

The Nature and Causes of Extreme Events: An Application to Subprime Market Spillovers

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Abstract

What drives extreme and rare economic events? Motivated by recent theory, and events in US subprime markets, we begin to open the black box of extremes. Specifically, we build a taxonomy of extremes, then extend standard economic analysis of extreme risk in two ways. First, we model the potentially relevant dimensions of dynamics and endogeneity. In characterizing individuals' endogenous propagation of extremes, we relate the latter to public goods. Second, using over a century of daily stock price data, we construct empirical probabilities of extremes. We document that extremes are relatively frequent and persistent. We also find evidence that extremes are endogenous for some reference periods, which raises the possibility that control of extremes is a public good.

Keywords: Extreme event; Subprime Market; Dynamics; Endogeneity; Public Good

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1 Introduction and Motivation

For whoever knows the ways of Nature will more easily notice her deviations;
and ... whoever knows her deviations will more accurately describe her ways.

F. Bacon: *New Organum*.

In the spring and summer of 2007, the aftershock from the subprime market, a relatively small part of US financial markets, has reached over to touch hedge funds and international markets. In the US, credit spreads have widened ominously, even for safer debt, and in Britain the interbank rate has reached its highest level in 9 years, as shown in Figure 1. Modern economies are repeatedly subject to such financial extremes, sometimes contemporaneously or in rapid succession, as in the contagion episodes in East Asia during the late 1990s. Extreme events often seem unpredictable, but are they? This paper begins to open the black box, and proposes a positive theory of extremes, based on externalities. By way of motivation, let us discuss two current puzzles.

A mortgage market puzzle: Recent events in the US subprime mortgage market are difficult to explain using standard economic analysis. The years leading up to 2007 featured a large demand for housing loans by US households of various credit levels. At the same time, lenders offered a large supply of low rate loans to prospective homeowners with extremely poor credit history, and high likelihood of default. According to standard information theory, the housing loan market features moral hazard and adverse selection, because borrowers know more about their ability to repay than do lenders. A standard solution to such information asymmetry involves credit rationing by lenders, or signalling quality by prospective borrowers (see Stiglitz and Weiss (1981) and Riley (2001)). However, during the period from 2000 to 2007, the opposite happened. As mentioned above, availability of credit to low credit history individuals increased, and potential borrowers did not have to signal.¹ Despite improvements in risk management by banks and regulatory authorities, such issues have recurred in recent years, in many developed economies. For example, a similar situation existed in Japan's Jusen loans, and in Norway during the 1990s. This puzzle suggests that there might be other factors at work in addition to information issues, especially during extreme events.

A hedge fund spillover puzzle: A second puzzle relates to the recent hedge fund debacles. Two issues convey the essence of this puzzle. The first issue concerns spillovers from

¹For example, borrowers without collateral were encouraged to apply for loans, and therefore did not have to signal quality.

the subprime market to hedge funds and other domestic or international investors. There have been a number of instances, so we only consider one of the more outstanding. In July and August of 2007 hedge funds suffered such severe losses that Goldman Sachs had to infuse US\$3 billion into one of its funds, Global Equity Opportunities. This fund lost 30 per cent of its value in the week between August 3 and August 10. This seems to be the first time that Goldman Sachs has assisted a hedge fund, especially in that magnitude. A major reason cited for the severe hedge fund losses was that the extremes that occurred in markets were '25 standard deviation' events (New York Times, August 13, 2007). These incidents are puzzling because hedge funds did not seem directly exposed to heavy enough risk to warrant such drops in value.² The second issue concerns extreme surprises. Most hedge funds and investment banks have risk management systems that are stress tested against extreme market events such as terrorism risk, banking crises, and interest rate changes. So what sort of event could surprise such respected hedge funds enough to lose as much as one-third of their value? A potential answer is that our approach to understanding "surprise" extreme events is incomplete. One source of incompleteness is that both information economics and current risk management are generally silent about time variation in the probability of extremes. Another issue is that they do not always account for endogenous spillover effects from one economic sector to others (such as mortgage market effects on hedge funds), especially in the face of extreme events.

A possible solution to both puzzles is to extend existing theory to include explicit, positive analysis of extremes. Existing theory acknowledges that individual agents' incentives or behavior can affect individual outcomes, for example, in insurance markets. This framework is usually restricted to individual agents or sectors, and typically requires asymmetric information between borrowers and lenders. The current issues, however, potentially affect numerous sectors and regions. Moreover, especially in the case of subprime mortgages, it is difficult to argue that lenders were oblivious to asymmetric information issues, and did not understand the potential for default when supplying loans to borrowers with poor credit history or no collateral. Therefore, current experience suggests that an extension of existing approaches may be valuable, especially for analyzing extreme outcomes. In this paper we attempt to provide such an extension. In particular, we will illustrate that under some conditions, aggregate spillover effects can happen even in the absence of asymmetric information. A graphical depiction of our approach is in Table 1. This table shows that our view of endogenous probability is similar to that of moral hazard. The only difference is that

²Moreover, the spillover effect on international markets was large enough to prompt unscheduled central bank interest rate cuts in the US and Europe.

we consider more general settings, where there may be spillovers and general information structures.

Discussions of extreme economic events often assume that extremes are generated exogenously by nature, and have a constant probability of occurrence.³ But is the likelihood of extreme and rare events affected, at times, by our behavior? And do we sometimes observe spikes in the frequency of extremes? The answer to both questions is yes. Dynamic, endogenous extremes occur in economics and in nature, including the effect of human activity on both the likelihood of extreme financial events, and extreme climate changes.⁴ Importantly, when human activity endogenously increases the likelihood of extremes, they may become less rare. In this paper, we explore a possible explanation for endogenous extremes, namely, externality effects. Externalities occur when one agent's actions directly affect the environment of other agents. Financial crises and extremes have externality features, since they affect many individuals in the national or global financial system, even though often precipitated by a small number of individuals. It is well known that externalities cause inefficiency of the price system.⁵ Consequently, if extreme events are due to externalities, society may not pay the appropriate price for the extremes that it generates.

How does this formulation of extreme externalities help us? It does so in two ways. First, it allows us understand the origin of some extremes (the endogenous ones), thereby giving us insight into which we can plausibly try to avert. Second, it gives banks and regulatory authorities an additional set of tools from public finance—subsidies, property rights, and so on—that may help to address extreme events before and during their occurrence.

1.1 Related Research

Several recent papers discuss issues related to extreme and rare events. Barro (2006) constructs a Lucas (1978) model with rare extreme events. Upon calibrating the model to twentieth century data on extreme events, he finds that it allows him to address the equity premium and riskfree rate puzzles. Weitzman (2007) develops a Bayesian model of asset returns. He discovers that when agents consider the possibility of extremes, there is a reversal of all the major asset pricing puzzles. Chichilnisky and Wu (2006) present a model

³See, for example, Barro (2006) and Friedman and Laibson (1989).

⁴See the cover story of *Time*, March 30, 2007; and *Stern* (2007); and Grossman (1988). The *Economist's* June 29 issue discusses climate change, where extremes become more normal over time.

⁵For textbook expositions of externalities, see Varian (1992), Mas-Colell, Whinston, and Green (1995) and Harris (2003) Chapter 9.

of endogenous uncertainty where increased financial innovation leads to greater likelihood of default. Chichilnisky (2007) shows that if we axiomatically extend expected utility to account for extreme responses to extreme events, then we can overcome decision theory paradoxes, such as those due to Allais (1953) and Ellsberg (1961). Danielsson and Shin (2003) discuss a scenario where unanticipated coordination of agents' behavior leads to an endogenous increase in risk. The research of Bazerman and Watkins (2004) suggests that certain "surprise" events in modern society are predictable, since there may exist sufficient information to know that these events are imminent.⁶ Our paper is similar to the above papers in that we discuss the importance of extreme events in socioeconomic life. However, our paper is different in several ways. First, unlike previous research, we explicitly construct a taxonomy of extremes, then develop a simple model to explain the origins of endogenous extremes. Second, we derive the "signature" of endogenous extremes, and devise simple empirical tests. Third, we test our model on recent US data, providing both in-sample and out-of-sample evidence on the causes of current market extremes. Finally, the model allows us to discuss new policy solutions to extreme events, using a standard public finance toolkit.

The rest of the paper is organized in the following manner. Section 2 discusses general information on extreme events and proposes a taxonomy. Section 3 outlines a positive approach to analyzing dynamic, endogenous extremes. Section 4 addresses some policy implications for current financial markets. Section 5 applies the model to current data, and Section 6 concludes.

2 Nature and Causes of Extremes

Knowing the origin of extremes is evidently valuable for investors and policymakers. In attempting to provide a glimpse of the origin, we now outline our positive approach to analyzing extremes. There are two aspects to this approach.

⁶According to Bazerman and Watkins (2004), predictable surprises have six characteristics: leaders know about a problematic issue, which will not go away; the issue worsens over time; the issue is costly to fix now, and benefits would occur later; fixing the issue entails a certain cost, but uncertain reward; addressing the issue changes the status quo; a small vocal minority benefits from lack of preventive action.

2.1 Temporal Nature of Extremes

The first aspect concerns dynamic behavior of extremes. In typical economic applications it is often implicitly assumed that the likelihood of extremes is constant over time. This assumption is useful for analytical tractability. Evidently economic and natural systems change and grow over time, which may affect the probability of extremes. There is some evidence that extreme probabilities change over time, such as record-breaking stock market levels in the 1990s, and increased numbers of Atlantic hurricanes since 2000. As shown in Figure 2, both the number of natural disasters and their impact seem to have varied over the past generation.⁷ For stress testing in hedge funds, for example, the likelihood of large price deviations is very important to estimate. A mistaken assumption of constant likelihood of extreme price changes is clearly dangerous at many levels, to central bankers as well as individual and institutional investors. Thus, we might allow the temporal nature of extremes to be static or dynamic. For static extremes, the likelihood of extreme events p_t is constant, and $p_t = p$ for all time periods. Dynamic extremes, by contrast, can be of two varieties, either random or with a discernible dynamic pattern. We shall discuss this in Section 3.

2.2 Causes of Extremes

The second aspect is an understanding of the distinction between exogenous and endogenous extremes, each of which has a different policy response.⁸ Exogenous extremes arrive from outside the economic system and are truly acts of nature, from the perspective of the domestic economy. For example, in a crop-based economy, the probability p of extreme changes in crop value could depend on exogenous swings in weather.⁹ Since weather is generally unpredictable beyond a few days, and exogenous to an individual farmer, we can represent the probability of extremes as essentially random. In order to obtain bounded probabilities, we may consider a random variable z_t and p_t that are related in the following manner:

⁷The definition of disaster by EM-DAT is "A situation or event which overwhelms local capacity, necessitating a request to the national or international level for external assistance, or is recognized as such by a multilateral agency or by at least two sources, such as national, regional or international assistance groups and the media".

⁸In practice, there is likely to be a spectrum of extremes, with some being a mixture of exogenous and endogenous. The idea here is to give us tools to assess the dominant influence on extremes.

⁹Other causes of exogenous extremes may include foreign wars, natural catastrophes, and uncertainty about new technology.

$$\begin{cases} z_t = z_{t-1} + \varepsilon_t \\ p_t = \frac{\exp(z_t)}{1 + \exp(z_t)}, \end{cases} \quad (1)$$

where $\varepsilon \sim i.i.d N(0, \gamma)$, with $\gamma > 0$, for example.

Endogenous extremes, by contrast, are generated and perhaps amplified within the economic system, by agents' activity and interaction. This activity persists because extremes have externality-like attributes, and therefore agents may 'over-produce' the amount of extremes in the system. For example, stock market crashes and banking panics may stem from excessive risk taking and borrowing of a segment of the economy (Fisher (1933), excessive credit creation Allen and Gale (2000)), and excessive reliance on computer-based trading (Grossman (1988)).¹⁰ Since each agent has an incentive to borrow or risk too much from the social point of view, competition leads to overproduction of extremes. Hence, the probability of extremes may no longer be a random walk. We will derive the relevant expression for extreme probability in Section 3, after developing a concrete definition of extremes below.

2.3 A Simple Taxonomy

Why do we need definitions of extreme and rare events? The main reason is that extremes occur in many disciplines. Therefore, each has developed its own terminology, which may be incompatible with that of other disciplines. For example, the concept of rare event is used in at least four ways in decision-related sciences. First, in statistics and econometrics, rare refers to a record-breaking phenomenon, one that has never occurred before (de Haan and Sinha (1999)). Second, in political science, it denotes a low probability event with a high impact, which may have occurred before (King and Zeng (2001)). Third, in the theory of risky choice, it refers to a low probability event, which may have occurred before, but not necessarily with a high impact (Hertwig, Barron, Weber, and Erev (2005)). Fourth, in finance the closely related peso problem denotes an infrequent regime that is unobserved but anticipated by economic agents (Evans (1996)).

