We analyze the impact of different trading network topologies on the emergence of money as a medium of exchange. This work represents an extension of the Kiyotaki & Wright’s model \[24\] in which traders must have a relationship, or “link”, to engage in exchange. We explore a simple economy using an agent-based simulation where traders are matched in pairs and the exchange can only be bilateral and quid pro quo. Agents update their trading strategies using an evolutionary approach. Properties of the trading network topology can drive the system towards monetary equilibria with either one or multiple currencies. In particular, we discover a significant and positive correlation between the shortest path, the diameter of the trading network and the number of currencies in the system. If the network is highly connected, a single good emerges as the medium of exchange. As connectivity decreases, more than one medium of exchange can exist simultaneously. When agents are allowed to reconfigure their links to other traders without additional costs, the networks tend toward specific characteristic configurations, regardless of the initial topology, where the majority of agents have only a few linkages to others and the rest are quite connected. These latter traders own the same characteristics of the Middlemen agents reported in Rubinstein & Wolinsky \[40\] and Gilles et. al. \[13\]: high exchange rate, number of links and goods in their inventory. They also have the power to break up communication among other players in the network.

Keywords: Social Networks; Money; Middlemen.

JEL Classification: E00, D85, C63.

1. Introduction

The analysis of network structures in economic studies is becoming a key factor for understanding economic relationships (Jackson & Wolinsky \[20\]). Various aspects of human interactions have been investigated, i.e. firms organization (Boorman \[4\], Keren & Levhari \[23]\), employment search(Montgomery \[32\]), information transmission (Goyal \[15\]), airline routes structure (Hedricks et al \[18\]), Starr and Stinchcombe \[44\]), market trades (Furusawa & Konishi \[12\], Goyal & Joshi \[16\]) \[a]. This

\[a\]A substantial portion of this model has been developed during the 2006 Graduate Workshop in Computational Social Science Modeling and Complexity, Santa Fe Institute.

Centre for Computational Finance and Economic Agents, University of Essex, Wivenhoe Park, Colchester, CO4 3SQ, Essex, United Kingdom. Tel:+44 (0) 1206 874876, Fax:+44 (0) 1206 87272, email: sgians@essex.ac.uk.

See Jackson & Wolinsky \[20\] for a more detailed literature review.
paper intends to investigate the relationship between trade network structures and traders behavior in evolving one or multiple commodities monies.

The reason for the emergence of a commodity as a medium of exchange that we have observed historically is still not well understood. More than two centuries ago, Adam Smith said:

"In order to avoid the inconvenience of such situations, every prudent man in every period of society, after the first establishment of the division of labour, must naturally have endeavoured to manage his affairs in such a manner, as to have at all times by him besides the peculiar produce of his own industry, a certain quantity of some one commodity or other, such as he imagined few people would be likely to refuse in exchange for the produce of their industry". (Smith [42], p. 221)

The situations that Smith pointed out refer to what Jevons [21] called "double coincidence of wants", required condition for barter trade equilibrium. According to him, "The first difficulty of barter is to find two persons whose disposable possessions mutually suit each others wants. There may be many people wanting, and many possessing those things wanted; but to allow an act of barter, there must be a double coincidence which will rarely happen." (Jevons [21], p. 31). Medium of exchange has been pictured out as the "oil" in the economic machinery, the "shunting locomotives" in the process of trade (Wicksell [46], p. 5, 19). It has also been defined as a "dynamic phenomenon" (Niehans [34], p. 773) which allows traders to escape from the double coincidence constrain, and this is what the evolutionary framework presented in this paper tries to prove.

The specific characteristics of this particular commodity, i.e. size, transportability, rarity, durability, social approval, could explain the phenomenon of commodity money. In particular, Menger [30] identified the degree of saleableness in commodities as the key factor of this process. The higher the degree of saleableness, the higher the probability of that particular commodity to become medium of exchange.

The properties of the market and its structure could also drive the system towards the evolution of a medium of exchange. In this direction, "the driving force behind the use of money" for Smith [42], chap. 4] was "specialization, which implies that agents do not necessarily consume what they produce. Here, they will also meet randomly over time in a way that implies that trade must be bilateral and quid pro quo". In a bilateral match between i and j, an exchange respects the quid pro quo constrain if and only if each delivers to the other the same value of goods (Ostroy & Starr [36], p. 1096). Niehans [33][34] analyzed the market structure underlining the crucial role of transaction costs in the emergence of commodity monies. Ostroy [35] and Feldman [11] described the matching structure as an iterative

\footnote{He said that "money is essentially a dynamic phenomenon, related to speculation and uncertainty, which can not be analyzed satisfactorily in terms of equilibrium theory". Including transaction costs "this theory is able o explain the emergence of different exchange arrangements, ranging}
bilateral match under the quid pro quo constrain (not in every exchange); money in that case is the commodity that minimizes the number of meetings in order to accomplish the traders’ ultimate exchange. An extension of this model has been proposed by Jones [22] who proved that "all individuals accomplish their ultimate exchanges through two-stage trades rather than through direct one-stage trades or through longer, more indirect trading sequences" (Jones [22], p. 758). This theory is based on the probabilistic beliefs of each agent to find other traders wishing to buy each good as wishing to sell it.

At last, the importance of a central power that guarantees the value of the commodity money, operating also as a value of every other goods, is threaten in particular by Grieson [17] and Wray [47].

