Organizational Design with Endogenous Information

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Abstract

Recent empirical work suggests that the strength of incentives, the scope of delegation, and the level of environmental uncertainty are positively related. Standard theories of agency and delegation do not provide adequate explanations for these relationships. We argue that these relationships are consistent with a model in which agents are motivated to acquire costly information and to use the information in the principal’s interest. In this setting the optimal organizational design of incentives and delegation must solve both an information acquisition problem and a subsequent agency problem. By overlooking either one of these two problems, standard models come to partial or misleading conclusions.

In our model, an agent first exerts effort to learn the success probabilities of a set of projects, and then chooses one of these projects to implement. The principal’s mechanism consists of the set of projects to allow the agent to study and choose from (delegation), and the amount to pay him as a function of the outcome of the project he implements (incentives). Consistent with the empirical evidence, the model exhibits complementarities among the level of environmental uncertainty, the optimal scope of delegation and the optimal strength of incentives. We then extend our analysis to another key element of organization, communication. The key finding here is that while an increase in the level of uncertainty still increases the strength of optimal incentives, it decreases the optimal amount of reports. Lastly, we allow the principal to use all three instruments: communication, delegation, and incentive pay. We find that the principal will not include communication in his design if the conflict of interest between him and the agent is either zero or, alternatively, sufficiently large.

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1 Introduction

Modern economic organizations operate in uncertain and information-rich environments. In an organizational hierarchy, much of the information critical to decision-making is acquired and used by lower-level agents. Hayek (1945) famously argues that the economic problem of society is to efficiently utilize knowledge. On the other hand, Demsetz (1988) forcefully contends: “Economic organization, including the firm, must reflect the fact that knowledge is costly to produce, maintain, and use.” Bringing both views together, this paper asks the question: How should an agent be motivated to acquire costly information and to use the information appropriately in the principal’s interest? Standard models of delegation and agency have extensively studied each issue in isolation. However, in view of recent empirical evidence on the relationships among environmental uncertainty, monetary incentives, and delegation of authority, explanations from standard models are inadequate or even misleading.

A central prediction of standard agency models (e.g. Holmstrom, 1979; Holmstrom and Milgrom, 1991) is that there should be a negative trade-off between the level of uncertainty and the strength of monetary incentives. More uncertainty causes a risk-neutral principal to provide more insurance against risk to a risk-averse agent, and thus to reduce the incentives to exert effort. However, the empirical evidence suggests otherwise. Prendergast (2002) surveys two dozen empirical studies on performance pay and uncertainty in various occupations. He finds that the data suggest a positive relationship between measures of uncertainty and monetary incentives rather than the posited negative trade-off. In a very recent empirical study, Foss and Laursen (2005) also document that there is a positive and significant relationship between uncertainty and the use of performance pay.

Why is the prediction of standard agency models inconsistent with the evidence? Our answer is that they ignore the acquisition of information in uncertain environments. Standard agency models treat uncertainty as observational error to performance measures. This view misses an important role of uncertainty, namely, that of increasing information value. For example, uncertainty is higher in environments where an industry is growing fast, innovation occurs rapidly and market demand is volatile. In these cases, there is a large potential value to the information that agents can acquire. Hence, the information acquisition behavior of agents figures more prominently in firms’ fortunes, and stronger incentives should be provided to encourage information acquisition.

Delegation of authority plays a key role in the information acquisition process. Recently,
its relationships to incentives and uncertainty have attracted much attention from empirical research. Both across and within organization studies indicate a positive relationship between the amount of authority delegated to agents and the strength of incentives. In retail banking, Nagar (2002) finds that branch managers with more authority receive more incentive-based pay.\footnote{In another study of US banking practice, Magnan and St-Onge (1997) investigate the relationship between executive compensation and managerial discretion, i.e., the decision-making latitude held by executives. They find that executive compensation is more contingent on firm performance in situations of high managerial discretion than in situations of low managerial discretion.}