¹⁰The above authors and other related researchers consider some form of extreme event or crisis, but vary in their emphasis on endogeneity. Some model a closed economy or a single sector, others an international setting. Therefore the applications differ, although endogeneity or externality issues are common to all. Our paper seems to be the first to use this framework explicitly in a general setting, in order to develop a positive theory with testable implications.

We therefore need to develop a common language to discuss extreme and rare events, since they arise in a wide variety of settings.¹¹ Possessing a common language, we can address implications for describing, forecasting and controlling extremes, a task that we begin to pursue in the next section. Based on previous research as well as what we feel to be intuitively appealing aspects of extremes, we now begin to develop a taxonomy. We will first provide a set of heuristic definitions of typical, extreme and rare events, in turn. Given the focus of this paper, we use definitions for quantitative data, such as security returns.¹²

Typical events are those that are normal in some sense, or that we encounter frequently. In previous economic literature, typical events have been conceptualized in two ways. First, they are near the center of the distribution, for example, within 2 or 3 standard deviations. This intuitive definition is useful in the case of the normal distribution, where 3 standard deviations around the mean capture 99.7% of the distribution.¹³ Second, another way to think of typical events is in topology. In this sense, an event is typical if it fills up the space of events.¹⁴ Rare or extreme events can be heuristically visualized as the complement of typical events, one in topology, the other in probability. Extreme events are 'far away' from the median, while rare events are 'small' in the set of all events, respectively. Armed with these heuristic descriptions, we suggest the following, simple taxonomy.

Consider a variable X with domain $\mathbf{X} \subset R$. Define a relevant sample $X_s \subseteq \mathbf{X}$, comprising n realizations of this variable, $X_s = X_1, \dots, X_n$, with median \bar{X}_s , and standard deviation σ_s . If X_s is a time series, assume that the relevant sample data are covariance stationary. In the following, superscripts T , R , and E indicate 'typical', 'rare' and 'extreme', respectively.

Definition 1: A typical event $X^T \in X_s$ is in a range X_{range} that contains more than 1/2 of the observations in the relevant data sample:

$$\frac{\text{Number of } X_i \in X_{range}}{\text{Number of } X_i \in X_s} \geq \frac{1}{2}.$$

¹¹The study of extreme and rare events increasingly affects so many disciplines that it has the potential to be considered a field in its own right. Examples of some associated disciplines include decision theory, international relations, finance, insurance, climatology, astrophysics, chemistry, and statistics.

¹²Financial data may be categorical or quantitative. Categorical data are measured in terms of a set of categories. A simple example of categorical extremes is a "Yes-No" type occurrence, such as whether there is a war. Quantitative data are reported numerically, such as the level of the DJIA. The distinction between categorical and quantitative extremes is relevant for quantitative modelling. For example, in quantitative data, extremes are very large and small values, which are not always definable for categorical data.

¹³ For arbitrary non-normal distributions with finite variance σ^2 , we can provide deviation bounds in a similar way, using Chebyshev's inequality.

¹⁴This may be expressed as the typical events form an open dense set in the set of all events (Debreu (1970)).

We now turn to rare events. The benchmark case for rare events is $1/5$, to match the psychologically motivated definition of Hertwig, Barron, Weber, and Erev (2005). However, other researchers estimate different values for rare events.¹⁵ Therefore, in order to give the researcher flexibility in deciding just how rare is rare, we index the definition by a multiplicative parameter $\delta \geq 1$, that reduces the size of the rare set accordingly.

Definition 2: A δ -**Rare event** $X^{R(\delta)}$ is in a range that contains less than $1/5\delta$ of the observations in the relevant sample, in the presence of another (nonoverlapping) range that occurs more frequently than itself:¹⁶

$$\frac{\text{Number of } X_i \in X_{range}}{\text{Number of } X_i \in X_S} \leq \frac{1}{5\delta}.$$

Before defining extreme events, we observe that much research in finance, political science and statistics uses the terms extreme and rare interchangeably. This custom is misleading for at least two reasons. First is the possibility of extreme clusters, where extreme events occur relatively frequently. For example, during bubbles or periods of high financial market volatility, it is possible for the stock index to reach levels far from the recent median, routinely. Second, for highly skewed or heavy-tailed distributions, extremes can occur more frequently than central observations. Consequently, it is important to define extreme events in a way that does not assume, a priori, that they are either typical or rare. It is also helpful to employ a definition that is related to the current practice of using standard deviation or volatility. We therefore adopt the following definition.¹⁷

¹⁵For example, the empirical values all fall below 2% in the research of Barro (2006), Chollete, de la Pena, and Lu (2006), Jansen and de Vries (1991), and King and Zeng (2001). Weber (2007, personal communication) explains that in experimental research on decisionmaking, the level $1/5$ is the threshold at which the divergence between decisions based on experience and description becomes pronounced.

¹⁶We require there to be a more frequent event, since the notion of rare is relative. Typical and rare events are mutually exclusive but not exhaustive. An event can be neither typical nor rare, for example if its range has an empirical likelihood between 0.1 and 0.5.

¹⁷This is related to terminology of extreme value theory, where extremes are usually phrased in terms of closeness to the maximum or minimum. The median is used instead of the mean or extrema because it is robust and has a higher breakdown point. Psychologically, people may take time to adjust their concept of normal, and the median embodies this more than the mean. Note that we choose a slightly different definition from extreme value theory because in finance we might worry about deviations from what is typical, even if they are not record-breaking events. For large ω , the definition will be identical, by choosing ω such that $\omega\sigma = |X_{(1)} - \bar{X}|$, where $X_{(1)}$ is an extreme order statistic.

Definition 3: An ω -**Extreme event** $X^{E(\omega)}$ is an event that is at least $\omega \geq 1$ standard deviations away from the relevant median:

$$|X^{E(\omega)} - \bar{X}_s| \geq \omega\sigma_s.$$

For financial time series, the benchmark median of the relevant dataset can easily be computed dynamically, to capture the notion that over time, what once was extreme may become commonplace.¹⁸ We are now ready to implement a workable definition of the empirical probability of extremes, p_t , for later use.

Definition 4: The **Empirical probability** $p(\omega)$ of an ω -extreme event $X^{E(\omega)}$ measures the relative frequency of observations exceeding ω standard deviations from the relevant median:

$$p(\omega) = \frac{\text{Number of } [X_i^* \in X_s : |X_i^* - \bar{X}| \geq \omega\sigma_s]}{\text{Number of } X_i \in X_s}.$$

Evidently what is typical, extreme, or rare may change over time, and our definitions above are designed to reflect this notion.¹⁹ We therefore emphasize that our definitions are conditional: we condition on the relevant data sample, which is chosen with the guidance of scientific theory and knowledge of the question at hand. This approach makes sense from a social science perspective, recognizing that when the world changes, we take some time (one period) to realize it. The conditional approach is a strength and a potential challenge. The strength is that it frees researchers in various disciplines or with different questions to choose their concept of rareness or extremeness, with alternative values of δ and ω . A challenge is potential lack of comparability across different studies. If comparability is an issue, one might use the data sample suggested by scientific theory and then compare with results using the entire data available.

¹⁸For example, one could compute extremes relative to the previous quarter's benchmark median, to capture individuals' lag time in learning and computing the benchmark. The notation ω is chosen since it is often used in definitions of oscillation.

¹⁹Our definitions compare current events to past medians. The reason is that individuals' notions of extreme is often relative to what they have learned in previously. This can be motivated by psychology, where we take time to learn about rare events by experience (Hertwig, Barron, Weber, and Erev (2005)), or by disaster myopia (Herring and Wachter (2005)). It can also be motivated by econometric considerations, since we gather data at the end of the period before we can compute sample statistics.

3 Dynamic, Endogenous Extremes

3.1 Dynamic Extremes

As mentioned in Section 2, dynamic extremes can be random or display patterns. The random case is represented by equation (1). Patterns may have many possible representations. For parsimony, and in order to relate our formulation to existing work in time series analysis, we consider a simple stationary pattern. One such model is a simple autoregressive representation, $p_t = \alpha + \sum_{j=1}^J \theta_j p_{t-j} + \varepsilon_t$. Although many lags are possible, we focus on the first order case:

$$p_t = \alpha + \theta_1 p_{t-1} + \varepsilon_t, \quad (2)$$

where $|\theta_1| < 1$. Expression (2) permits us to capture the potential clustering in extremes mentioned above.^{20 21}

What is the *signature* of dynamic extremes? According to equations (1) and (2), dynamic exogenous extremes have a frequency p_t that depends either on a random arrival ε , or else only on some function of its own past values.

3.2 A Simple model of Endogenous Extremes

Above, extreme probabilities are exogenous, and do not depend directly on variables under the control of economic agents. We now formalize the arguments of Section 2.2, and consider the possibility that economic agents and the economic environment influence the frequency of extremes. While exogenous extremes are statistically unrelated to the eco-

²⁰The focus of our discussion is on the *empirical* properties of p_t . Therefore, the regression residual ε in (2) must be compatible with bounded probabilities, because the p_t data used in our estimation will lie in the $[0,1]$ interval. If we were interested in modeling the theoretical properties of the process, we could impose boundedness in a standard way by using some variant of a logistic function, as we illustrated in equation (1).

²¹We could also consider simple nonstationary models, for example a regime switching generalization. Such models can be motivated by the real world consideration that extremes are sometimes likely, and at other times unlikely, reflecting aggregate external factors such as the state of the economy and seasonality in exogenous natural disasters. A simple regime switching model allows the intercept α to depend on time, related to the realization of a 2-state Markov chain. Specifically,

$$p_t = \alpha_{s_t} + \theta p_{t-1} + \varepsilon_t, \quad (3)$$

where s_t is a random variable that can assume two values, $s_t \in \{1, 2\}$. The transition probability between states i and j is represented by $\Pr\{s_t = j | s_{t-1} = i, p_{t-1}\} = \pi_{ij}$.

conomic environment, endogenous extremes (since they are generated by economic agents) should be related to the optimizing or equilibrium behavior of agents.

More formally, consider an economy comprising a large number l of lenders, and a large number m of mortgage borrowers. $m = f \cdot l$, where f is some positive integer. Each lender deals with an equal number of borrowers, $f = \frac{m}{l}$. The m mortgages are drawn from the same distribution, and of similar term. Their economic activity affects other agents in the financial system, including other banks, investment firms, hedge funds, and non-borrowing investors, domestically and internationally. We denote these other agents O , for other. In the following analysis we represent other, lenders and borrowers by subscripts 0, 1 and 2, respectively. Borrowers and lenders are both in the market for borrowed funds. Effective demand for borrowing is b_d and supply of borrowing is b_s . Investors and banks consider themselves small enough that their own borrowing and lending does not affect asset prices. As in the literature on credit cycles (Kiyotaki and Moore (1997)), the financial sector alternates between periods of easy and hard credit. The credit regime is denoted Cr and varies continuously between 0 and 1, where $Cr = 0$ denotes the hardest credit regime and $Cr = 1$ denotes the easiest credit regime. In keeping with the spirit of credit cycle literature, we let the effective demand and supply of borrowed funds b_d and b_s depend continuously and positively on the availability of credit. That is, for each agent, $b_d = b_d(Cr)$, and $b_s = b_s(Cr)$, with positive derivatives $b'_d > 0$ and $b'_s > 0$.²²

We focus on a representative mortgage lender and borrower with utility functions u_1 and u_2 , respectively. Both lender and borrower have concave differentiable utility functions with standard properties, that depend continuously on the level of borrowing: $u_1 = u_1(b_s)$ and $u_2 = u_2(b_d)$. Moreover, both u_1 and u_2 are increasing in the amount of borrowing, $u'_1(b_s) > 0$ and $u'_2(b_d) > 0$.²³ In order to control for contemporaneous costs, we consider utility to be net of current costs. Each agent knows there is a possibility of systemwide

²²Since borrowing depends on the credit cycle, our model has an important complementarity, delivering aggregate effects. Strategic complementarity means that it is more attractive to borrow (or offer credit) if other agents are doing the same. Strategic complementarities tend to arise in situations of imperfect competition, costly search for trading partners and preference externalities (Cooper (1999)). These situations can plausibly exist for lenders, borrowers and hedge funds, respectively. The credit regime summarizes for each agent how attractive other agents find it to engage in extensive borrowing or lending. Thus, when one agent borrows or lends, so do many others in the economy. Therefore an endogenous cause of extremes could be time varying incentives to offer and accept easy credit, perhaps related to the real estate cycle (Pavlov and Wachter (2007)).