The importance of the trade network structure, that is the connections between traders and their business partners, about media of exchange has been analyzed by few researchers. One of these is Iwai [19] who underlined the connectedness property as "the minimum requirement for an association of individuals to form "a" self-contained economy, not a mere collection of isolated individuals and/or disjoint communities". The medium of exchange described here does not require any intrinsic properties, nor specific market structures and central power: "money is money just because it is used as money" (Iwai [19], p. 4, 6). This is the way that this paper is going to follow.

The Kiyotaki & Wright’s model [24] as a medium of exchange is one of the most promising environment which investigate the endogenous determination of the good(s) that serve as media of exchange. A model including commodities with different intrinsic properties (in particular storability costs) and the quid pro quo constrain has been implemented. Starting from this framework and others which apply evolutionary behaviors on it (they will be discussed in the next section, in particular Marimon et al. [28] and Marimon & Miller [29]), this paper tries to prove that commodity money is a dynamic process driven by the evolutionary learning process of traders with bounded rationality in which no agents type or intrinsic properties of commodities are applied. The trade network analysis allows us to investigate which network properties can drive the monetary system towards the emergence of one or multiple stable commodity monies. We recognize that if the network is highly connected, a single good emerges as the medium of exchange. As connectivity decreases, more than one medium of exchange can exist simultaneously. When agents are allowed to reconfigure their links to other traders without additional costs, every initial network structure tends toward specific structure with similar properties, regardless of the initial topology, where the majority of agents have only a few linkages to others and the rest are quite connected. These latter traders own the same characteristics of the Middlemen agents reported in Rubin-
stein & Wolinsky [40] and Gilles et. al. [13]: high exchange rate, number of links and goods in their inventory. They also have the power to break up communication among other players in the network.

The rest of the paper is organized as follows. Section 2 introduces the Kiyotaki & Wright’s framework and many further variants of it. The definition of graphs, trader links and network properties are formalized in Section 3. Section 4 describes the artificial framework presented in this paper. The main results of two different implementation with and without dynamic links are shown in Section 5. Section 6 concludes the work.

2. The Kiyotaki-Wright framework

The Kiyotaki & Wright [24] model is made up of three different goods and three agents type: agent $i$ can consume only good $i$ and can only produce good $i+1$. There is an equal proportion of each type of agent in the model. Agents can store only one unit of one of the three goods, incurring in a specific storability costs. In particular, Kiyotaki & Wright depict two different economies, one where $c_1 < c_2 < c_3$ (Economy A) and the other where $c_1 < c_3 < c_2$ (Economy B). When agent $i$ is able to trade with his consumption good, he will obtain utility $u$ and produce a unit of his production good that will be carried in his inventory for the next match. In every step agents incur in the storability cost of the good they carry. Agent’s matches are pairwise and random. Exchanges take place if and only if both traders accept the trade. Obviously, agent $i$ will always be willing to exchange with agents that carry good $i$. Otherwise, he may not be willing to exchange his production good $i+1$ with good $i+2$. Kiyotaki & Wright analyze the two possible pure strategies: the fundamental strategy and the speculative strategy, chosen by a maximization process of agents’ “expected present value of discontinuity over an infinite horizon” (Sethi [41], p. 236). The convergence towards the fundamental strategy identifies a barter economy in which the double coincidence of wants is required, while monetary economy emerges from the convergence of the trader’s behavior towards the speculative strategy, where just a single coincidence is required (Ostroy & Starr [36]), that is the same commodity money. They analytically proved that with some parameter values one(two) of the agents play speculative strategy in Economy A(B). They thus showed that “certain goods emerge endogenously as a media of exchange, or commodity money, depending both on their intrinsic properties and on extrinsic beliefs” (Kiyotaki & Wright [24], p. 927).

Many extensions of this model can be found in literature. Marimon & Miller [29]

---

Footnotes:

*d*The notation and organization of this section follow Sethi [41].

*The notion of a non-consumption good is accepted if and only if its storability cost is lower than the storability cost of his production good.*

*The notion of a non-consumption good is accepted regardless its storability cost, just because agents believe that it is more marketable than his inventory good. This marketability of good is closely related to the Menger’s concept of saleableness [30].
implemented a pure genetic algorithm which updates the only one decision rule that agents can choose every time. Very close to the last model, Marimon et al. [28] introduced a huge classifier system as agents’ strategy controller, updated via genetic algorithm. In the same way Gintis [14] implemented an artificial life simulation that, with respect to the previous ones, supports stable convergence even to speculative equilibria. More close to the original version is Wright [48] who allowed arbitrary distribution of types among agents. His evolutionary framework updates the agent types “in such a way that types with an high payoff relative to the average payoff increase their relative numbers” (Wright [48], p. 200). He discovered that the distribution of agents’ types is also a key factor on the evolution of media of exchange. Experiments in laboratory environments made by Brown [5] examined the possibility that individuals can adopt a good as a medium of exchange. He found that “while the results generally support the hypothesis that individuals are willing to utilize a good as a medium of exchange, the trading patterns of a portion of the individuals deviated from the predicted patterns” (Brown [5], p. 583). Also Duffy & Ochs’s [9]’s experimental subjects failed to accept a commodity money with an higher storability cost than their inventory commodity. On the contrary, Luo [26]’s artificial process with a double trading meeting per step produced the same Kiyotaki & Wright [24] and Wright [48] results, confirming the important rule of intrinsic values and proportion of agents’ types. The results shown by Sethi [41] prove that fundamental, speculative, and polymorphic states can all be stable, and that there may exist a multiplicity of stable states (Sethi [41], p. 233). This mechanism increases the degree of agents’ specialization forcing the system in evolving a commodity money. Finally, Duffy [8] produced stable speculative strategy equilibria combining simple agent-based simulations with humans subject experiments.

