Business fluctuations in an evolving network economy

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1. Introduction
Asymmetric information theory deeply affected economic discipline. The basic paradigm in mainstream economic theory (i.e., the Arrow-Debreu version of the general equilibrium model) includes one imperfect information directly challenges: the individuals take decisions in isolation, using only the information received through some general market signals such as prices. Macroeconomists tried to avoid the problem of uniqueness and stability of equilibrium of the general equilibrium model by resorting to what has become the standard paradigm in modern macroeconomics, that is, the representative agent framework. The use of such an approach has lately been frequently contested. There is no simple, direct, correspondence between individual and aggregate regularities. It may be that, in some cases, aggregate choices correspond to those that could be generated by an individual. However, even in such exceptional cases, the individual in question cannot be thought of as maximizing anything meaningful from the point of view of society’s welfare. It means that one has to ignore communication and direct interaction among agents, which, ultimately, defines away the problem of co-ordination (Leijonhufvud, 1992; Hahn, Solow, 1995): once again, in the general equilibrium model interaction and co-ordination occur only through prices.

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The role of prices is undoubtedly important, but the price mechanism alone can work only if information is complete; in such a case, one can ignore the influence of other co-ordination and interaction mechanisms. The literature on imperfect information has shown that: (i) agents have to be heterogeneous and, (ii) markets are incomplete, i.e. the future is uncertain. If (i) and (ii) hold true, *interaction among agents* and *credit among agents and periods of time* become constitutive elements of the picture. Central to this query is information: information not only about the economic status of the party, but also about the incentive structures that the party faces. Every time, the party providing the credit has to ascertain both the risks involved and her ability and willingness to bear that risk. In effect, all of the questions crucial to modelling banks' supply of credit are relevant in determining the supply of credit by firms, households, or non-bank financial institutions. All in all, direct, non-price interaction happens through the credit channels.

In the following we take seriously these hypotheses by considering the possibility of bankruptcies. In fact, in an equity-constrained world where the phenomenon of informative imperfection is pervasive, unexpected events on future revenues may affect bank-firm relation as well as inter-firms credit link. In other words, we model an *inside credit* (*commercial credit*) between firms of the different productive sector, and an *outside credit* (*bank credit*), which is provided by the bank, which lends credit to the industrial sector (Stiglitz, Greenwald, 2003).

A central determinant of the level of economic activity is the supply of credit and that the most important institution in determining the supply of credit is the banking system. Whenever one delivers a good to someone else without an immediate exchange of money or goods of full value, credit is extended. Traditionally, suppliers provide credit to their customers, who supply credit to theirs. In some cases, purchasers extend credit to suppliers, to enable them to produce the inputs that they wanted.

Failure of fulfilling debt commitments could lead to *bankruptcy chains*. If debt commitments are not fulfilled, *bad debt* increases and the rate of interest may increase.
as well. The interest rate increase leads to more bankruptcies: “the high rate of bankruptcy is a consequence of the high interest rate as much as a consequence of it” (Stiglitz, Greenwald, 2003: 145).

If future is uncertain, what probability is there that the contract will be fulfilled? Are there major consequences of a small shock? This paper will show in what way a business cycle is linked to firm bankruptcies and how a domino effect (a crescendo in terms of bankruptcy) can arise (and as such be avoided). At the same time, the main facts of firm demography (like power law distribution of size and Laplace growth rates of firms) emerge endogenously (and are more resistant to external shocks.) As opposed to previous models of business cycles, in our model, avalanches are due to the interdependence of the output of a firm on supply and payments from other firms.

Heterogeneous agent interaction has a second major implication. We will see that the structure of aggregate behaviour (macro) actually emerges from the interaction among the agents (micro). In other words, statistical regularities emerge as a self-organised process at the aggregate level. Complex patterns of interacting individual behaviour may generate certain regularity at the aggregate level (Delli Gatti et al., 2005). The idea of representing a society by one exemplar (a representative agent) denies the fact that the organizational features of the economy play a crucial role in explaining what happens at the aggregate level (Kirman, 1992).

The model is presented in section 2. The model simulation and the discussion of results are shown in section 3. Section 4 concludes.

2. The model

We model a three-sector sequential economy (t = 1,2,...,T) populated by a multitude of heterogeneous agents. The three sectors of the economy are the following: a downstream sector with a number I of firms (labeled by the index i = 1,2,...,I), an upstream sector with J firms (j = 1,2,...,J) and a banking sector with Z banks (z = 1,2,...,Z).
Downstream firms produce a perishable goods using labor and intermediate goods as inputs. They sell all the produced output at a stochastic price. Upstream firms produce the intermediate goods “on demand” with a technology that requires only labor as input. The financial side of the economy is characterized by two lending relationships: (i) downstream and upstream firms obtain credit from banks to finance the wage bill (the fraction not covered by internal resources); (ii) downstream firms buy intermediate goods from upstream firms by means of a commercial credit contract. The network structure of the productive and credit relationships endogenously emerges from the decentralized interaction among agents, on the basis of a preferential partner choice: for example, downstream firms look for upstream firms with low prices; at the same time, firms search for banks with low interest rates on lending contracts.