²³The assumption of increasing utility must hold in the relevant range, otherwise there is no interesting economic problem: borrowers and lenders would just be automatically prudent, and never in danger of over-borrowing or over-lending. This assumption helps convey the nature of the economic problem in the US subprime market.

extreme events occurring, captured by the probability p , whose functional form is common knowledge. In the spirit of Fisher (1933) and Allen and Gale (2000), the probability of future extreme events increases with the average level of borrowed funds, $p = p(b_d, b_s)$, where $\partial p / \partial b_d > 0$ and $\partial p / \partial b_s > 0$.²⁴ If an extreme event occurs in the future, agent i incurs a positive cost c_i , $i = 0, 1, 2$. This cost is financial, social or psychological discomfort suffered in an environment of extremes or financial instability.²⁵ There is no asymmetric information about the likelihood of extremes. Each agent knows that this likelihood increases with average borrowing or lending.²⁶ Now consider the lender's problem. We include subscripts to clarify the time of decisionmaking. At period t the lender decides how much to lend this period by solving the problem:

$$\max_{b_s} u_1(b_{s,t}(Cr_t)) - p_{t+1}(b_{d,t}, b_{s,t}) \cdot c_{1,t+1}.$$

First order conditions for an interior solution are $u'_1(b_{s,t}(Cr_t)) - \frac{\partial p_{t+1}(b_{d,t}, b_{s,t})}{\partial b_{s,t}} \cdot c_{1,t+1} = 0$, which can be rewritten as

$$\frac{\partial p_{t+1}(b_{d,t}, b_{s,t})}{\partial b_{s,t}} = \frac{u'_1(b_{s,t}(Cr_t))}{c_{1,t+1}}. \quad (4)$$

Equation (4) says that optimally the (derivative of) extreme probability is related to the marginal utility of borrowed funds, discounted by expected costs. An implication, given positive marginal utility and the positive relation between borrowing and Cr , is that the responsiveness of extreme probability to the level of borrowing is positive, and increases

²⁴This summarizes the intuition that excessive borrowing is destabilizing, without emphasizing the particular channel of destabilization. Channels through which borrowing leads to increased likelihood of extremes are explored by a number of authors, including Fisher (1933) and Allen and Gale (2000).

²⁵A financial cost is that trading suffers because prices are relatively uninformative Harris (2003), Chapter 9). A social or psychological cost is increased Knightian uncertainty in an unstable economy Caballero and Krishnamurthy (2007), Weitzman (2007)). Implicit in our work is the notion that this pattern of excess borrowing may recur because of time variation in not just financial but also moral and psychological costs of overborrowing (Agarwal, Driscoll, Gabaix, and Laibson (2007)). Learning may not occur, since different individuals are involved, given the time of the asset cycle. For related ideas, see Kiyotaki and Moore (1997), Andvig and Moene (1990), and Minsky. This notion parallels concepts in corruption and tax evasion where supply of corruption increases due to lower moral costs of taking bribes. Corruption literature suggests that a cure can be increasing wealth. Analogously, we can consider arguments for increasing minimum wage or property rights.

²⁶Similar assumptions occur in many other economic contexts, such as the idea of price taking, competitive agents (Arrow (1954), Debreu (1959), Chichilnisky and Wu (2006), even though the demand of each agent will affect the price to some extent.

with the easiness of credit.²⁷ Similarly, the borrower's problem yields first order conditions which can be rewritten as

$$\frac{\partial p_{t+1}(b_d, b_s)}{\partial b_{d,t}} = \frac{u'_2(b_{d,t}(Cr_t))}{c_{2,t+1}}.$$

As in equation (4), the above expression says that the future probability of extremes is dynamic, and depends positively on the current level of credit availability.

Equilibrium: In equilibrium, the demand and supply of borrowed funds will be equal, $b_d = b_s \equiv b$. Therefore we can rewrite the optimality expression for the borrower and lender as

$$\frac{\partial p_{t+1}(b_t)}{\partial b_t} = \frac{u'_1(b_t(Cr_t))}{c_{1,t+1}} = \frac{u'_2(b_t(Cr_t))}{c_{2,t+1}}. \quad (5)$$

Equation (5) constitutes the *signature* of endogenous extremes. The responsiveness of extreme probability to borrowing is proportional to marginal utility of agents in the market for borrowed funds.²⁸ If extremes are truly exogenous, there will be no statistical relation between extreme probability and b , and $\partial p(b)/\partial b = 0$. The difference between equations (1) and (5) gives a sense of the estimation error from assuming extremes are exogenous, when they are in reality endogenous.

We are not just saying there is a link between over-borrowing and extremes. Instead, we are showing that even without asymmetric information, over-borrowing may arise as an equilibrium phenomenon. This phenomenon occurs due to the failure of *both* borrowers and lenders to internalize an important externality, the excessive probability of systemwide future financial crashes. An easy way to see that the probability of crashes is excessive is to consider what happens if the lender considers the effect of her lending on other agents O . Then her problem is

$$\max_{b_{s,t}} u_1(b_{s,t}(Cr_t)) - p_{t+1}(b_{d,t}, b_{s,t}) \cdot (c_{0,t+1} + c_{1,t+1}).$$

²⁷Utility increases with borrowing, which in turn increases with the credit regime. It follows that utility increases with the credit regime, since for finite real numbers an increasing function of an increasing function is also increasing.

²⁸ Our result is intuitive: agents affect extreme probability by their optimizing behavior over a certain variable with external effects. Therefore, optimally their marginal utility equals the responsiveness of extreme probability to this variable. It also captures the notion that when credit is easier, the financial system becomes more likely to suffer future crashes.

The first order conditions are $u'_1(b_{s,t}(Cr_t)) - \frac{\partial p_{t+1}(b_{d,t}, b_{s,t})}{\partial b_{s,t}} \cdot (c_{0,t+1} + c_{1,t+1}) = 0$. Rewriting this expression as before and imposing equilibrium $b_d = b_s \equiv b$, we obtain the condition for a socially optimal level of extremes:

$$\frac{\partial p_{t+1}(b_t)}{\partial b_t} = \frac{u'_1(b_t(Cr_t))}{c_{0,t+1} + c_{1,t+1}}. \quad (6)$$

The numbers are smaller, since the denominator on the right hand side of (6) exceeds that of (5). Thus, when the lender takes into account the future costs of other agents, optimal behavior involves a lower extreme probability for a given level of borrowed funds. A similar logic exists for borrowers. It is in this sense that competitive markets may lead to endogenous, excessively high probability of crashes.²⁹ There are two ways to express this situation. First, as before, we can recognize that overborrowing due to easy credit has a negative externality, and is therefore overproduced. Second, in language perhaps closer to regulators' concerns, we can say that financial system stability (control of extremes) is a public good, which suffers from classic underprovision.

The most important implications from equation (5) relate to the likelihood and persistence of extremes. For a given level of borrowing, the likelihood of future extremes increases with the ease of credit, and based on our previous discussion, with any instruments related to strategic complementarity, such as consumer confidence. It decreases with expected social, financial and psychological costs of extremes. Persistence of extremes is higher when the marginal utility of borrowing and costs are persistent. We shall explore this further with parametric preferences in Section 5.

3.2.1 Extreme Spillovers

The above formulation gives little intuition on spillovers, or the expected breadth of extremes. To tease out this information, one possibility is to observe that the breadth of spillovers depends on the strength of aggregation and complementarity. These effects may be assessed using indices of imperfect competition and search costs, for example (see Cooper (1999)). However, we feel a more realistic approach is to examine a situation where spillovers are typically very unlikely to begin with, and then ask what drives spillovers? In today's markets, spillovers are increasingly important to consider, since globalization and

²⁹Note that optimality will not necessarily entail complete elimination of extreme events. Rather, the extreme level is adjusted to the point where the marginal benefit to lenders of an additional unit of the externality-generating activity, $u'_1(b)$, equals its marginal cost to other agents, $-u'_0(b)$.

financial innovation not only spread risk, but also *hide* risk—investors in one sector might unknowingly bear some part of the risk from agents in distant sectors and nations. These risks are diversified away in normal times, but may be significant in periods of correlated returns and dependent defaults. Until now, our model has suggested that spillovers happen to hedge funds and all other sectors automatically in the subsequent period. This can occur if other sectors are directly exposed to default risk by purchasing high risk debt from lenders. In modern financial markets, however, there are numerous ways of diversifying such risk, for example by securitizing debt into a new instrument. Consequently, even if several borrowers default, their risk will be spread over many buyers, and therefore have little impact on each buyer of the securitized asset. What could cause a spillover in this case? The main channel is a systematic comovement in defaults or selling, as in the LTCM case during summer of 1998, and in many US financial markets in spring and summer of 2007. Such comovement may result in sharply reduced value of securitized assets. If this occurs, then securitized assets become highly undiversified, and may propagate the effects of extreme events. The question therefore becomes, what could cause a systematic comovement in defaults or selling? A compelling answer is liquidity. If economic agents face sharp, simultaneous liquidity drops in many asset classes, they may be forced to default (borrowers) or sell assets (lenders), which will simultaneously reduce the value of many securitized assets.³⁰ According to this logic, the incidence of extreme spillovers is determined at least partially by the extent of liquidity comovement.

We offer a simple, stylized formalization of the above arguments, describing a channel through which other economic agents experience increased extreme probability p . An important aspect of the economic environment is securitization. Loans are pooled into a diversified security S , like a CDO, then resold to the other sector at a competitive price equal to its discounted value. The reason for supply of this asset is evident, since lenders wish to diversify away their risk. Why does the other sector demand this asset? The reason is that S may dominate other risky assets, or provide diversification benefits.³¹ As in spring and summer of 2007, extreme spillovers are hastened by liquidity demands. To meet these demands, hedge funds and other investors may sell off a liquid, unrelated tranche such as municipal bonds. This selloff inhibits liquidity in that tranche, which leads to further

³⁰Since our focus is on extremes, for simplicity we model liquidity as exogenous. In practice liquidity may respond to changes other variables such as collateral prices (e.g. real estate) for collateralized instruments.

³¹In practice, CDOs are often bought by fixed income investors in search of high yields. CDOs will be at least as attractive as high yield bonds because the former are generally uncorrelated. In the present situation, sub-prime CDOs had relatively low risk when the real estate market was going up, since the collateral was extremely valuable. We are grateful to Arjun Jayaraman for discussions on this point.

selloffs in one tranche after another. Therefore, the exogenous driving force behind extreme spillovers is liquidity comovement—rare but high impact contemporaneous drops in liquidity across various securities. These liquidity shocks increase asset correlations and default dependence, making them move ‘in step’, all selling or defaulting at the same time. In addition, an important endogenous source of spillover risk is excessive diversification, which implies the other sector is highly exposed during periods of correlated default.³² This endogenous risk is amplified by inordinate, perhaps unknown, exposure to securitized assets like CDOs, by various market participants. Why is this endogenous risk left unchecked? There are three reasons. First, agents cannot fully understand rare events. They may have disaster myopia or otherwise underestimate the likelihood of rare events (Herring and Wachter (2005)), Hertwig, Barron, Weber, and Erev (2005), King and Zeng (2001)). It is well known that myopic behavior can arise for rational preferences, for example in the case of *logarithmic utility*. Other reasons why this may arise include limited computational ability, or statistical issues—it is hard to estimate the probability of rare events with limited data.³³ Second, even if some take rare events seriously, when liquidity is high, the ease of resale makes them more comfortable with elevated risk, since they feel they can resell if necessary. Third, during some stages of the asset cycle the collateral for securitized debt has a relatively high price. In the subprime market case the collateral is real estate, which featured rising prices around the turn of the century. Such a scenario can be another cause of complacency, since the value of collateral is high.

More formally, the mortgages are packaged into m securitized assets $S_1, \dots, S_j, \dots, S_m$, each with $1/m$ of the original securities, for maximal diversification. Each original mortgage i yields a stream of payments with liquidity-adjusted excess returns $r_{i,t}^e$. For simplicity, we assume symmetric correlations and standard deviations: $\rho_{ij,t} = \rho$, all $i \neq j$, and $\sigma_i = \sigma$.

³²Paradoxically, therefore, diversification can increase risk during extreme periods, since investors have a wider network of obligations, and are thus more likely to affect others and be affected by an extreme event in any sector (Chichilnisky (2004), Danielsson and Shin (2003), Ibragimov and Walden (2007), Duffie (2005)).

³³Underweighting of rare events has been documented in at least three ways. First, agents may discount rare events when they estimate probabilities based on experience (Barron and Erev (2003), Hertwig, Barron, Weber, and Erev (2005), and Rabin (2002)). Second, econometrically there is a bias to under-estimate rare events (King and Zeng (2001), and de Haan and Sinha (1999)). Third, expected utility does not effectively incorporate low probabilities (Bhide (2000) and Chichilnisky (2000)).

The return on a securitized asset is $r_{S,t}$, with mean $\bar{r}_{S,t} = \frac{1}{m} \sum_{i=1}^m r_{i,t} = \bar{r}_{i,t}$. The variance is denoted $\sigma_{S,t}^2$, and computed as³⁴

$$\sigma_{S,t}^2 = \frac{1}{m}(\sigma^2 + 2\rho). \quad (7)$$

We now define an extreme spillover, then discuss the role of liquidity.