All the model shown above were completely or partially able to endogenously evolve a commodity as a medium of exchange, finding more or less stable speculative strategy equilibria. However, stringent constrains, in particular storability costs and agent types, are needed in those contexts for obtaining these results, playing a crucial role on the dynamic process called money. What this paper tries to prove is the possibility to evolve one or multiple media of exchange from commodities that are no different in terms of storability or transportation costs, exchanged by traders with no difference in terms of types. The next section will present an agent-based model in which traders do not have inventory limits in terms of units they can store in their stocks. They consume one unit of all commodities in the system in order to survive. No complementarity between goods exists, then every good is needed for traders, in particular their rare goods. The composition of each inventory will differentiate then the agents’ preference about commodities and it will change overtime on the basis of the traders’ actions (unlike Wright [48] in which the evolution process

---

1Agents can store one unit of goods only from the mornig to the afternoon meeting. No speculative strategy is admitted in the afternoon trading match and no unit of goods can be stored for the next day. For more detail, see Luo [26]
Simone Giansante

updates agents type). Like Marimon & Miller [29], traders have bounded rationality expressed by a single strategy rule mapped into their inventory and updated by a pure genetic algorithm. Additionally, agents can trade only with their business partners (identified by networks links). This possibility allows us to study both the effect of different trading structures on the evolution of one or multiple commodity monies when we impose static links (they can not change over time and are chosen at the beginning of the simulation) and the evolution of middlemen trader positions when we implement dynamic links (they can change overtime in an endogenously way based on the traders’ exchange activity with their partners).

3. Definitions

Let \( N = \{1, \ldots, N\} \) be the set of finite traders. The partner positions among traders are represented by undirected graphs whose traders are represented by nodes.

3.1. Links

An undirected link between \( i \) and \( j \) is defined by the set \( \{i, j\} \). We will use the shorthand notation \( ij \) in order to identify this link. As we adopted undirected graphs, \( ij \) is clearly equivalent to \( ji \) (Gilles et al. [13], p. 4.). In the set of traders \( N \), the maximum number of connections will be \( \frac{1}{2}n(n - 1) \), where recursive links are not allowed (players can not be connected to themselves as the trade process requires at least two traders).

3.2. Graphs

The complete graph is the set of all potential links on \( N \) and is denoted by

\[
g^N = \{ij | i, j \in N \text{ and } i \neq j\}
\]

A graph is now defined as any collection of links in \( g^N \), that is \( g \subset g^N \). The collection of all possible graphs on \( N \) is denoted by \( G^N \{g | g \subset g^N\} \). The empty graph is denoted by \( g_0 = \emptyset \) consisting of no links on \( N \).

Let \( g + ij \) denote the graph obtained by adding link \( ij \) to the existing graph \( g \) and \( g - ij \) the graph obtained by deleting link \( ij \) from the existing graph \( g \). Formally,

---

8 The graph notation used in this paper follows Jackson & Wolinsky [20] and Gilles et al. [13].
9 According to Gilles et al., "the two players forming a link are assumed to be "equals" within the relationship" (Gilles et al. [13], p. 4.). Although both traders have to decide whether or not dropping of staying in the relationship, in our model with dynamic links players can individually form other links without the permission of the new partner. The equal power into the relationship is interpreted here in terms of trading power in exchanging goods, in particular the power to decide whether the exchange has to take place or not. More details about it will be shown in the next section.
For every network $g \in G^N$ and every player $i \in N$ we denote $i$‘s trade partner neighborhood in $g$ by

$$N_i(g) = \{j \in N| j \neq i \text{ and } ij \in g\}$$

(3)

Trader $i$ therefore can directly exchange with those in his link set denoted by

$$L_i(g) = \{ij \in g| j \in N_i(g)\} \subset g$$

(4)

We also define $N(g) = \bigcup_{i \in N} N_i(g)$ (Gilles et al. [13], p. 5.).

3.3. Path

According to Gilles et al., a path connecting $i$ and $j$ is a collection of players

$$\{i_1, i_2, \ldots, i_p\} \subset g|i_1 = i, i_p = j \text{ and } \{i_1i_2, i_2i_3, \ldots, i_{p-1}i_p\} \subset g$$

(5)

with $p \geq 2$. It means that $i$ and $j$ are not directly connected.

They also define the shortest path between two distinct players $i, j \in N$ (assuming that a path exists between $i$ and $j$) as a path "of a minimal number of players", which "contains one and only one member of the neighborhood set $N_i(g)$, as well as one and only one member of the neighborhood set $N_j(g)$" (Gilles et al. [13], p. 5.). All the possible shortest paths are denoted by $P_{ij}(g)$. In case of no path between $i$ and $j$, $P_{ij}(g) = \emptyset$.

3.4. Graphs properties

The statistic properties of graphs that we investigate are the shortest path (defined above), the diameter and the cluster coefficient.