The production function of downstream firms is given by the equation:

\[ Y_t = \phi A_t \]

where \( \phi > 1 \), and \( A_t \) is the net worth of the firm \( i \) at time \( t \). Downstream firms have the following labor and intermediate goods requirement functions: \( N_t = \delta_d Y_t \), \( Q_t = \gamma Y_t \), with \( \delta > 0 \) and \( \gamma > 0 \). Final goods are sold at a stochastic price \( u_t \).

Upstream firms produce intermediate goods required by the downstream sector using a technology employing only labor as input: \( Q_t = \delta_u N_t \), where \( \delta_u > 0 \). In each period \( t \) an upstream firm \( j \) receives a demand of intermediate goods from downstream firms the number of which depends on the price \( p_t = 1 + r_t \), where \( r_t \) is the interest rates charged on the commercial lending contract. The level of \( r_t \) depends on the financial condition of the \( j \)-th upstream firm:

\[ \text{ta}^1 \text{ That is a random variable that, for the sake of simplicity, is uniformly distributed in the interval (0,2).} \]
that is the interest rate charged on downstream firms decreases as the financial soundness of the upstream firm increases.

The wage bill exceeding firms’ net worth is financed by bank loans. Accordingly, the demand of credit is equal to $B_{xt} = W_{xt} - A_{xt}$, where $W$ is the firm’s wage bill ($x=i$ for downstream, $x=j$ for upstream firms). Clearly, firms with a level of net worth sufficient to finance the wage bill does not demand bank credit.

Banks set the interest rates on loans as follows:

$$r_{jt} = \alpha A_{jt}^{-\beta}$$

(2)

where $\beta>0$, $\theta>0$, and $B_{xt}$ is the amount of the bank loan for firms ($x=i$ for downstream, $x=j$ for upstream). Accordingly, the level of bank interest rates is inversely related with the financial soundness of the bank (proxied by $A_{zt}$) and is proportional to the firms’ debt ratio.

Each downstream firm has a (productive and credit) relationship with an upstream firm. Initially (at time $t=1$) the links among downstream and upstream firms are established in a random way. After that, each downstream firm $i$ looks at the price of a randomly selected upstream firm $j$ and if the observed price is lower than the price of the upstream firm $j'$ with which $i$ interacted in the precedent period, then $i$ establishes a relationship with $j$; otherwise, $i$ continues to interact with $j'$. According to this preferential partner choice process, an endogenous network of relationships among downstream and upstream firms evolves over time. In a similar way firms interact with banks generating a network of firm-bank relationships: each firm looks at the interest rate set by a bank picked at random and if this is higher than that of the bank with which the firm interacted in the previous period then the firm continues to
interact with the same bank; otherwise the firm changes the bank and obtains a loan with a lower interest rate.

As we will see in the following, the structure of credit interlinkages has important implications with respect to the extent of bankruptcies diffusion and business fluctuations, because of the interdependence of firm and bank behaviors: the default of one agent (e.g., a downstream firm) can cause the default of another agent by decreasing its financial soundness (e.g., an upstream firm linked with the bankrupted downstream one) and so on, depending on the number of links among agents (the default of an agent with many links implies a high probability of bankruptcy diffusion across the network).

Downstream firms’ profit is equal to:

\[
\pi_u = u_d Y_n - (1 + r_{zt}^i)B_n - (1 + r_{jt}^i)pQ_{jt}
\]

where \( r_{zt}^i \) is the interest rate set by bank \( z \) on the loan to downstream firm \( i \), \( r_{jt}^i \) is the interest rate on commercial credit set by upstream firm \( j \), and \( p \) is the price of intermediate goods.

Upstream firms’ profit is equal to:

\[
\pi_j = p(1 + r_{jt}^j)Q_{jt} - (1 + r_{zt}^j)B_j
\]

where \( r_{zt}^j \) is the interest rate set by bank \( z \) on the loan to upstream firm \( j \).

Banks’ profit is equal to:

\[
\pi_z = \sum_{i=1}^{I_j} (1 + r_{zt}^i)B_n + \sum_{j=1}^{J_j} (1 + r_{zt}^j)B_j
\]
where \( I_z \) and \( J_z \) are, respectively, the number of downstream and upstream firms interacting with bank \( z \).