Definition 4: An Extreme Spillover is an ω -extreme event in S_j . That is, a situation where $|r_{S,t} - \bar{r}_{t-1}| \geq \omega\sigma_{S,t-1}$.

In order to implement Definition 4, we need explicit expressions for portfolio returns, which we now develop. Previous research has documented that the correlation of asset returns, and correlation of defaults tend to increase together during extreme periods.³⁵ We formalize this empirical observation starkly by saying that individual mortgages are uncorrelated in general, but highly correlated a small fraction δ of the time. This is represented as a regime shift,

$$\rho = \begin{cases} \rho^{hi}, & \text{with probability } \delta \\ 0, & \text{with probability } 1 - \delta, \end{cases} \quad (8)$$

where δ is close to zero. Given assumption (8), the securitized asset variance in equation (7) is nonlinear, equalling $\frac{\sigma^2}{m}$ most of the time, and equalling $\frac{\sigma^2}{m} + \frac{2\rho^{hi}}{m}$ a small fraction of the time. Thus, the benefits of diversification accrue to the securitized asset in typical times, when its variance is much smaller than the sum of the individual security variances. By contrast, during extreme times correlations become important for returns, and diversification benefits evaporate.

³⁴To obtain equation (7), note that the security variance $\sigma_{S,t}^2$ is the variance of a sum of random variables,

$$Var \left[\sum_{i=1}^m \frac{1}{m} r_{i,t}^e \right] = \sum_{i=1}^m \frac{1}{m^2} Var(r_{i,t}^e) + 2 \sum_{i \neq j} \sum_{i \neq j} \frac{1}{m^2} Cov(r_{i,t}^e, r_{j,t}^e).$$

This can be computed as

$$\frac{1}{m^2} \sum_i \sigma_i^2 + 2 \sum_{i \neq j} \sum_{i \neq j} \frac{1}{m^2} \rho_{ij} = \frac{m\sigma_i^2}{m^2} + 2 \frac{m\rho_{ij}}{m^2} = \frac{\sigma^2}{m} + \frac{2\rho}{m},$$

where the last equality invokes the identical variance and correlation assumption from above.

³⁵See Mcneil, Frey and Embrechts (2005) p331; Ang and Bekaert (2001); and Engle, Capiello and Shepard (2006).

These extreme correlations are driven by liquidity. In particular, most of the time $(1 - \delta)$ liquidity is plentiful in some markets. However, a small fraction δ of the time, liquidity dries up in most markets, which forces multi-market margin calls and flight to quality. Consequently, there is an increase in defaults and delinquent payments across many assets, and their returns enter a high correlation regime. We assume liquidity drives the correlation regimes in (8) directly.³⁶ Specifically, liquidity is an additive cost or benefit to gross returns $r_{i,t}$ —mortgage securities have higher excess returns $r_{i,t}^e$ if idiosyncratic liquidity $L_{i,t}$ is positive, and a lower return if it is negative:

$$r_{i,t}^e = r_{i,t} + L_{i,t}.$$

Average gross returns are equal across the individual mortgages, and over time, $\frac{1}{m} \sum_{i=1}^m (r_{i,t} = \bar{r}_{i,t} = \bar{r}_{i,t-1})$. Thus the most important dynamics come from liquidity.³⁷ Every period there is an exogenous liquidity shock $L_{i,t}$ to the return on each mortgage. Most of the time these shocks are uncorrelated and zero-mean, $\frac{1}{m} \sum_{i=1}^m L_{i,t} = 0$. Somewhat rarely they are perfectly correlated, as in the following structure:

$$L_{i,t} = \begin{cases} \sim i.i.d. \text{ Uniform}[-1, 1], & \text{with probability } 1 - \delta \\ L_t^-, & \text{with probability } \delta \end{cases} \quad (9)$$

where L_t^- is a large negative number.³⁸ This setup produces spillovers because, in the rare regimes, liquidity costs reduce the value of every component of the securitized assets S_j . That is, the S_j inherits the liquidity costs of all its component securities:

$$r_{S,t} = \begin{cases} \frac{1}{m} \sum_{i=1}^m (r_{i,t} + L_{i,t}) = \bar{r}_{i,t} + 0, & \text{with probability } 1 - \delta \\ \frac{1}{m} \sum_{i=1}^m (r_{i,t} + L_{i,t}) = \bar{r}_{i,t} + L_t^-, & \text{with probability } \delta. \end{cases} \quad (10)$$

To see how this can lead to an extreme spillover, consider a correlated liquidity shock in period t (the δ -rare regime), while period $t - 1$ features the typical uncorrelated shock. Recall from Definition 4, that an extreme spillover occurs when

$$|r_{S,t} - \bar{r}_{t-1}| \geq \omega \sigma_{S,t-1}. \quad (11)$$

³⁶Our liquidity cost is in similar spirit to Amihud and Mendelson (1986), and Jacoby, Fowler and Gottesman (2002), who model liquidity as a proportional cost relative to the security price.

³⁷Alternatively, we can think of this as the gross returns being net of other effects, since we wish to concentrate on liquidity shocks.

³⁸Alternatively, we could say there is a shift of the distribution, for example to $L \sim U[2L_t^-, 0]$. Since L_t^- is negative, the mean is now negative.

From equation (10), the left side of (11) satisfies

$$|r_{S,t} - \bar{r}_{t-1}| = L^- . \quad (12)$$

The right hand side of (11), using equation (7), satisfies

$$\omega \sigma_{S,t-1} = \omega \frac{1}{m^{1/2}} [\sigma_{t-1}^2 + 2\rho_{t-1}]^{1/2} = \omega \frac{\sigma_{t-1}}{m^{1/2}}, \quad (13)$$

where the last equality uses the fact that $\rho_{t-1} = 0$ before the onset of the correlated liquidity shock. Combining (12) and (13) yields the condition for extreme spillover: $L_t^- \geq \omega \frac{\sigma_{t-1}}{m^{1/2}}$, or

$$\ln L_t^- \geq \ln \omega + \ln \sigma_{t-1} - \frac{1}{2} \ln m. \quad (14)$$

This expression is intuitive. In an environment where returns depend on liquidity shocks, extreme spillovers will happen if a liquidity shock is large enough relative to average return volatility σ_{t-1} . The term $-\ln m$ is also natural, since the larger the number of borrowers, the more sources of hidden risk in S and therefore the lower the liquidity shock needed to set off an extreme event.³⁹ This expression predicts that extreme events will tend to spill over and persist when liquidity shocks are large relative to average volatility of individual instruments.

A key question for investors and regulators is what determines the incidence and persistence of extreme spillovers? According to equation (14) this depends on the average number m of borrowers in a securitized debt instrument, and the amount of liquidity comovement.⁴⁰ Thus, the breadth and persistence of extreme spillovers depends on liquidity comovement and persistence of liquidity shocks. If we are interested in predicting the extent of endogenous versus exogenous extremes, note that exogenous extremes should be fairly contained, *ceteris paribus*. Endogenous extremes are only limited by the extent of liquidity comovement and diversification.

³⁹Having a large enough L^- will reduce diversification benefits because it involves adding a highly *dependent* risk to a portfolio. The condition may also be implemented empirically by taking time derivatives d/dt on both sides of equation (14), which yields the condition

$$\hat{L}_t \geq \hat{\sigma}_t,$$

where the hat symbol denotes the logarithmic derivative, representing the growth rate of a variable. Empirically it will be valuable to connect this expression to equilibrium and extreme probability p .

⁴⁰Another influence will be the degree of diversification, that is, the average share CDOs held by individuals.

4 Potential Policy Implications

What well-defined question can this framework help us answer? While we have provided a simple analysis of economic extremes in general, we feel our approach is especially suited to address the puzzles discussed in the introduction. Our framework suggests that the subprime market and spillover puzzles can be understood as the result of an uninternalized externality, the effect of excessive borrowing on financial stability. This is not a simple externality, but a multilateral, public externality. It is generated by and affects many agents, for example mortgage lenders, borrowers, hedge funds, and even global investors. To solve this type of public externality the introduction of a standard sort of market will not be optimal, due to the free rider problem and the impact of liquidity.⁴¹ We now discuss liquidity and externalities, in turn.

Our model suggests that liquidity is important and should be monitored carefully.⁴² There are many aspects to liquidity's importance. Perhaps the most crucial is our implicit notion that liquidity has a dual role: high liquidity helps initiate extremes by providing a market for securitized debt; and correlated liquidity drives the subsequent spillovers. Regarding initiating extremes, the risk of counterparty default is unimportant if there is enough liquidity and innovation for a lender to repackage and resell her debt. Regardless of information structure, once individuals feel they have enough liquidity, they may take excessive risk. Thus, information asymmetry may become less relevant in an environment of plentiful liquidity. Regarding spillovers, in the face of correlated liquidity events, all diversification benefits might disappear, and even pooled securities can experience a sharp drop in value. Therefore, liquidity amplifies endogenous extremes, which aggregate from one sector to the larger economy because of complementarities due to the credit cycle, and externalities. Since control of these extremes has public externality aspects, it is unlikely that an individual agent will provide the public liquidity necessary to avert systemwide extremes.⁴³ Why do agents produce endogenous extremes in the first place? There are two main reasons, disaster myopia and externalities: the effects of excessive borrowing happen somewhat rarely, in the future, and mainly to others. Individuals may not fully internalize the cost of elevated future extremes because this probability is considered relatively low

⁴¹Private externalities are depletable, while public externalities are non-depletable, and retain potency for all who are affected. For an exposition of multilateral externalities, see Mas-Colell, Whinston, and Green (1995), Chapter 11.

⁴²For a discussion of what are the best liquidity measures, see Cholleto, Nas, and Skjeltorp (2007).

⁴³An exception is John Rockefeller in the Wall Street panic of 1907.

and distant, and individuals have a bias to underweight low probability events. These two reasons are reinforced for both borrowers and lenders by liquidity, as described above. Why do individual regulatory authorities allow excess extremes to happen, once started? An important reason is that it is difficult to predict extremes. Furthermore, there is a free riding problem at the domestic and international level. Finally, there is moral hazard— there is a knife edge aspect to central bank supplying liquidity to calm extremes, since liquidity is what can initiate extremes in the first place.

Understanding the externality aspect of extremes may enable us to address not just the effects but also the causes of extremes. The externality issue gives lenders and borrowers inadequate incentives, leading to an overproduction of extremes. In order to correct this, a standard public finance solution involves giving agents pecuniary or property right incentives to reach the optimality condition in (6). Given the diversity of agents and nations in the current situation, two further suggestions that we think plausible are to create an international institution responsible for monitoring global system stability, and to delegate responsibility reciprocally, for providing aggregate liquidity and control of extremes. The former would ensure information sharing and avoid costly international duplication of effort. The latter would be a global market maker, ensuring public provision of a global public good.⁴⁴ Let us discuss these three solutions. First, ex ante incentives include subsidizing housing and borrowing, and assigning or auctioning tradeable extreme production rights. A related incentive involves educating borrowers and lenders on the nature of liquidity shocks, and the consequences of endogenous, extreme price changes.⁴⁵ Second, a monitor of lending and borrowing activity, and diversification of portfolios, would have special tasks. It would assess adequate levels of borrowing and lending (given average liquidity), and provide indices of current and expected liquidity levels, and average risk in all securitized assets that could be subject to hidden risk during extremes.⁴⁶ Third, a global market maker would be used as an ex post measure. Since systemwide liquidity is a public good, each actor has an incentive to free ride, therefore liquidity effects may be persistent, once extremes begin. Public provision of systemwide liquidity is thus necessary to disrupt

⁴⁴For literature on delegated monitoring, see Diamond (1984) and Sheard (1994). This would involve coordination of international regulatory authorities, for example, each taking turn as delegated monitor. Regarding global market makers, a related suggestion has been made by Buitier and Siebert, at <http://maverecon.blogspot.com/2007/08/central-banks-in-time-of-crisis.html>. For a global market maker, there is of course a moral hazard problem.

⁴⁵Similar to pollution permits, these would allow the market participants to engage in reasonable levels of borrowing/lending even during times of low aggregate liquidity.

⁴⁶Such global monitoring of liquidity, hidden risk and extremes, should be delegated because of its public good aspects, and to avoid costly duplication (Diamond (1984), Sheard (1994)).

persistence of liquidity shocks. Reciprocal delegation of the role of global market maker will allow for efficient provision, and remove the free rider issue at the individual, national and global level.⁴⁷ These three prescriptions together may reduce extremes, and ensure that financial markets pay an appropriate price for the extremes they produce.

5 Empirical Application

An important pre-condition for the policy analysis described in the previous section is empirical documentation of the properties of extreme probabilities, to which we now turn. Since each asset price series may have individual characteristics, we focus on one that summarizes aggregate security performance, and for which there is a relatively long time series of daily observations, the Dow Jones Industrial Average.