The diameter of $g$ is defined to be the maximum, over all pairs of nodes $ij$, of the length of the shortest path from $i$ to $j$. Formally,

$$D(g) = \max_{ij}(\#P_{ij}(g)) \forall i, j \in g$$

(6)

The cluster coefficient is defined by the fraction of $i$’s neighbors that are also neighbors of each other. Formally

$$CC_i(g) = \frac{\#\hat{N}_i(g)}{\binom{\#N_i(g)}{2}(\#N_i(g)-1)}$$

(7)

where
\[ \hat{N}_i(g) = \{ j \in N | j \neq i, z \text{ and } ij, iz, zj \in g \} \] (8)

4. Artificial environment

The set of traders \( N \) are localized in a specific undirected graph \( g \subset g^N \) in which all the individual business relationships are specified (\( \#N = 100 \)). Traders play a variant of the Kiyotaki & Wright \(^{24} \) model shown above. Pairwise matches are made on the basis of each trader’s partner neighborhood. \( G \) different types of commodities are implemented in the system with a fixed quantity of units per each commodity, denoted by \( Q_i \) (where \( i = 1, \ldots, G^3 \)). These units are randomly distributed among traders at the beginning of the simulation. No inventory limit in storing goods has been implemented. Traders can carry as many units of commodities as they want. Also, no intrinsic property of commodities has been imported from other monetary models, regarding specifically storability and transportation costs. The traders’ perception of different commodities in terms of their intrinsic properties is therefore exactly the same. This is confirmed by the fact that traders have to consume one unit of every commodity every time period. This is an important variant of the original Kiyotaki & Wright framework which underline the key role of these properties on the endogenous emergence of commodity monies. It also allows us to show that stringent properties are not needed for the emergence of media of exchange. In order to have constant \( Q \) over time, the units consumed in the previous time are randomly redistributed among agents\(^1 \).

No direct utility is provided to agents that consume commodities. The utility function that traders have to maximize is mapped into traders’ inventory. Traders are homogeneous in terms of utility function. For a generic agent \( n \), it is denoted by

\[ U_n = \min_i (q^n_i) \] (9)

where \( q^n_i \) identifies the units of commodity \( i \) in the inventory of trader \( n \). This utility measure directly refers to the survival basket of goods (the agent’s inventory) that each trader has to optimize. With this particular formula where there is no complementarity between commodities (every good is necessary for the agent wealth) zero units to any good can threaten our agent survival. The optimization process of the survival basket leads traders to balance the number of units of their inventory commodities\(^2 \).

\(^1\)Like the Kiyotaki & Wright model and many other extension of it, we fix \( G = 3 \). There are 1000 units of each commodity distributed in the system.

\(^2\)This redistribution process can be view as the luck of traders in finding commodities in the environment without a trading action.

\(^3\)The utility formula rewards traders who exchange with their rare commodity.
Traders have two way for obtaining commodities they need. First, they can receive units from nature as result of the redistribution process explained previously and for which they have no control. Second, traders can obtain commodities from their business partners by exchanging units appropriately. Each time period, trader $i$ is randomly matched with one of his partners included in the set $N_i(y)$. He makes an exchanging proposal, that is he chooses the commodity he is willing to offer and the commodity he would like to receive in exchange. On the basis of the proposal, the second trader accepts or rejects the offer. If the offer is accepted, the exchange takes place. Like Kiyotaki & Wright \[24\] and many of the variant of it discussed above, the exchange process follows the quid pro quo constrain with exchange rate 1 : 1.

### 4.1. Strategy of traders

The exchange behavior of traders is controlled by vector of weights. The generic trader $i$’s vectors are denoted by

$$A_i = \{\alpha_{i1}, \ldots, \alpha_{iG}\}$$  \hspace{1cm} (10)$$

$$O_i = \{\beta_{i1}, \ldots, \beta_{iG}\}$$  \hspace{1cm} (11)$$

$$R^A_i = \{a_{i1}, \ldots, a_{iG}\}$$  \hspace{1cm} (12)$$

$$R^O_i = \{o_{i1}, \ldots, o_{iG}\}$$  \hspace{1cm} (13)$$

and represent the asked good weights, the offered good weights, the accepted weights for asked and offered good respectively.

Trader $i$ chooses the commodity he is willing to offer and the commodity he would like to receive on the basis of the following formula:

**Asked Good**

$$x_{\alpha} = \{x_{\alpha}^x < \sum_{z=1}^{G} \alpha_{i}^z \times \hat{q}_z^x \forall y \neq x\}$$  \hspace{1cm} (14)$$

**Offered Good**

$$y_{\beta} = \{y_{\beta}^y < \sum_{z=1}^{G} \beta_{i}^z \times \hat{q}_z^y \forall x \neq y\}$$  \hspace{1cm} (15)$$

where $\hat{q}_x^y = \frac{\beta_{i}^y}{\sum_{z=1}^{G} \beta_{i}^z}$.

The second trader will accept the offer if and only if

$$\begin{align*}
  a_{i}^x \hat{q}_x^x - \frac{\sum_{z=1}^{G-1} a_{i}^z \hat{q}_z^x}{\alpha_{i}^G} &> 0 \\
  o_{i}^y \hat{q}_y^y - \frac{\sum_{z=1}^{G-1} o_{i}^z \hat{q}_z^y}{\beta_{i}^G} &< 0
\end{align*}$$  \hspace{1cm} (16)$$
where $x$ identifies the asked good and $y$ the offered one. Traders’ weights are randomly fixed at the beginning of the simulation in the range $[0, 1]$.

