 1 > P = S = 0$.

Each simulation started with equal share of cooperative and non-cooperative agents. The final shares of cooperators in the population for every combination of network type, imitation strategy and synchronization are displayed in table 2. The numbers in the table were calculated as averages over last 1000 periods in the case of synchronous updating and over last 4000 periods in the case of asynchronous updating and then averaged over all 50 experiments and all values of parameter b . More detail results are displayed on figures 1-5.

Figure 1: **Ring Network**

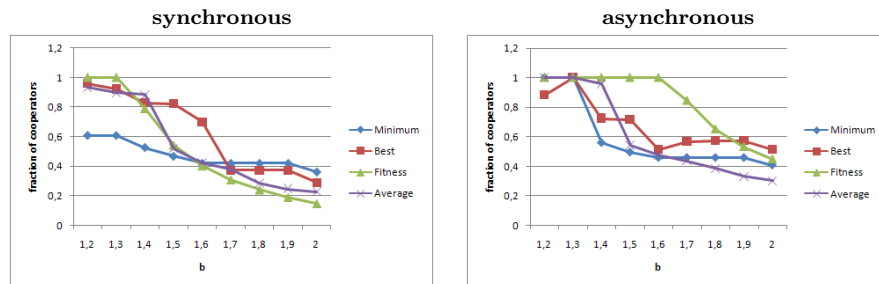


The average fraction of cooperators in the ring network was 0.71 for the synchronous case and 0.68 for the asynchronous case. But values for different combination of parameters varied greatly and we can observe interesting phase transition from high to low cooperative states.

First, we will consider ring topology that shows highest variations in cooperation levels for different updating rules. [13] have shown that cooperation on the ring persists thanks to the fact that concentrated cooperators earn high payoffs. Each agent considers only two immediate neighbors and uses mean rule for decision about the strategy in the next round. The authors have shown that in this case final outcome depends in fact only on the border agents separating clusters of cooperators from clusters of defectors. [47] uses the max rule and his results indicate that ring network is most supportive for the emergence of cooperation with level of cooperation very slowly (very large number of rounds) increasing to levels above 95% for both synchronous and asynchronous updating. Both cooperators and defectors will survive if there is at least one string of more than three cooperators and at least one defector at the beginning of the simulation.

Our results suggest that ring network is highly sensitive to updating rule and also intensity of the dilemma measured by the value of parameter b . The average fraction of cooperators in the ring network was 0.71 for the synchronous case and 0.68 for the asynchronous case. But individual results for different combination of parameters varied greatly and we can observe interesting phase transition from high to low cooperative levels. The strong dependence of the results on the timing of the updating should serve as another piece of evidence that this topic should be taken seriously by the researchers.

Figure 2: **Grid Network**



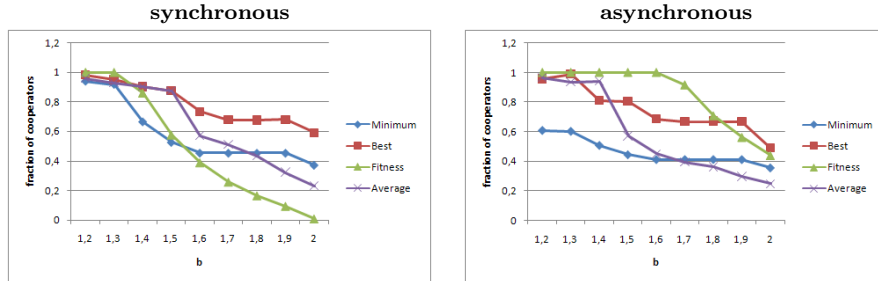
The average fraction of cooperators in grid network was 0.54 for synchronous and 0.67 for asynchronous case. The individual results for synchronous and asynchronous cases are both qualitatively and quantitatively quite close.

Grids with both the Moore (eight neighbors) and Von Neumann (four neighbors) neighborhoods were largely studied by [29], [30] and [31]. One of their main findings is that updating rules that give more weight to the most successful neighbors favor cooperation (max rule should lead to higher cooperation levels than fitness rule). Our results confirm these outcomes both for the synchronous and asynchronous update. Curves for either max or fitness rule lie above the others for most of the values of parameter b .

Small-world networks are derived from the ring by random rewiring (we get

ring for the rewiring probability $p = 0$; small-world networks for $p \in (0, 1)$; and random network for $p = 1$). There is a non-monotonic relationship between rewiring probability and the level of cooperation in the stable state. [1] have found (using max rule) that the number of cooperators has clear bottom at the rewiring probability equal to 0.1 for the temptation parameter equal to 1.2. The explanation of high cooperation levels for low rewiring is obvious; small compact groups are able to benefit from mutual cooperation without being exploited by individual defectors. With rising interconnectedness of the network these clusters are torn apart and defectors are able to exploit cooperating groups. However, with further increase in the rewiring probability cooperators are again able to reconnect and cooperation survives.