Our main data comprises the following series: daily Dow Jones Industrial Average series (DJIA), from May 26, 1896 to September 28, 2007; the degree of securitization in US financial markets (SEC), computed monthly from January 1989 to December 2006; the liquidity measure (LIQ) of Pastor and Stambaugh (2003) available monthly from April 1962 to December 2006; the value of real estate loans in the US (REALLOAN) available monthly from January 1947 to July 2007; and a measure of investor sentiment (DSENT1), used in the study of Baker and Wurgler (2007). This data is available from January 1966 to December 2005, kindly provided at Professor Jeffrey Wurgler's website.⁴⁸ Unless otherwise noted, the data are obtained from WRDS and Datastream.

There are three steps to our empirical approach. First, we examine whether our p series are significantly different from zero. Second, we examine their dynamics by considering autoregressive time series models. Third, we begin to analyze endogeneity by using simple VAR models. We discuss each of the above in turn.

⁴⁷This is similar to what is already done by individual regulator authorities, pumping in liquidity. The difference is that it would be coordinated internationally in a systematic way.

⁴⁸Since the empirical probability series are between 0 and 1, and SEC and REALLOAN are in billions of dollars, we use percentage changes in SEC and the ratio of real estate loans to total loans, in order to scale them down comparable.

5.1 Computing the p_t Series, and Summary statistics

The main series we compute is $p_t(\omega)$, according to Definition 4 in Section 2.3. Using the DJIA described above as our base series, we compute the proportion of times each month that there is an observation more than ω standard deviations away from the median. Both the median and standard deviation are computed over the preceding k months, where $k = 12, 24, 60$ and 120 . This procedure is done on a rolling basis.⁴⁹ An example of the p_t series for the 12 month reference period is shown in figure 2. As should be expected, the probability of extremes becomes much less active as we move from 1 or 2 sigma events to 3-sigma and beyond. Evidently, the series move around quite a bit, so even from a visual perspective the series are not constant.

Figures 3 and 4 display histograms of our extreme probabilities for 1 and 2 sigma events. In both cases there is a u-shaped pattern for all reference periods. Moreover, the longer reference periods tend to have more mass concentrated at 0 and 1. Thus, if economic agents compute extremes in this range (1 and 2 sigma events), their probability estimates would tend to be more volatile. Figures 5 to 7 show histograms for extreme probabilities of 3- to 5-sigma events in the DJIA. As we would expect, the distributions become more concentrated at zero, with the shortest horizon (12 months) being the last to have all probabilities at zero.

We now turn to summary statistics and more formal tests. Table 2 shows that as the level of ω increases, both the mean and standard deviation decrease. However both the t-test and nonparametric sign rank test generally have minute p-values until the level of $\omega = 5$. This suggests that the likelihood of extreme events beyond 2 standard deviations may be non-zero, regardless of agents' reference periods.

Is there a difference in extreme probabilities within reference periods? We test this hypothesis in Table 3. Except for some marginal significance between 4 and 5 sigma events, there is very little evidence of similarity between the various extreme probabilities within a given reference period. A different pattern emerges, however, when we examine tests for differences across reference periods, in Table 4. This latter table reports mixed evidence about the similarity of average extreme probability depending on the reference period. This

⁴⁹Recall that the series begins in May 1896. Thus, for example, to compute the $p_t(1)$ for a 12 month reference period, we count the number of times in June 1897 that the DJIA exceeded 1 standard deviation from the median, calculated from May 1896 to May 1897. We then do the same for July 1897, where the median and standard are calculated from June 1896 to June 1897, and so on.

result may be of practical relevance, if different investor groups have different time horizons when deciding whether a particular event is extreme.

5.2 Dynamics

As we discussed in Sections 1 and 3, the dynamic behavior of extreme probabilities has important impacts for risk management and stress testing. We therefore examine the time series behavior of our $p_t(\omega)$ series. The results are displayed in Table 5.⁵⁰ Importantly, except for the very extreme 5-sigma events, the Q-test of white noise is rejected. This suggests that there are important dynamics in the likelihood of extreme events. The best-fitting models generally range from AR(1) to AR(3), although there are a few models with higher lags. Thus, our empirical probabilities seem to exhibit memory—extreme events cluster over time, regardless of our reference period.

5.3 Endogeneity

While extreme dynamics relate to investment, extreme endogeneity also relates to policy analysis and financial stability. The results of this section are exploratory, since our theoretical guidance is quite general. Ideally, we would like to assess whether p_t depends on plausible aspects of economic behavior, suggested by theory. Based on our Section 3 discussion, some examples are borrowing, and investor sentiment. However, our ideas about endogeneity come from equations (5) and 14, which are not easily testable. For example, the empirical probability of extremes will depend on other factors besides borrowing behavior and liquidity. Moreover, for simplicity we model the cause of period $t+1$'s extremes to occur in t , while in practice, there may be substantial lag affects.

We therefore use a simple 3-variable vector autoregression to examine both the effect of borrowing (extremes redux) and liquidity (spillovers). In light of our Section 3 discussion, we examine whether borrowing and the general state of investor sentiment affect the probability of extreme events. Our proxy for borrowing is REALLOAN, and for sentiment is DSENT1, described above. The results from these tests are displayed in 6, which produces orthogonalized impulse responses for REALLOAN and DSENT1 on extreme probabilities.

⁵⁰Note that some series had insufficient variance to compute time series models. This was particularly the case with the 5-sigma extremes, since much of it consists of zeros.

For all reference periods the VAR is either first or second order. In general, the standard errors are large, therefore it is not easy to make strong statements. An important finding from this table is that the effects of a shock in either REALLOAN or DSENT1 persist. Interestingly, there is sometimes a change of sign by the sixth month. For example, in Panel A, the effect of investor sentiment on 2- σ event probabilities is around 0.0214 in the first month ($k=1$), but switches to around -0.0038 by the sixth month. These patterns may reflect a complex nonlinear relation between sentiment and extreme probabilities. Another issue is that sometimes the shocks increase over time. For example, again in Panel A, the effect of a shock in real estate loans on 2- σ event probabilities is 0.0001 in the first month, and increases to 0.0003 by the sixth month.

Our final table for the VARs is Table 7, which examines the spillover effect discussed above. Specifically, in the spirit of equation 14, we estimate a vector autoregression that contains extreme probabilities, liquidity (LIQ) and percentage change in securitization (PCTSEC), as described above.⁵¹ Similar to the previous table, there are switches in sign, and amplification of shocks over time. For example, in Panel A, a shock to LIQ has an effect in the first month ($k=1$) of 0.0038 on the likelihood of a 2 - σ event, which has switched sign to -0.0088 by the sixth month. Moreover, again for Panel A, the effect of PCTSEC in the first month is -0.0001, which has increased and changed sign to 0.0015 by the sixth month. Again, the standard errors are large, reflecting the relatively short sample. This suggests the importance of finding better instruments for liquidity and securitization, with longer sample.

One issue in the above results is the large autocorrelation of our empirical probabilities. From a practical point of view, these point estimates are less useful than 'early warning signals' that indicate whether it is likely to have 'high' levels of extreme events. With this in mind, we take another look at endogeneity, by dividing the empirical probabilities into three ranges, Low, Medium and High. Low corresponds to less than 0.33, Medium to the range 0.33 to 0.67, and High to the range 0.67 to 1. We then estimate a cumulative logistic model for all the various reference periods. The results are reported in Tables 8 to 11. In many cases the level of real estate borrowing is significant, and the level of securitization is also important. Moreover, for 1 and 2 sigma events, a low level of liquidity is also important for predicting high likelihood of extremes. This provides some evidence of endogeneity. Specifically, a low level of liquidity coupled with a high level of real estate borrowing is an important explanatory variable for high likelihood of extremes.

⁵¹We exclude the 4-sigma events since their estimation generally does not converge.

To summarize our empirical exploration, The main findings are that extreme probabilities are dynamic and persistent, as documented in Tables 2 through 5. The evidence on endogeneity is mixed. A preliminary VAR analysis suggests that instruments related to borrowing, investor sentiment and securitization, may play a small role in the dynamics of extreme probabilities, although to establish this would require a long time series of good instruments. More encouragingly, a cumulative logistic analysis shows that the level of real estate borrowing and market liquidity are related to the likelihood of a large number of extreme events.

6 Conclusions

In light of recent developments in financial markets, our paper develops and tests a positive approach to extreme events. This approach suggests that some extreme events may vary systematically over time, and might be explained and forecasted on the basis of economic theory. In addition to creating a taxonomy, we have three main contributions. First, we distinguish exogenous from endogenous extremes, the latter of which can be understood in the framework of externalities. This distinction is particularly important when there is the possibility of large spillovers, and has immediate policy implications: for truly exogenous extremes, we must often focus on ex post protection, while for endogenous extremes, we can use economic incentives. Therefore, in tackling subprime issues (or anywhere that individuals' actions spill over to harm economic stability) we have at our disposal a new set of public finance tools in addition to the traditional solutions of interest rates. Second, we show the 'signature' of different types of extremes, and suggest a test for each type. In relation to the signature of extremes, we provide some insight on their incidence and size. According to equation (5), they are more responsive to borrowing if expected costs are lower and credit is easier. According to equation (14), extremes have a higher incidence if correlated liquidity shocks dominate volatility, and if m is large. Third, we propose coordination of global regulatory authorities in controlling extreme events. In addition to providing incentives for borrowers and lenders, this global coordination will involve monitoring and sharing information on liquidity and extremes, activities that have global public good qualities. Finally, on the empirical side, we have computed extreme probabilities for various reference periods, and shown that in a number of cases they are neither rare nor constant. In most cases extreme probabilities are significantly different from zero, and have strong autoregressive components—there is memory in extremes. An exploratory vec-

tor autoregression analysis suggests some weak evidence that instruments based on theory may have effects on extreme probability that evolve over many months. More encouragingly, a cumulative logistic analysis shows that real estate borrowing, market liquidity and the degree of securitization can help to explain the likelihood of being in a high-extreme probability economy.

Our central concepts, embodied in equations (5) and (14), are the externality argument, and the notion that information-based incentives are counteracted by liquidity and credit cycles before extreme events. These concepts yield quite new and unexpected implications for economic behavior and regulatory policy in securities markets. A major implication is that if we can identify the signature of endogenous extremes, we may potentially counter them using other tools in addition to traditional interest rate management by central banks. Thus, it is not necessary to try to remove extremes ex post, agents may do it themselves, given the right ex ante incentives. Another implication is that anything driving the credit cycle will increase the likelihood of extremes, for example, innovation or loose interest rates. A final implication is that global coordination of regulatory bodies extends the idea of 'too big to fail' to an international setting, where the institution in question is the entire global economy. In the future, such coordination may help ensure effective domestic and global diversification.

This paper may be seen as a first step on the road to incorporating extreme events into standard economic analysis. Even if the particular channel of endogeneity differs from the one we focus on (borrowing), the message remains: endogenous extreme events may be prevented using tools from public finance. By viewing extremes as the outcome of optimizing behavior, we can attempt to address the proximate cause, for example, reducing the demand for overborrowing by subsidizing certain house purchases. Our approach differs from previous work because we give a method for detecting and predicting excessive extreme events. Another difference is that we suggest ex ante and ex post methods for dealing with extremes. Important extensions to this work include dynamic modelling, and identifying the various channels of endogenous extremes encountered in practice. Refinements of this approach are an exciting task for future research.

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Figure 1: UK banks' price of borrowing.

The figure shows the price of interbank borrowing in the UK. The solid (red) line is the 3-month interbank rate and the dotted (green) line is the base rate. Source: DataStream.

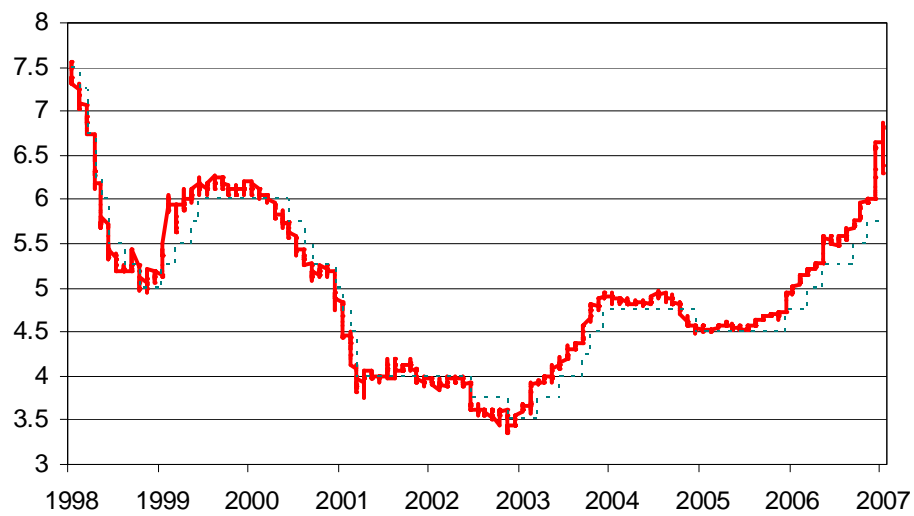


Table 1: Two examples of endogenous probabilities

	<i>Effects felt mainly in one market or transaction</i>	<i>Spillover effects in many markets</i>
<i>Asymmetric Information</i>	Moral Hazard	
<i>Symmetric or Asymmetric Information</i>		Endogenous Extremes

Figure 2: Frequency and Impact of Extreme Events in Nature, 1974-2003.