The idea behind the weight vectors is the following. Weights represents the individual preferences about commodities. On the basis of the utility function, traders are able to increase their wealth exchanging one unit of their abundant commodity with one unit of their rare commodity. This strategy is relatively close to the Kiyotaki & Wright [24] fundamental strategy where agents exchange their inventory commodity only with a cheaper commodity in terms of storability costs. The situation that guarantees this result is when agents have the same preferences about commodities and their strategy is just focused on the inventory balancing. Technically, it happens when

\[
\alpha_x^i = \alpha_y^i \vee \beta_x^i = \beta_y^i \vee a_x^i = a_y^i \vee o_x^i = o_y^i \forall y \neq x
\]  

(17)

This commodity $x$ becomes the medium of exchange of the trader $i$.

In all other configuration of weights values we can find what Sethi [41] called polymorphic states.

4.2. Evolutionary process

Traders’ behavior is updated by a pure genetic algorithm every $f$ time periods\textsuperscript{1} following the so-called replicator dynamic technique (Mailath [27]). This assumes that a portion of the population with the highest wealth does not change at all (in this paper it is the 70%), while the rest of the population updates its weights trying to imitate the best agents (Basci [2]) on his trade partner neighborhood.

Technically, the genetic algorithm creates new vector of weights from trade partners belonging to the population of best traders. It applies the crossover technique and then mutates these new values in a range $\pm m$ with probability $pm$ (Mitchell [31], Mailath [27])\textsuperscript{m}.

\textsuperscript{1}The GA implementation is relatively close to Marimon & Miller [29] and Staudinger [45]. $f$ in this framework is equal to 100.

\textsuperscript{m}Results from many simulations with different $m$ and $pm$ values allowed us to find the best calibration of these values, in terms of convergence speed to the trader’s strategy, in $m = .05$ and $pm = .1$. 

\[ \alpha_x^i = \alpha_y^i \vee \beta_x^i = 1 \vee \beta_y^i = 0 \vee a_x^i = a_y^i \vee o_x^i = 1 \vee o_y^i = 0 \forall y \neq x \]  

(18)
Inventory units of the worse agents are reorganized with those of the best agents, transferring part of the inventory units from the best agents to the worse ones\(^a\).

### 4.3. Conditions for monetary equilibrium

We define a monetary equilibrium as the condition in which at least \( \frac{3}{4} \) of the traders converge towards a speculative strategy in which one commodity is used as a medium of exchange. Technically we check the propensity, in terms of weights, of one good to be offered and accepted in an exchange. Values close to one prove that trader has found his commodity money. Different traders can choose different commodity monies and this possibility has been investigated in this paper in terms of trading network properties.

### 5. Results

The agent-based framework presented in this paper has been implemented using a java library called JAS (Java Agent-based Simulator\(^b\)). The simulation results have been organized in the following way. The first paragraph of this section presents a simulation context in which links are static (they can not change over time). It means that traders are not able to modify their trade partner neighborhood \( N_i(g) \). This is a benchmark test that allow us to study the properties of trade networks structures on the monetary equilibrium. The second paragraph investigates both the monetary equilibria and the networks structure in a context in which links are dynamic. It means that traders are able to modify their partner neighborhood when exchanges do not occur. In particular, it investigates the endogenous evolution of middlemen positions among traders.

#### 5.1. Static links context

##### 5.1.1. Simulation results

Results shown in this paragraph refer to simulations based on different \( g \subset G^N \). In particular, the dataset contains more than a hundred different graphs starting from random topology towards small world and hubs configurations.

The first important result in terms of evolutionary learning is that the traders’ utility increases over time up to very high values close to its maximum value. Traders then are able to balance their inventory evolving efficient strategies. Although different trade network structures do not affect the positive evolution of the utility mean, they can drive the monetary equilibrium towards either one or multiple commodity monies. Imposing \( G = 3 \), four possible monetary equilibria can be observed in this

\(^a\)Technically, worse agents inherit from two agents of the other population (that should also be partners) exchange strategies and 15% of their inventory in order to make them able to trade.

\(^b\)http://jaslibrary.sourceforge.net/
Simone Giansante

evolutionary framework. Figure 1 shows examples of simulation results where the frequency of every commodity to be offered in an exchange is pictured out.

Figure 1a refers to the equilibrium with only one commodity money. Traders quickly converge to the speculative strategy in which all of them select good 1 as a medium of exchange. The frequency of this commodity raises to 1 after 200,000 time periods. Traders were able to coordinate their trading behavior choosing one commodity as a medium of exchange. The exchange in which agents offer one unit of their abundant good versus one unit of their rare good is the efficient strategy in terms of utility in a one-shot trade. However, in repetitive matchings when the probability to satisfy the double coincidence of wants becomes lower and lower, exchanges in which traders always offer the same commodity as medium of exchange increase exponentially and result to be a strategy more efficient than the fundamental one. This equilibrium also appears robust and stable over time. This confirms the results reported in Kiyotaki & Wright $^{24}$ and in many other variants of that original model. Despite the absence of storage costs as well as specific trader typologies, money as a medium of exchange emerges as a result of a spontaneous selection process in which traders need to find an universal language of coordination for reaching their aim. This language is money.

Figure 1b refers to the equilibrium in which only one commodity money emerges as a medium of exchange, but it is not the same one over time. Cyclical evolution among commodities can be observed during the evolution making the equilibrium unstable.

Finally, Figure 1c and 1d shows equilibria with multiple monies that exist in the

Fig. 1. Equilibria in terms of offered goods’ frequency
same system, in particular equilibria with two or three monies respectively.