If a firm goes bankrupt in the period \( t \) then it cannot pay back for the credit obtained by an upstream firm and/or a bank; consequently, the lender has a “bad debt”, that is a reduction of its net worth. In this way, defaults in a sector can diffuse to other sectors possibly generating bankruptcy avalanches with a magnitude that depends on the configuration of the interaction credit network.

At the end of the period, firms’ net worth is equal to \( A_{xt+1} = A_{xt} + \pi_{xt} \), and a firm goes bankrupt if \( A_{xt+1} \leq 0 \) (\( x=i \) for downstream, \( x=j \) for upstream, \( x=z \) for bank).

Finally, we assume a simple mechanism of entry-exit: bankrupted firms/banks are replaced with new entrants on the basis of a one-to-one replacement.2

3. Simulations

The dynamic properties of the network economy described above are analyzed by means of computer simulation, with a time span \( T=1000 \) and the following number of agents: \( I=500 \) (downstream firms), \( J=250 \) (upstream firms) and \( Z=100 \) (banks). The configuration of the parameter setting and initial conditions are described in the appendix.

\[ ^2 \text{Accordingly, the total number of agents in the economy is constant along time. New agents are endowed with an initial amount of net worth given by the uniform random variable with mean 1 and finite variance.} \]
Figure 1. Downstream sector aggregate production

Figure 2. Firm size distribution
(downstream and upstream; in terms of net worth)
Figure 1 describes the fluctuating behavior of downstream sector’s aggregate production. Starting from identical initial conditions firms become very heterogeneous and a power law distribution of firm size emerges over time (see figure 2). In addition, the number of links per firm (or bank), that initially is randomly distributed, tends to become more asymmetric over time due to the preferential partner choice mechanism governing the interaction among firms and banks: the degree distribution of credit interlinkages suggests that some firms and banks, in the right tail of the distribution, have a number of customers higher than that generating by a normal distribution (see figure 3).

The default of an agent can generate a diffusion of bankruptcies across the network and the likelihood of this phenomenon depends on the structure of the network. In fact, agents with a high number of links are robust because can offer good conditions about credit contract. Upstream firms which are financially sound set intermediate goods’ prices lower than that of other firms; in this way, even with an imperfect information partner choice process, they attract more downstream firms and increase their financial soundness making profits. In a similar way banks increase the number of their customers. All in all, the preferential attachment operating through the choice of partners by lower prices and interest rates increase the number of links of financially sound firms and banks (in this way they can offer lower prices and interest rates also in the following periods) that improve their financial positions, increasing the “robustness” of the network. On the other hand, the evolution of a scale invariant degree distribution increases also the “vulnerability” of the network because the default of a highly connected agent can have serious consequences on the stability of the networks generating an avalanche of bankruptcies.

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3 Obviously, the aggregate production of upstream sector shows the same behavior given that they produce intermediate goods required by downstream firms “on demand”.
A typical story is the following. The failure of one or more downstream firms generates a deterioration of the financial condition (bankrupted firms do not reimburse the commercial debt) of the upstream firms with which they interact in a given period of time. It is possible that the upstream firm is quite robust and can survive after this episode. In the opposite case also the upstream firm goes bankrupt and so even this firm does not pay back the debt to the bank. Depending on the size of the loan the deterioration of the bank’s financial condition may cause the default of the bank, and so on. In this model the principal channel through which bankruptcies diffuse across the network starts from the failure of downstream firms that causes the failure of upstream firms; the unfulfilling of debts by upstream firms generates the default of banks (see figure 4). The seriousness of agents’ defaults with respect to bankruptcy avalanches and business fluctuations depends on the size of agents and their connectivity. In fact, even a high number of failures can be absorbed by the system without generating a domino effect. Accordingly, the extent of bankruptcy events depends also on the amount of bad debt (see figure 5).
crises amplifies business fluctuations and, as consequence, the distribution of aggregate growth rates is far from being Gaussian, showing instead a double exponential behavior with a noticeable asymmetry for negative events (see figure 6).

Figure 4. Cross-correlation in sector bankruptcies: downstream vs. upstream (top-left panel); downstream vs. banks (top-right panel); upstream vs. banks (down-left panel); downstream and upstream vs. banks (down-right panel).
Figure 5. The distribution of bad debt per period

Figure 6. The distribution of aggregate growth rates
APPENDIX: Parameter setting and initial conditions

$\phi = 1.5$
$\delta_d = 0.5$
$\gamma = 0.5$
$\delta_u = 1$
$\alpha = 0.1$
$\beta = 0.1$
$\theta = 0.05$
$w = 1$
$p = 1$
$A_{ij} = A_{ji} = 1$
$A_{ii} = 1000$