Aggarwal and Samwick (2003) classify managers into four mutually exclusive groups according to the authority they have: (1) CEO, (2) executives with oversight authority, (3) executives with divisional responsibility, (4) executives with neither oversight authority nor divisional responsibility. By controlling for the level of compensation, the authors find that executives’ pay performance sensitivity increases with the authority they have. These findings are generally complemented by Barron and Waddell (2003) and Wulf (2005 b).\footnote{Using the same ExecuComp data, Barron and Waddell (2003) rank executives by compensation and find a systematic increase in the use of incentive pay, and especially in the use of equity-based compensation, at higher-level positions. Moving from the bottom rank of their sample to the top rank (within the same firm) increases the proportion of total compensation that is incentive pay by 49.1%, and increases the proportion of incentive-pay that is equity-based by 70.9%. Using the data from Hewitt Associates, Wulf (2005) evaluates how incentives vary across divisional managers. She finds bonuses for officers at the high level are more sensitive to both divisional and firm performance relative to managers at the low-level.}

Furthermore, the evidence suggests a positive relationship between delegation of authority and environmental uncertainty. Nagar (2002) finds that the delegation of decision rights to branch managers is significantly higher in volatile banks than more stable ones. Foss and Laursen (2005) also find evidence that delegation and environmental uncertainty are positively correlated.

However, standard models of delegation (e.g. Aghion and Tirole, 1997) do not speak to the positive relationship between delegation and monetary incentives. Monetary incentives are largely ignored in these models.\footnote{Aghion and Tirole (1997), Dessin (2002), and Stein (2002) largely ignore monetary incentives by assuming either that the agent is infinitely risk-averse, or that outcomes are unverifiable.} Although the importance of information acquisition is emphasized, the efficient use of this information, once acquired, is taken for granted. Regardless of monetary incentives, agents always choose their favorite project after information acquisition, and therefore monetary incentives have no impact on the information value of the chosen project. So standard delegation models cannot explain the complementary roles of delegation and incentives in information acquisition. By oversimplifying the subsequent
agency problem of using information, nor do standard delegation models adequately explain how measures of uncertainty should relate to measures of delegation.⁴

The above discussion suggests that the agent should be simultaneously motivated to acquire costly information and to use the information in the principal’s interest. Optimal organizational design of incentives and delegation must solve both an information acquisition problem and a subsequent agency problem. By overlooking either one of these two problems, standard models come to partial or misleading conclusions.

To explore these ideas, we construct a delegation model in which an agent is employed to exert effort in order to learn the success probabilities of a set of projects, and then to choose one of these projects to implement. The agent’s payoff depends on the outcome of the project he implements. In addition to an incentive contract that determines the agent’s pay as a function of project performance, the principal decides how many projects should be delegated to the agent. This number measures the scope of delegation. We model the conflict of interest between the principal and the agent by assuming that the agent’s favorite project gives him an additional private benefit.

Our first result is that if the conflict of interest is either zero or sufficiently large, although the optimal strength of incentives increases with uncertainty, the optimal scope of delegation is independent of incentives and uncertainty. In both cases, incentives increase the agent’s initiative to acquire information, but have no effect ex post on his choice of project. This result shows the inadequacy of standard delegation models. Since the agent’s choice is assumed to be unaffected by incentives in standard models, delegation is unrelated to uncertainty and incentives. Consistent with the evidence, our main result shows that if the agent’s choice is affected by incentives, then the level of environmental uncertainty, the optimal scope of delegation and the optimal strength of incentives are complementary with each other. These positive relationships arise from the complementarity between the information acquisition problem and the subsequent agency problem. The more favorable choice the agent makes in the principal’s interest, the higher is the information value of the chosen project, and the more is the initiative to acquire information.

We then extend our analysis to another key element of organizational design, commu-

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⁴As far as we know, Dessin (2002) is the only model that addresses the relationship between uncertainty and delegation. He studies delegation as an alternative to communication. He finds that the larger the uncertainty about environment, the larger is the range of biases for which the principal delegates control. But this is because uncertainty makes communication less desirable. Uncertainty does not affect delegation per se.
nication. In the communication model, the principal delegates all projects to the agent to study. But the principal decides how many projects the agent should report to him ex post. Although an increase in the number of reports reduces both the agent’s initiative and the value of information, it reduces the probability of conflict of interest. Here uncertainty is still complementary with incentives, but communication is a substitute for both. Therefore, an increase in the level of uncertainty still increases the optimal strength of incentives, but decreases the optimal amount of reports. Lastly, we allow the principal to use all three instruments: communication, delegation, and incentive pay. We find that the principal will not include communication in his design if the conflict of interest is either zero or sufficiently large. The reason is as follows. Although communication recovers loss from the conflict of interest, it diminishes even more the agent’s initiative and the value of information.