Stable speculative equilibria can emerge in a context in which the evolution of traders’ behavior is a dynamic coordinated process between trading partners. It does not require any stringent constrain, but needs a dynamic framework able to manage a high level of complexity. Agent-based simulation are one of these.

An intuitive interpretation of this dynamic process is the following. As the initialization of traders’ weights is random, the initial strategies of agents can be very heterogeneous. Suppose that, during the evolution, the genetic algorithm creates some new vectors of weights with a very strong preference to a particular good as a medium of exchange. Traders with these new strategies are able to exchange using the same commodity money and increase their wealth. But this is not enough. The dynamic process called money also requires a critical mass of well connected traders in terms of trade network structure in order to start the exponential convergence towards the monetary equilibrium. In very connected trade networks, this dynamic can produce an huge coordinating process affecting all traders in the system and evolving an unique commodity as the universal medium of exchange. This is what Iwai [19] called bootstrap mechanism: “Even if there is no "real" demand for it and even if there is no "real" supply of it, this long-run "bootstrap mechanism" endows any good in the economy with all the characteristics that money should have”. It is just a matter of time that the economy will find its monetary equilibria. Our results also confirm that “since our choice of good m is completely arbitrary,” (in this case the genetic algorithm process) “we have indeed established the existence of N different equilibria, each using one of N goods as money” (Iwai [19], p. 472). When the connectivity of the trade network decreases the bootstrap mechanism effect is localized in particular portion, of connected subset, of the entire network evolving equilibria with multiple commodity monies.

In the next paragraphs properties of trade networks are investigated with respect to monetary equilibria.

5.1.2. Trade path

We define a trade path as a particular set of agents that follows the properties described in (5) and uses the same medium of exchange.

Figures 2 and 3 display social trading networks in which equilibria with two and three currencies respectively evolve as a result of the evolutionary learning. They can be obtained from a random graph with probability equal to .02 and .01 respectively. The color of each node/trader represents the commodity the trader offered in every exchange, that is the medium of exchange used for exchanging goods. The black nodes identifies traders that did not exchange during the last period of the simulation.

The speculative strategy that makes use of a commodity as a medium of exchange appears very clear in these two figures. We can notice traders on the same trade path exchanging with the same commodity money, particularly in Figure 3.
where the network density in terms of number of links is very low and therefore it is much more simple to follow trade paths. Different media of exchange co-exist in the
same global network and are localized in specific trade paths among trade partners. We can also observe that traders who did not exchange during the last period of the simulation are those situated on the border of the network, where the learning process is much more difficult. In fact these traders who have the smallest number of partners have problems in coordinating their behavior and therefore finding the right currency. Indeed, the probability that these traders will update their strategies by the genetic algorithm is higher than the probability of other traders.

5.1.3. Network structure and properties of equilibrium

We tested different network structures in order to investigate the correlation between network properties and equilibrium. Results from nine different configuration are observed and data reported in Table 1.

<table>
<thead>
<tr>
<th>Graphs</th>
<th>CC</th>
<th>D</th>
<th>average</th>
<th>#P</th>
<th>Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>complete</td>
<td>1</td>
<td>1</td>
<td>.96</td>
<td>.13</td>
<td></td>
</tr>
<tr>
<td>rnd(p=.1)</td>
<td>.19</td>
<td>3</td>
<td>1.776</td>
<td>.0</td>
<td></td>
</tr>
<tr>
<td>rnd(p=.03)</td>
<td>.06</td>
<td>5.375</td>
<td>2.73</td>
<td>.07</td>
<td></td>
</tr>
<tr>
<td>rnd(p=.02)</td>
<td>.11</td>
<td>8</td>
<td>3.37</td>
<td>.31</td>
<td></td>
</tr>
<tr>
<td>rnd(p=.01)</td>
<td>.30</td>
<td>inf</td>
<td>5.25</td>
<td>.75</td>
<td></td>
</tr>
<tr>
<td>sw(k=2 p=.01)</td>
<td>.03</td>
<td>6.8</td>
<td>3.42</td>
<td>.28</td>
<td></td>
</tr>
<tr>
<td>sw(k=2 p=.005)</td>
<td>.01</td>
<td>10.3</td>
<td>4.60</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>regular</td>
<td>0</td>
<td>50</td>
<td>24.27</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>hub</td>
<td>1</td>
<td>3</td>
<td>2.54</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

where inf (infinity) identifies the case in which $\exists i, j \in g | P_{ij}(g) = \emptyset$.

The value $p$ in random graphs (rnd) identifies the probability that every node has to be connected to each other in the network. The lower the probability of connectivity, the smaller the number of trading partners. For a small world graph (sw) the value of $k = 2$ identifies the connection to her previous and following node (creating a regular graph). Furthermore nodes can be connected to each other node with probability $p$. Finally, we also tested a particular graph structure called "hub". This represents an architecture in which we find four nodes called "principals". The other 96 nodes have only one connection to one of the four principals. Principal nodes are also fully connected among them. This is a particular representation of a network with very short path and very high clusterization.

The variable Equilibria reported in the last column is a standardized variable that identifies the equilibrium described above. Indeed, this variable is equal to zero when only one currency emerges as a medium of exchange. It results to be equal to one when the number of simultaneous media of exchange is maximum, that is equal to the number of commodities that can be exchanged. Therefore, values close to 0, .5 and 1 identify equilibria with one, two and three commodity monies respectively.