Figure 3: **Small-World Network**



The average fraction of cooperators was 0.62 for synchronous and 0.66 for asynchronous case. The individual results both for different imitation strategies and updating mechanisms varied greatly.

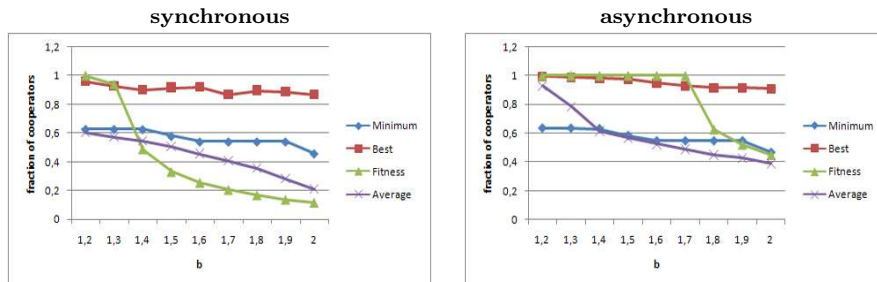
We can observe two processes when increasing rewiring probability p - decreasing average shortest path and changes in the degree distribution of the network from homogeneous to heterogeneous. [36] examined one-shot PD played on the heterogeneous and homogeneous small-world networks to distinguish between pure small world effects and concludes that the role of pure small-world effect is quite subtle. Imitation strategy is based on pair wise asynchronous comparison of the total payoffs assigning higher probability of being imitated to agents with relatively higher total payoffs. The level of cooperation increases for the high temptation to defect and decreases for small temptation as a consequence of pure small-world effect.

[25] identified three regimes for the ring network with the homogeneous degree distribution. For the temptation parameter sufficiently small (large) the system converges towards the full cooperation (defection) state. For the temptation parameter between 1.3 and 2.3 the number of cooperators is closely related to the clustering coefficient of the network. They further investigated the speed of convergence towards the stable state and conclude that the convergence is faster for larger p .

In our experiment the average fraction of cooperators was 0.62 for syn-

chronous and 0.66 for asynchronous case. The individual results both for different imitation strategies and updating mechanisms varied greatly.

Figure 4: **Scale-Free Network**



The average fraction of cooperators was 0.58 for synchronous and 0.73 for asynchronous case. The individual results were quite different for different imitation strategies. While the max rule resulted in high levels of cooperation other rules led more often to low levels of cooperation. We suggest that scale-free network can hardly be seen as a "unifying network for the emergence of cooperation" ([37]).

Scale-free networks has attracted a lot of attention within the scientific community mainly thanks to their interesting properties such as vanishing epidemic threshold ([9]) or high robustness to random failure (e.g. [17]). The emergence of cooperation on the scale-free networks has been stressed in [47] using max rule as well as in [37] using updating mechanism favoring strategies with higher payoff dominance. Our results are not fully in line with these conclusions. We show that using max rule we really achieve high cooperation levels. However, for other four updating rules the scale-free topology displays ambiguous results. Average cooperation level did not exceed 60% for asynchronous updating and was 73% for synchronous case. The variation of the cooperation levels are second highest among updating rules (the highest variations were exhibited by ring topology). Hence, we suggest that scale-free topology is one of the topologies sensible to the updating rules.

5 Conclusions

In this article the effect of topological structure of interactions on the evolution of cooperative behavior was systematically examined. We constructed a simple model of a prisoner's dilemma game played on networks and let the strategies evolve according to particular evolutionary mechanisms. We use four different types of networks that are both often used in the theoretical literature and exhibit some features of real world interaction structures: grid, ring, small-world and scale-free network. Along with the similar studies on this topic our results confirm that under certain conditions evolutionary dynamics in the structured population can favor cooperative behavior.

To gain deeper understanding on whether certain topology can solely by itself explain existence of cooperation we introduce to each topology four different mechanisms (imitation strategies) through which strategies can spread in the population. As the results of the extensive Monte-Carlo simulations showed the evolutionary dynamics in our model is influenced by the combined effect of both the topological features of particular interaction structure and other evolutionary mechanisms. Even if none of the examined networks represents a "unifying network for the emergence of cooperation" ([37]) the effect of interaction structure is undoubtedly important and deserve further study in the social sciences. There are also many ways in which our model could be extended or modified to get another interesting results. For example one can assume that individuals use more complex strategies that map the history of the previous plays into the actions in the future, combine individual learning with the imitation or use different strategies for each partner in the game.

Evolutionary models like the one presented here represent very useful tools for understanding interesting phenomena around us. However, like every other tool it should be used with the caution. All models are built on assumptions. If one have to choose between different types of assumptions it is useful to carefully look for the empirical or experimental findings that can help to bring the model closer to real situations. For example we used experimentally observed imitation strategies and incorporate them into our model. We see this approach more useful than just using maybe intuitively acceptable but ad-hoc chosen assumptions.

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