Source: EM-DAT.

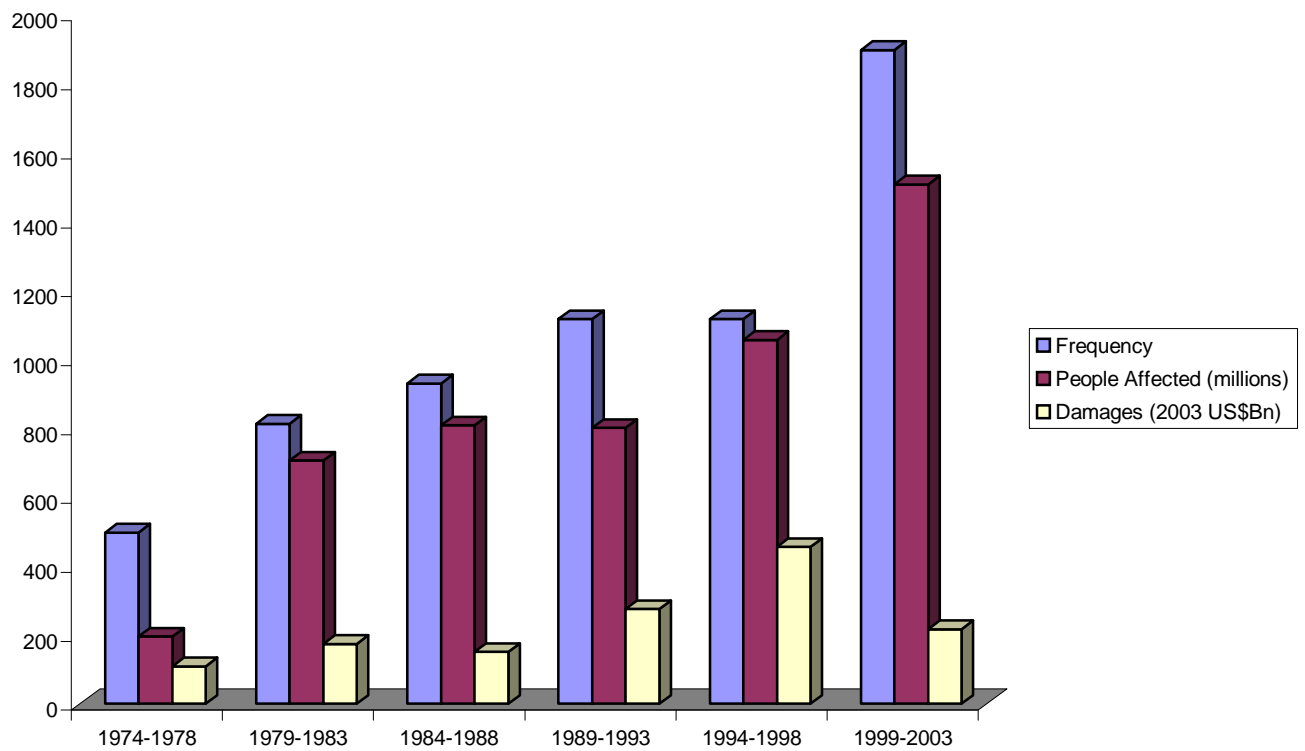


Figure 3: Time Series of Extremes

The figure shows a sample of the time series for various levels of extreme probabilities, from 1967 to 2007. The relevant reference period is 12 months.

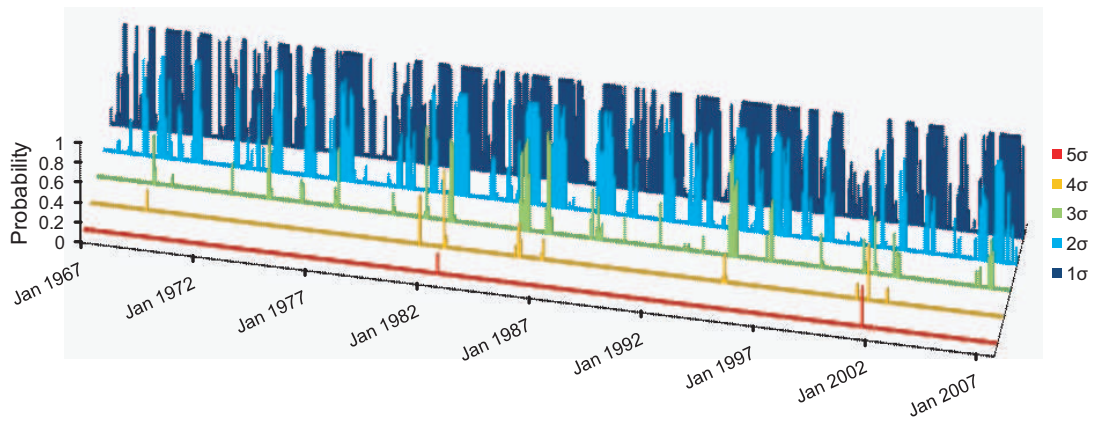


Figure 4: The Distribution of 1-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed one standard deviation from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

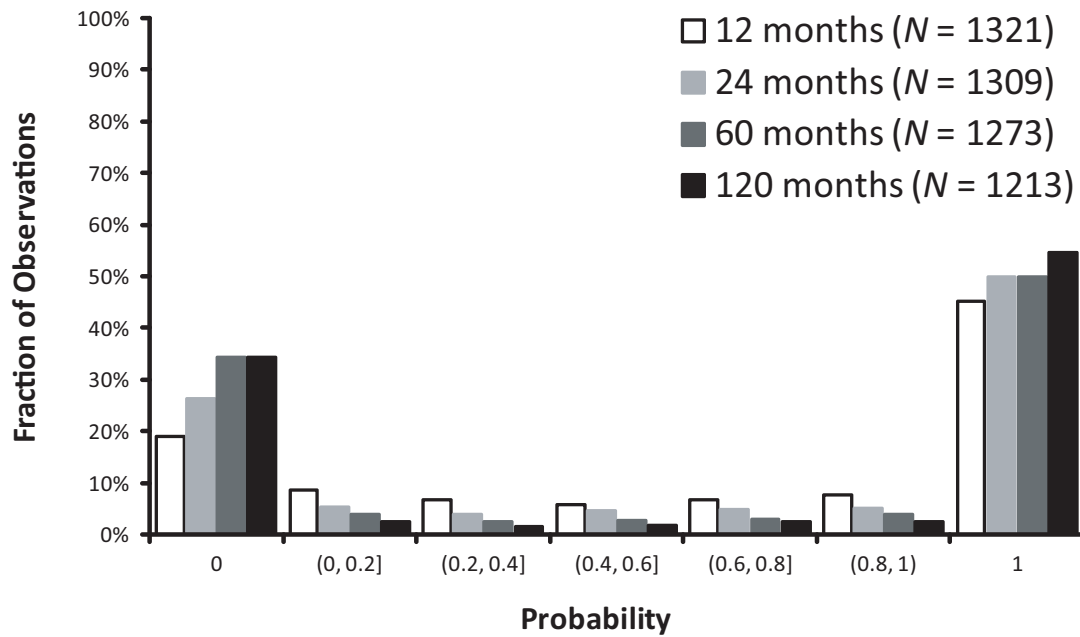


Figure 5: The Distribution of 2-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed two standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

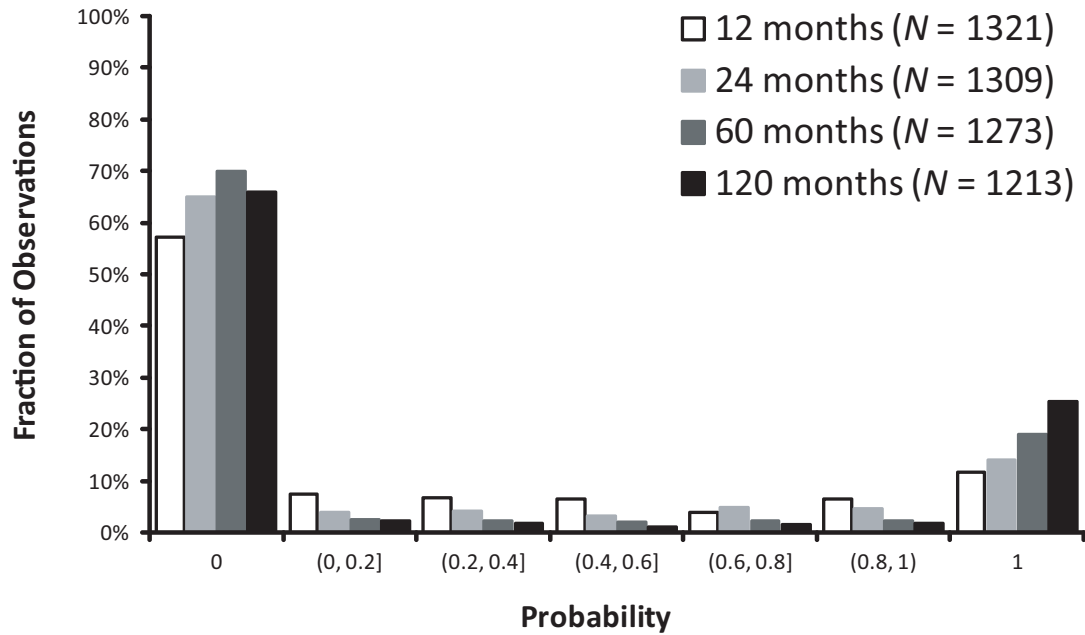


Figure 6: The Distribution of 3-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed three standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

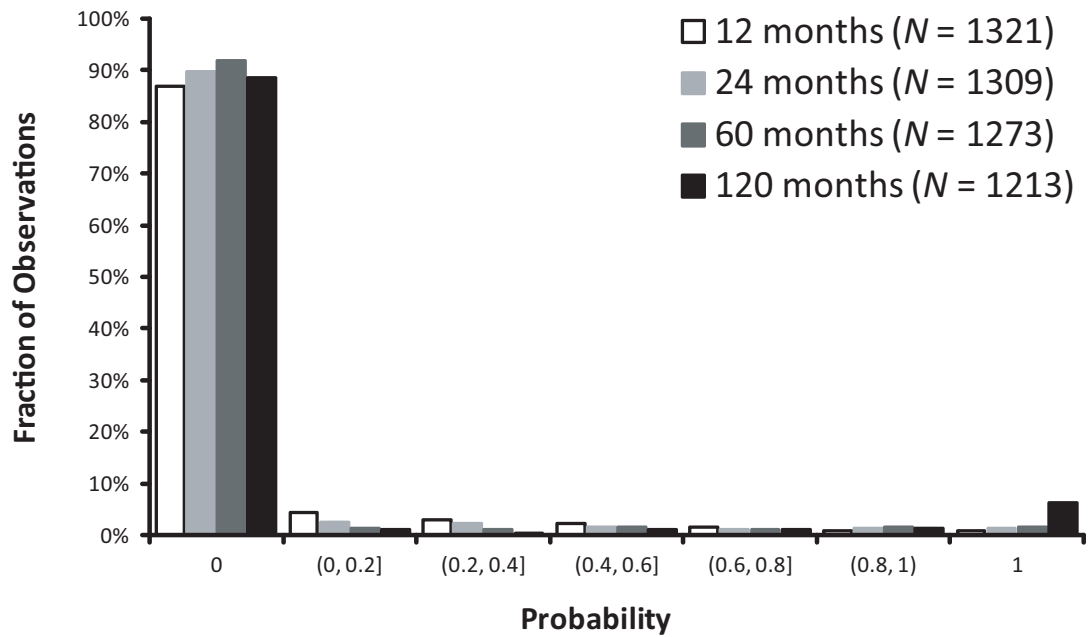


Figure 7: The Distribution of 4-sigma Events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed four standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

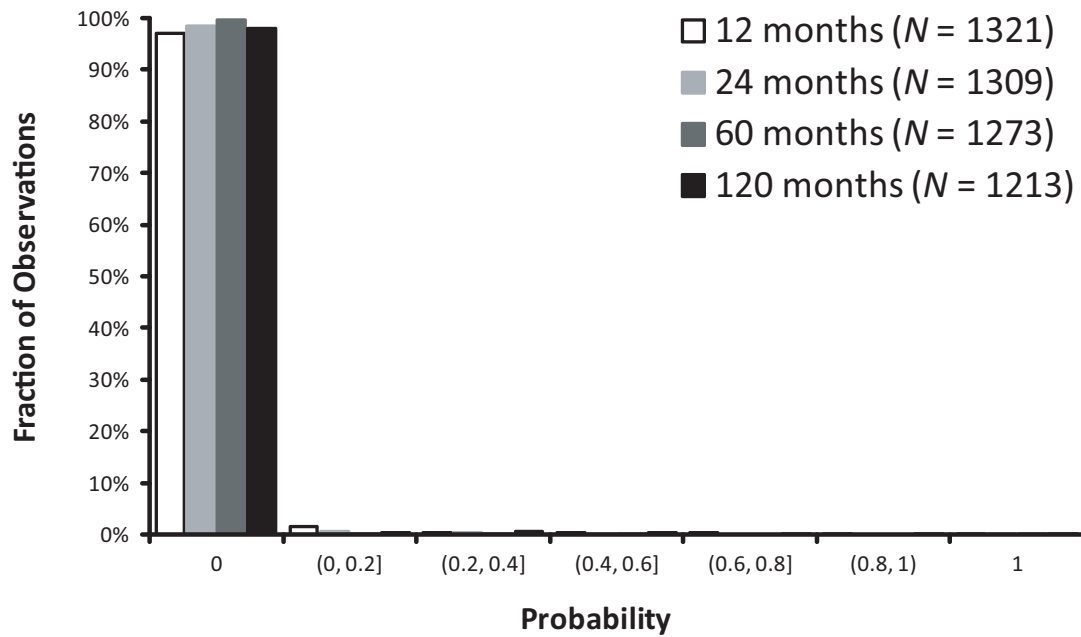


Figure 8: The Distribution of 5-sigma events.