We observe that whether the density of the social trading network is high, a single good emerges as the medium of exchange (Figure 1a). As connectivity decreases, the medium of exchange alternates across epochs. That is what we observed fixing
Simone Giansante

At extremely low levels of connectivity, more than one medium of exchange can exist simultaneously. Table 1 clearly shows a positive relation between shortest path, diameter of the network and equilibria. On the contrary, the cluster coefficient seems not to be so relevant in moving the equilibrium from one towards multiple currencies. Table 2 proves our intuition about the effect of each network property to the equilibrium

The Pearson correlation coefficients confirm that the shortest path and the diameter of the network are the only significant variables which are correlated, in a positive way, to the standardized variable equilibria. Indeed, increasing the value of these two variables the equilibria value increases up to the maximum.

These results can explain some processes observed in the reality, in which the growth of connectivity in trade connections among States and/or Nations produced the birth of an unique money.

5.2. Dynamic links context

This second section of the paper analyzes the co-evolution of exchange behaviors and network structure. Traders are now able to randomly find new trading partners if and only if they did not exchange with their previous ones before the genetic algorithm updating mechanism. The searching technique of new trading partners is completely random. Indeed, traders select new partners from the entire trade

---

**Correlations**

<table>
<thead>
<tr>
<th></th>
<th>Cluster Coefficient</th>
<th>Diameter</th>
<th>ShortestPath</th>
<th>Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cluster Coefficient</strong></td>
<td>1</td>
<td>-.790**</td>
<td>218</td>
<td>-.089</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.</td>
<td>.030</td>
<td>0.35</td>
<td>.497</td>
</tr>
<tr>
<td>N</td>
<td>60</td>
<td>39</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td><strong>Diameter</strong></td>
<td>-.790**</td>
<td>1</td>
<td>970**</td>
<td>.740**</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.003</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>N</td>
<td>39</td>
<td>39</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td><strong>Shortest Path</strong></td>
<td>.218</td>
<td>.978**</td>
<td>1</td>
<td>.640**</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.035</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>N</td>
<td>60</td>
<td>39</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td><strong>Equilibria</strong></td>
<td>-.003</td>
<td>.740**</td>
<td>649**</td>
<td>1</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.497</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>N</td>
<td>60</td>
<td>39</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

**. Correlation is significant at the 0.01 level (2-tailed).**

---

a random graph with p=.02 (Figure 1b). At extremely low levels of connectivity, more than one medium of exchange can exist simultaneously.

Table 1 clearly shows a positive relation between shortest path, diameter of the network and equilibria. On the contrary, the cluster coefficient seems not to be so relevant in moving the equilibrium from one towards multiple currencies. Table 2 proves our intuition about the effect of each network property to the equilibrium.

The Pearson correlation coefficients confirm that the shortest path and the diameter of the network are the only significant variables which are correlated, in a positive way, to the standardized variable equilibria. Indeed, increasing the value of these two variables the equilibria value increases up to the maximum.

These results can explain some processes observed in the reality, in which the growth of connectivity in trade connections among States and/or Nations produced the birth of an unique money.

5.2. Dynamic links context

This second section of the paper analyzes the co-evolution of exchange behaviors and network structure. Traders are now able to randomly find new trading partners if and only if they did not exchange with their previous ones before the genetic algorithm updating mechanism. The searching technique of new trading partners is completely random. Indeed, traders select new partners from the entire trade

---

The sample used for computing the correlation coefficients lacks the results obtained in the simulations with regular and hub network structure that represent extreme network structures. This simplification has been needed in order to eliminate the outliers in the correlation calculus.

We leave to other works the historical analysis of this events. More information can be found in Bayoumi & Eichengreen [3], Cooper [6], Portes et al. [37] and Rousseau [39].
network using a pure random criterium. This process is also totally costless, without any additional cost. Finally, the number of new trading partners is equal to the number of the old partners who were eliminated from the trading network in order to preserve $\# N_i(g) \forall i \in g$.

Suppose that trader $i$ did not exchange with his partner $j$, the searching technique modifies $g$ in the following way:

$$g - ij \text{ and } g + iz \text{ where } iz \notin g$$ (19)

5.2.1. Simulation results

We initialized simulations starting from the same network structures we tested in the previous section. In order to make traders able to coordinate their exchange behavior with their trading partners before the searching technique of new trading partners, we set the frequency of it five times bigger than the genetic algorithm frequency (in this case it will be 500).

As a first interesting result, we observed that in every run of simulations and in every different network structure traders coordinate their strategy towards the equilibrium with only one commodity money (Figure 1a). The searching mechanism increases the coordination process among traders and permits them to drive the equilibrium towards only one unique medium of exchange.

This mechanism also modifies the properties of the initial network structures. Some examples are shown in Table 3.

<table>
<thead>
<tr>
<th>Graphs</th>
<th>CC</th>
<th>D</th>
<th>#P</th>
<th>CC</th>
<th>D</th>
<th>#P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_0$</td>
<td>$t_{end}$</td>
<td>%Δ</td>
<td>$t_0$</td>
<td>$t_{end}$</td>
<td>%Δ</td>
</tr>
<tr>
<td>$\text{rnd}(p=.1)$</td>
<td>.18</td>
<td>.26</td>
<td>.46</td>
<td>3</td>
<td>4</td>
<td>.33</td>
</tr>
<tr>
<td>$\text{rnd}(p=.03)$</td>
<td>.09</td>
<td>.22</td>
<td>1.33</td>
<td>5</td>
<td>inf</td>
<td>/</td>
</tr>
<tr>
<td>$\text{rnd}(p=.02)$</td>
<td>.11</td>
<td>.15</td>
<td>.38</td>
<td>8</td>
<td>inf</td>
<td>/</td>
</tr>
<tr>
<td>$\text{rnd}(p=.01)$</td>
<td>.35</td>
<td>.25</td>
<td>-28</td>
<td>inf</td>
<td>inf</td>
<td>/</td>
</tr>
<tr>
<td>$\text{sw}(k=2\ p=.01)$</td>
<td>.01</td>
<td>.2</td>
<td>14.25</td>
<td>7</td>
<td>inf</td>
<td>/</td>
</tr>
<tr>
<td>$\text{sw}(k=2\ p=.005)$</td>
<td>0</td>
<td>.27</td>
<td>60.3</td>
<td>11</td>
<td>inf</td>
<td>/</td>
</tr>
</tbody>
</table>

where $t_0$ is the starting time of the simulation, that is the first time period, and $t_{end}$ the last time period.

The mechanism leads the clusterization effect towards cluster coefficient values around .22 in every run of simulation. The shortest path converges towards values around 2.55 and the diameter becomes infinity. It is the case in which at least one node results to be disconnected from the rest of the network and the calculus of these coefficients becomes impossible.

Even though we start from different network structures, the mechanism that allow traders to find more convenient trading partners makes the network change towards a structure characterized by same properties. This process appears very
clear in Figure 4 where examples of the network topologies we tested are displayed.

The six graphs in Figure 4 refer to the following criteria:

- a) random graph with \(p = 0.02\) \((t_0)\);
- b) random graph with \(p = 0.02\) \((t_{end})\);
- c) random graph with \(p = 0.01\) \((t_0)\);
- d) random graph with \(p = 0.01\) \((t_{end})\);
- e) small world graph with \(k=2\) and \(p = 0.01\) \((t_0)\);
- f) small world graph with \(k=2\) and \(p = 0.01\) \((t_{end})\);
5.2.2. Trading middlemen

A common evolution in the trader degree distribution appears very clear in Figure 5.

The final degree distribution results to be less egalitarian than the initial one. It appears very smooth with a long right tail. A small number of traders were able to increase the number of their connections with respect to the others that maintain a few number of linkage.

Differences in the distribution of wealth between the starting point and the end of the simulations is another interesting phenomenon that we briefly investigated. Figure 6 shows these results on the basis of the network structures shown above.

From the starting point in which the social wealth (that refers to the utility values) is very close to a normal distribution, the distributional shape of the social wealth changes a lot during the simulation. First of all, we observe a strong growing of the mean of the distribution: in average the exchange process increased the individual wealth. This process also increased the differences between traders in terms of utility values.

Beside the mean values that we have discussed above, the variance of the final
Fig. 6. Distribution of fitness among traders at the beginning and the end of the simulations distributions is very high and also the skewness and kurtosis values changed a lot. A common result that we underline looking at the distributional shape of the final wealth is the clear separation of the initial population in two sub populations, one with high values and the other one with low ones. The exchange process we described and the evolutionary learning mechanism we implemented lead the system to a situation in which differences between traders, in terms wealth and number of partners, are observed. Starting from a wealth distribution quite egalitarian, we look after the emergence of two different social castes, one richer than the other one.

This phenomenon is strictly correlated to the results obtained for the degree distribution shape. Looking at the simulation data, we observed a positive correlation between connectivity and wealth. Traders that increased their connectivity also
improved their wealth condition. As the latter condition refers to the composition of the inventory, this means that they accumulated a lot of units of goods caused by their high connectivity and, therefore, the intensive exchange activity. This population of high connected traders own the same characteristics of the Middlemen agents reported in Rubinstein & Wolinsky [40] and Gilles et. al. [13]: high exchange rate, number of links and goods in their inventory. They also have the power to break up communication among other players in the network.

The emergence of middleman position in bilateral trades, in particular in monetary economy, covers a crucial rule in understanding and analyzing the characteristic and properties of our current complex monetary markets. The simplicity of this model does not allow us to investigate these actual problems yet. However, our results clearly show that middleman positions can emerge from a simply monetary economy where homogeneous traders in terms of utility function and intrinsic properties self-organize their set of business partners optimizing their exchanges and increase their wealth.

6. Conclusion

This article analyzes the impact of trading network properties on the evolution of either one or multiple commodity monies. The agent-based simulation we implemented for studying trade dynamics allows us to obtain results from a model with an high degree of complexity in terms of observed variables.

The model clearly proves that speculative strategies introduced by Kiyotaki & Wright [24] result to be stable and the more efficient behavior for every trader in every exchange. The emergence of money increases the number of exchange and the social wealth of the system.

Different trading networks can evolve different commodities as a medium of exchange. In a multi-currency equilibrium we can observe an high clusterization of strategies. Trading partners coordinate their behavior in order to use the same money and, therefore, increase the number of exchanges.

When agents are allowed to reconfigure their links to other traders without additional costs, networks evolve structures with specific characteristics, regardless of the initial topology, in which the majority of agents have only a few linkages to others while a few agents become quite connected. These latter traders own the same characteristics of the middleman agents reported in Rubinstein & Wolinsky [10] and Gilles et. al. [13]: high exchange rate, number of links and goods in their inventory. They also have the power to break up communication among other players in the network.

\(^1\)For more information about this connectivity power, see Gilles et. al. [13].
References

24 Simone Giantsante

working paper, Technical University, Vienna, 1998.