The histogram shows the empirical probability of Dow-Jones Industrial Average levels that exceed five standard deviations from the relevant median. The median is calculated over different reference samples, ranging from 12 months to 120 months.

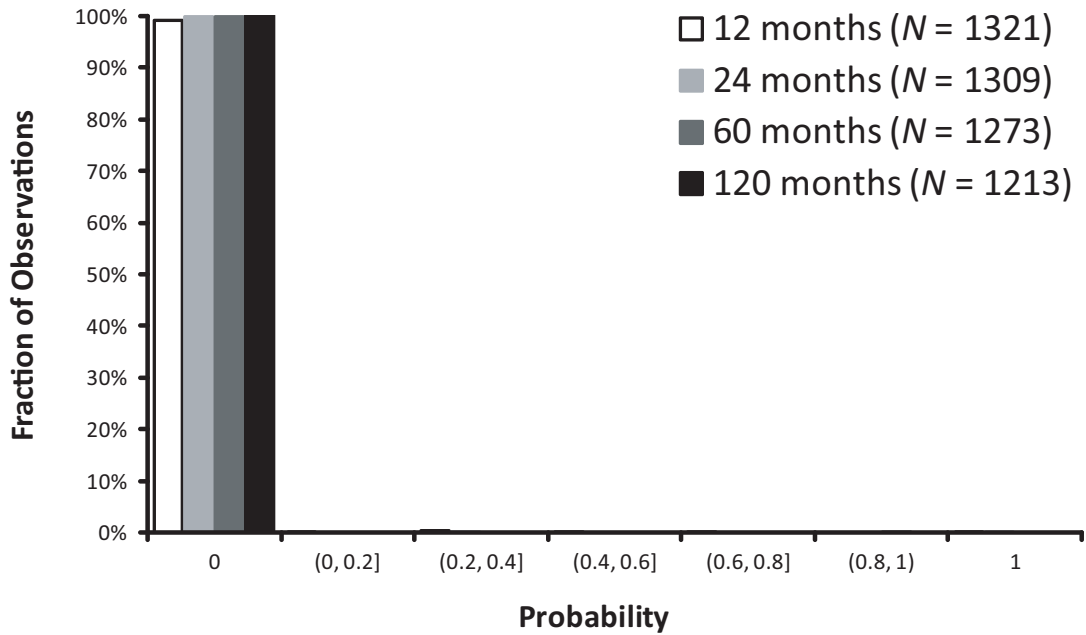


Table 2: Basic Properties of Extreme Probabilities $p_t(\omega)$

Panel A: 1-year reference period

	Mean	Standard Deviation	P-value for t-test	P-value for sign rank test
$(\omega = 1)$:	0.6288	0.4202	< 0.0001	<0.0001
$(\omega = 2)$:	0.2647	0.3806	< 0.0001	< 0.0001
$(\omega = 3)$:	0.0542	0.1768	< 0.0001	< 0.0001
$(\omega = 4)$:	0.0098	0.0739	< 0.0001	< 0.0001
$(\omega = 5)$:	0.0033	0.0437	0.0056	0.0020

Panel B: 2-year reference period

	Mean	Standard Deviation	P-value for t-test	P-value for sign rank test
$(\omega = 1)$:	0.6189	0.4439	< 0.0001	< 0.0001
$(\omega = 2)$:	0.2489	0.3906	< 0.0001	< 0.0001
$(\omega = 3)$:	0.0517	0.1844	< 0.0001	< 0.0001
$(\omega = 4)$:	0.0044	0.0502	0.0014	< 0.0001

($\omega = 5$): 0.0010 0.0291 0.2060 0.5000

Panel C: 5-year reference period

	Mean	Standard Deviation	P-value for t-test	P-value for sign rank test
($\omega = 1$):	0.5787	0.4680	< 0.0001	< 0.0001
($\omega = 2$):	0.2450	0.4083	< 0.0001	< 0.0001
($\omega = 3$):	0.0496	0.1899	< 0.0001	< 0.0001
($\omega = 4$):	0.0026	0.0445	0.0368	0.0313
($\omega = 5$):	0.0007	0.0259	0.3175	1.000

Panel D: 10-year reference period

	Mean	Standard Deviation	P-value for t-test	P-value for sign rank test
($\omega = 1$):	0.6038	0.4711	< 0.0001	< 0.0001
($\omega = 2$):	0.2961	0.4413	< 0.0001	< 0.0001
($\omega = 3$):	0.0908	0.2722	< 0.0001	< 0.0001
($\omega = 4$):	0.0086	0.0732	< 0.0001	< 0.0001

$(\omega = 5):$ 0.0000 0.0000 . .

The table shows stylized facts for the time series of extreme probabilities $p_t(\omega)$. As in the text, ω denotes the number of standard deviations away from the relevant median. The t- and sign rank tests examine whether the mean differs significantly from zero.

Table 3: P-values for Test of Differences *Within* Reference Periods

Panel A: 1-year reference period

	1- σ vs 2- σ	1- σ vs 3- σ	1- σ vs 4- σ	1- σ vs 5- σ	2- σ vs 3- σ	2- σ vs 4- σ	2- σ vs 5- σ	3- σ vs 4- σ	3- σ vs 5- σ	4- σ vs 5- σ
t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001

Panel B: 2-year reference period

t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0018
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001

Panel C: 5-year reference period

t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0530
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0313
SR test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0313

Panel D: 10-year reference period

t-test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Sign test:	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001

SR test: < 0.0001 < 0.0001 < 0.0001 < 0.0001 <0.0001 < 0.0001 < 0.0001 < 0.0001 < 0.0001 < 0.0001

The table shows the p-values from statistical tests for significant differences in the means of our $p_t(\omega)$ series, for various levels of extreme events. SR denotes the sign rank test.

Table 4: P-values for Test of Differences *Across* Reference Periods

Panel A: 1- σ Events

	1- vs 2-Year	2- vs 5-Year	5- vs 10-Year	1- vs 10-Year
t-test:	0.6024	0.0032	0.0408	0.1750
Sign test:	0.8438	0.0009	0.0049	0.4969
SR test:	0.6499	0.0028	0.0469	0.0710

Panel B: 2- σ Events

t-test:	0.1012	0.8694	<0.0001	0.0417
Sign test:	0.1198	0.1902	<0.0001	0.6960
SR test:	0.0998	0.9772	<0.0001	0.0146

Panel C: 3- σ Events

t-test:	0.9167	0.9404	<0.0001	<0.0001
Sign test:	0.3580	0.1679	<0.0001	0.9508
SR test:	0.8088	0.7975	<0.0001	<0.0001

Panel D: 4- σ Events

t-test:	0.0073	0.3936	0.0187	0.8629
Sign test:	0.0026	0.0490	0.0023	0.4885

SR test:	0.0004	0.2112	0.0114	0.7465
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Panel E: 5- σ Events

t-test:	0.0203	0.2306	0.3175	0.0111
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Sign test:	0.0156	0.5000	1.0000	0.0078
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SR test:	0.0156	0.5000	1.0000	0.0078
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The table shows the p-values from statistical tests for significant differences in the means of our $p_t(\omega)$ series, for various levels of extreme events. SR denotes the sign rank test.

Table 5: Time Series properties of Extreme Probabilities

Panel A: 1-year reference period

	$1-\sigma$	$2-\sigma$	$3-\sigma$	$4-\sigma$	$5-\sigma$
Q-Test of white noise on original series:	< 0.0001	< 0.0001	< 0.0001	<0.0001	0.4192
Selected Model:	AR(2)	AR(2)	AR(2)	AR(2)	AR(1)
Q-test of residuals from selected model:	0.1729	0.0266	0.3694	0.9654	0.9982

Panel B: 2-year reference period

Q-Test of white noise on original series:	< 0.0001	< 0.0001	< 0.0001	<0.0001	1.0000
Selected Model:	AR(2)	AR(2)	AR(2)	AR(4)	NA
Q-test of residuals from selected model:	0.0952	0.0949	0.8880	0.2330	NA

Panel C: 5-year reference period

Q-Test of white noise on original series:	< 0.0001	< 0.0001	< 0.0001	<0.0001	1.0000
Selected Model:	AR(1)	AR(4)	AR(4)	AR(3)	NA
Q-test of residuals	0.4002	0.0575	0.0166	0.1698	NA

from selected model:

Panel D: 10-year reference period

Q-Test of white noise	< 0.0001	< 0.0001	< 0.0001	<0.0001	NA
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on original series:

Selected Model:	AR(2)	AR(1)	AR(3)	AR(5)	NA
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Q-test of residuals	0.0327	0.0349	0.2157	0.5157	
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from selected model:

The table shows the results of time series estimation of the $p_t(\omega)$ series, for different reference periods. NA denotes 'not applicable', where estimation did not converge.

Table 6: Impulse Responses for Extremes: Effect of Investor Sentiment and Real Estate Borrowing

Panel A: 1-year reference period

	1-σ	2-σ	3-σ	4-σ
Selected Model:	VAR(2)	VAR(2)	VAR(2)	VAR(1)
k-period impulse response, DSENT1:				
<i>k=1</i>	-0.05897 (0.04603)	0.02139 (0.04586)	0.06479 (0.04618)	0.07833 (0.04625)
<i>k=6</i>	-0.00196 (0.00278)	-0.00381 (0.00331)	-0.00328 (0.00245)	0.00001 (0.00015)
<i>k=12</i>	-0.00001 (0.00006)	-0.00003 (0.00012)	-0.00003 (0.00026)	0.00001 (0.00016)
k-period impulse response, REALLOAN:				
<i>k=1</i>	0.00008 (0.00011)	0.00012 (0.00011)	0.00014 (0.00011)	0.00017 (0.00011)
<i>k=6</i>	0.00002 (0.00017)	0.00008 (0.00018)	0.00035 (0.00017)	0.00020 (0.00012)
<i>k=12</i>	0.00002 (0.00018)	0.00008 (0.00019)	0.00037 (0.00018)	0.00020 (0.00013)

Panel B: 2-year reference period

Selected Model:	VAR(2)	VAR(2)	VAR(1)	VAR(2)
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**k-period impulse
response, DSENT1:**

<i>k=1</i>	-0.00718 (0.04629)	0.07087 (0.04624)	0.03068 (0.03643)	0.02286 (0.04619)
<i>k=6</i>	-0.00969 (0.00783)	-0.00305 (0.00669)	0.00248 (0.00297)	0.00014 (0.00080)
<i>k=12</i>	-0.00175 (0.00159)	-0.00037 (0.00086)	0.00014 (0.00020)	-0.00001 (0.00027)

**k-period impulse
response, REALLOAN:**

<i>k=1</i>	0.00017 (0.00011)	0.00013 (0.00011)	0.00003 (0.00010)	0.00032 (0.00011)
<i>k=6</i>	0.00011 (0.00019)	0.00009 (0.00019)	0.00009 (0.00018)	0.00038 (0.00013)
<i>k=12</i>	0.00010 (0.00023)	0.00008 (0.00022)	0.00010 (0.00019)	0.00040 (0.00014)

Panel C: 5-year reference period

Selected Model:	VAR(1)	VAR(2)	VAR(1)	VAR(2)
k-period impulse response, DSENT1:				
<i>k=1</i>	-0.02476 (0.02344)	0.00608 (0.04595)	0.00997 (0.02964)	0.07863 (0.04503)
<i>k=6</i>	-0.01091 (0.00995)	-0.00619 (0.01102)	0.00248 (0.00734)	0.00241 (0.00228)
<i>k=12</i>	-0.00468 (0.00425)	-0.00294 (0.00524)	0.00053 (0.00159)	0.00003 (0.00006)

**k-period impulse
response, REALLOAN**

<i>k=1</i>	-0.00006 (0.00009)	0.00007 (0.00011)	0.00002 (0.00009)	0.00002 (0.00011)
<i>k=6</i>	-0.00025 (0.00019)	-0.00011 (0.00018)	0.00007 (0.00020)	-0.00005 (0.00014)
<i>k=12</i>	-0.00036 (0.00026)	-0.00024 (0.00026)	0.00009 (0.00024)	-0.00005 (0.00014)

Panel D: 10-year reference period

Selected Model:	VAR(1)	VAR(1)	VAR(2)	VAR(2)
k-period impulse response, DSENT1:				
<i>k=1</i>	-0.03066 (0.02124)	0.00413 (0.01622)	-0.00825 (0.04619)	0.00757 (0.04618)
<i>k=6</i>	-0.01547 (0.01019)	0.00282 (0.01044)	0.00090 (0.01067)	-0.00376 (0.00486)
<i>k=12</i>	-0.00774 (0.00513)	0.00200 (0.00710)	0.00037 (0.00446)	-0.00035 (0.00056)
k-period impulse response, REALLOAN:				
<i>k=1</i>	-0.00007 (0.00009)	0.00002 (0.00008)	0.00002 (0.00011)	0.00018 (0.00011)
<i>k=6</i>	-0.00034 (0.00018)	0.00012 (0.00016)	0.00013 (0.00018)	0.00036 (0.00018)

$k=12$	-0.00052	0.00021	0.00019	0.00041
	(0.00026)	(0.00026)	(0.00026)	(0.00021)

The table shows the results of vector autoregression estimation for our $p_t(\omega)$ series, from January 1967 to December 2005. DSENT1 is the investor sentiment measure of Baker and Wurgler (2002). REALLOAN is the ratio of real estate loans to loans in the US. Standard errors are in parentheses.

Table 7: Impulse Responses for Extremes: Effect of Liquidity and Securitization

Panel A: 1-year reference period			
	1-σ	2-σ	3-σ
Selected Model:	VAR(4)	VAR(4)	VAR(4)
k-period impulse response, LIQ:			
<i>k=1</i>	-0.00070 (0.00374)	0.00384 (0.00367)	-0.00057 (0.00359)
<i>k=6</i>	0.00013 (0.00204)	-0.00088 (0.00179)	0.00088 (0.00206)
<i>k=12</i>	0.00002 (0.00020)	0.00008 (0.00034)	-0.00006 (0.00037)
k-period impulse response, PCTSEC:			
<i>k=1</i>	0.00021 (0.00149)	-0.00015 (0.00145)	0.00051 (0.00143)
<i>k=6</i>	0.00016 (0.00074)	0.00152 (0.00069)	0.00130 (0.00077)
<i>k=12</i>	0.00003 (0.00016)	0.00013 (0.00022)	0.00002 (0.00024)
Panel B: 2-year reference period			
Selected Model:	VAR(3)	VAR(3)	NA

**k-period impulse
response, LIQ:**

<i>k=1</i>	0.00460 (0.00365)	0.00433 (0.00366)
<i>k=6</i>	0.00040 (0.00164)	-0.00077 (0.00164)
<i>k=12</i>	0.00007 (0.00041)	-0.00007 (0.00029)

**k-period impulse
response, PCTSEC:**

<i>k=1</i>	-0.00028 (0.00148)	-0.00175 (0.00143)
<i>k=6</i>	0.00035 (0.00067)	0.00149 (0.00069)
<i>k=12</i>	0.00009 (0.00018)	0.00018 (0.00024)

Panel C: 5-year reference period

Selected Model:	VAR(4)	VAR(3)	NA
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**k-period impulse
response, LIQ:**

<i>k=1</i>	0.00203 (0.00361)	0.00701 (0.00368)
<i>k=6</i>	-0.00031 (0.00185)	0.00010 (0.00161)
<i>k=12</i>	0.00100 (0.00130)	-0.00001 (0.00098)

**k-period impulse
response, PCTSEC**

<i>k=1</i>	0.00069 (0.00148)	-0.00012 (0.00150)
<i>k=6</i>	0.00117 (0.00090)	0.00043 (0.00065)
<i>k=12</i>	0.00130 (0.00077)	0.00030 (0.00041)

Panel D: 10-year reference period

Selected Model:	NA	VAR(3)	VAR(5)
k-period impulse response, LIQ:			
<i>k=1</i>		0.00104 (0.00364)	-0.00040 (0.00360)
<i>k=6</i>		-0.00128 (0.00113)	-0.00089 (0.00232)
<i>k=12</i>		-0.00106 (0.00096)	-0.00157 (0.00130)
k-period impulse response, PCTSEC:			
<i>k=1</i>		0.00249 (0.00144)	0.00190 (0.00143)
<i>k=6</i>		0.00128 (0.00043)	-0.00045 (0.00091)

$k=12$	0.00102	-0.00017
	(0.00038)	(0.00050)

The table shows the results of vector autoregression estimation for our $p_t(\omega)$ series, from January 1989 to December 2006. LIQ is the liquidity measure of Pastor and Stambaugh (2003). PCTSEC is the percentage change in the value of securitized loans in the US. NA denotes a model where estimation did not converge. Standard errors are in parentheses.

Table 8: Cumulative Logistic Estimation, 1-year Reference Period

Panel A: 1- σ events

	Intercept1	Intercept2	REALLOAN	DSENT	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
<i>Coefficient</i>	4.9653	4.2013	-19.8445	-0.00715	3.1311	-5.4992	-1.8287	24.5534	10.7404
	(0.0019)	(0.0081)	(0.0078)	(0.9571)	(0.6111)	(0.0222)	(0.3757)	(0.0247)	(0.2596)
Tests of Overall Fit (p-values):		LR Test	0.0261						
		Score Test	0.0226						
		Wald Test	0.0581						

Panel B: 2- σ events

<i>Coefficient</i>	8.4200	7.7323	-44.2233	-0.00792	-2.3560	-8.2313	-3.9923	39.6307	19.9052
	(0.0135)	(0.0231)	(0.0102)	(0.9579)	(0.7022)	(0.0390)	(0.3126)	(0.0425)	(0.3161)
Tests of Overall Fit (p-values):		LR Test	0.0020						
		Score Test	0.0146						
		Wald Test	0.0461						

Panel C: 3- σ events

<i>Coefficient</i>	14.8763	13.6937	-88.1325	0.0356	-12.6812	-21.3841	-12.9671	99.3204	67.8346
	(0.1581)	(0.1938)	(0.1097)	(0.9212)	(0.3366)	(0.0889)	(0.2420)	(0.1084)	(0.2399)
Tests of Overall Fit (p-values):		LR	0.1713						
		Score	0.3985						
		Wald	0.4655						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent variable is $p_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT is the investor sentiment measure of Baker and Wurgler (2002). LIQ is the liquidity measure of Pastor and Stambaugh (2002), SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other loans. P-values based on chi-square tests are in parentheses.

Table 9: Cumulative Logistic Estimation, 2-year Reference Period

Panel A: 1- σ events; Dependent variable is pt, lo med, hi

	Intercept1	Intercept2	REALLOAN	DSENT	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
<i>Coefficient</i>	8.0943	7.6624	-32.8161	0.0229	0.2950	-2.7495	-1.8432	11.9173	11.2734
	(<.0001)	(<.0001)	(0.0003)	(0.8768)	(0.9685)	(0.3271)	(0.4599)	(0.3483)	(0.3182)
Tests of Overall Fit (p-values):		LR Test	<.0001						
		Score Test	<.0001						
		Wald Test	<.0001						

Panel B: 2- σ events

<i>Coefficient</i>	1.9688	1.6283	-15.3030	-0.0169	7.4385	1.5592	-0.1817	-5.0872	1.3811
	(0.3934)	(0.4802)	(0.1751)	(0.9135)	(0.2383)	(0.6483)	(0.9515)	(0.7540)	(0.9245)
Tests of Overall Fit (p-values):		LR Test	0.0851						
		Score Test	0.1339						
		Wald Test	0.1613						

Panel C: 3- σ events

<i>Coefficient</i>	-1.4408	-2.3552	-6.4250	0.1366	2.2496	-0.6735	0.3425	4.2006	-2.9245
	(0.6920)	(0.5189)	(0.7127)	(0.6338)	(0.8520)	(0.8974)	(0.9489)	(0.8624)	(0.9094)
Tests of Overall Fit (p-values):		LR	0.9911						
		Score	0.9930						
		Wald	0.9943						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent variable is $p_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT is the investor sentiment measure of Baker and Wurgler (2002). LIQ is the liquidity measure of Pastor and Stambaugh (2002), SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other loans. P-values based on chi-square tests are in parentheses.

Table 10: Cumulative Logistic Estimation, 5-year Reference Period

Panel A: 1- σ events; Dependent variable is pt, lo med, hi

	Intercept1	Intercept2	REALLOAN	DSENT	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
<i>Coefficient</i>	16.9786	16.5095	-70.4190	0.1522	-10.5182	-1.8429	2.0033	10.0758	-6.1854
	(<.0001)	(<.0001)	(<.0001)	0.4344	(0.3580)	(0.7002)	(0.6823)	(0.6377)	(0.7765)
Tests of Overall Fit (p-values):		LR Test	<.0001						
		Score Test	<.0001						
		Wald Test	<.0001						

Panel B: 2- σ events

<i>Coefficient</i>	2.9659	2.7202	-18.6782	-0.0187	-0.8247	2.2365	1.8791	-10.9524	-7.0953
	(0.1951)	(0.2345)	(0.0962)	(0.9068)	(0.8959)	(0.5649)	(0.5405)	(0.5557)	(0.6379)
Tests of Overall Fit (p-values):		LR Test	0.0032						
		Score Test	0.0099						
		Wald Test	0.0326						

Panel C: 3- σ events

<i>Coefficient</i>	0.4406	0.00661	-16.0284	0.1180	-6.6053	8.8289	2.9406	-41.4283	-12.1881
	(0.9290)	(0.9989)	(0.5133)	(0.7092)	(0.5846)	(0.3623)	(0.6712)	(0.3911)	(0.7251)
Tests of Overall Fit (p-values):		LR	0.4572						
		Score	0.6365						
		Wald	0.7028						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent variable is $p_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT is the investor sentiment measure of Baker and Wurgler (2002). LIQ is the liquidity measure of Pastor and Stambaugh (2002), SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other loans. P-values based on chi-square tests are in parentheses.

Table 11: Cumulative Logistic Estimation, 10-year Reference Period

Panel A: 1- σ events; Dependent variable is pt, lo med, hi

	Intercept1	Intercept2	REALLOAN	DSENT	SECPCT	LIQ0	LIQ1	REALLOAN*LIQ0	REALLOAN*LIQ1
<i>Coefficient</i>	88.9107	NA	-349.3	-0.2846	-86.9502	189.6	79.0265	-834.0	-352.6
	(0.4142)	NA	(0.4126)	(0.9322)	(0.8750)	(0.5211)	(0.6861)	(0.5032)	(0.6674)
Tests of Overall Fit (p-values):		LR Test	<.0001						
		Score Test	<.0001						
		Wald Test	0.8770						

Panel B: 2- σ events

<i>Coefficient</i>	4.0076	3.9309	-21.7267	0.0876	17.3339	6.0011	3.4082	-25.7645	-17.0596
	(0.0612)	(0.0662)	(0.0367)	(0.5570)	(0.0204)	(0.1275)	(0.3536)	(0.1638)	(0.3492)
Tests of Overall Fit (p-values):		LR Test	<.0001						
		Score Test	<.0001						
		Wald Test	<.0001						

Panel C: 3- σ events

<i>Coefficient</i>	0.0380	-0.2100	-9.2003	0.0761	0.0676	7.7500	3.1538	-37.1530	-15.2701
	(0.9885)	(0.9367)	(0.4716)	(0.7288)	(0.9933)	(0.1886)	(0.4639)	(0.1975)	(0.4732)
Tests of Overall Fit (p-values):		LR	0.2074						
		Score	0.4223						
		Wald	0.5305						

The table shows the results of logistic regression estimation, from January 1989 to December 2005. The dependent variable is $p_t(\omega)$, which is ranked as Low (less than 0.33), Medium (between 0.33 and 0.67), and High (above 0.67). DSENT is the investor sentiment measure of Baker and Wurgler (2002). LIQ is the liquidity measure of Pastor and Stambaugh (2002), SECPCT is the percentage change in securitized loans, REALLOAN is the ratio of real estate loans to other loans. P-values based on chi-square tests are in parentheses.