1.1 Related Literature

Prendergast (2002) argues that the missing link between incentives and uncertainty is delegation. He focuses on the discrete choice between delegation (output-based contracts) and non-delegation (input-based contracts). But he does not analyze the strength of incentives when delegation occurs. In his model, the reason for delegation is attributed to exogenous input and output monitoring. Here we argue that endogenous information acquisition is the driving force. Moreover, in Prendergast’s delegation and output-based contracts, the principal actually sells the firm to the agent and so the first-best outcome is achieved. But selling the firm to the agent is seldom possible in reality, due to limited liability and wealth constraints.

If delegation is the explanation for the positive relationship between uncertainty and incentives, as stressed in Prendergast (2002), then we should see no relationship between uncertainty and performance pay after controlling for delegation. But Foss and Laursen (2005) find that uncertainty is still significant in explaining performance pay. In our model, there are complementarities among incentives, delegation and uncertainty. After controlling for delegation, uncertainty is still complementary with incentives.

Recently, Baker and Jogensen (2004), Raith (2004), and Shi (2005) also try to explain the positive relationship between incentives and uncertainty. A common and critical feature is that the agent can receive an informative signal of his marginal productivity. This feature is “volatility” in Baker and Jogensen (2004), “specific knowledge” in Raith (2004), “respond-
able risk” in Shi (2005).\textsuperscript{5} Intuitively, when the agent receives a signal of larger marginal productivity, his action will be more valuable to the principal and hence more incentives should be provided to him. Under particular probability structures on the informative signal and the marginal productivity, these models show that the optimal incentives may increase with the variance of the marginal productivity. In a broad sense, their key intuition is close to ours. Namely, stronger incentives should be provided to use more valuable information. Our analysis goes one step further. By introducing information acquisition, we endogenize their exogenous signal structures. Moreover, our model covers two other important issues in organizational design: delegation and communication.\textsuperscript{6} We carefully analyze the relationships among them, and thereby successfully account for more of the empirical evidence.

The above models more-or-less build on the standard moral hazard models. In contrast, our model is in line with the delegation models following Aghion and Tirole (1997). As mentioned before, monetary incentives are largely ignored in their analysis. When they consider monetary incentives in an extension of their model, they observe that adding monetary incentives increases the agent’s initiative to acquire information but reduces the principal’s surplus. But they do not determine which effect is stronger, nor characterize the optimal incentive and delegation design. Based on the extension, they argue that more uncertainty leads to less delegation and incentives, contrary to the evidence. Furthermore, their main concern is “whether to delegate or not”, whereas our question here is “how much to delegate”. In this sense, our model is a “partial delegation” or “optimal delegation” model. Szalay (2004) also studies an optimal delegation problem. He investigates the principal’s optimal design of the agent’s freedom of choice when information is costly to acquire. In his model, it may be optimal to force the agent to choose from extreme options in order to provide even stronger incentives for information acquisition, when contingent monetary compensation is infeasible.

In a broad sense, Cremer et al. (1998), Lewis and Sappington (1997), and Szalay (2004) address in different environments the same issue that we raise, the motivation of an agent both to acquire information and to subsequently use it in a principal’s interest. Lewis and Sappington (1997) and Cremer et al. (1998) investigate information acquisition in a

\textsuperscript{5}In the most recent version of the paper, Shi (2005) emphasizes the information-collection effort of the agent. But she does not analytically solve her model, stopping before characterization of the optimal contract.

\textsuperscript{6}In a two-task agency model, Raith (2004) also studies the optimal delegation decision. He does not consider the information acquisition and communication decision, but emphasizes the agent’s superior local knowledge, i.e., “specific knowledge”.

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procurement contract environment. The optimal contract simultaneously solves the problem of information acquisition and its subsequent revelation. In Szalay (2004) there is a conflict between ex ante information acquisition and ex post use of information. He shows that curtailing the agent’s authority may be optimal even if the agent and the principal share the same ex post objectives. In our model, the agent’s ex ante information acquisition and his subsequent choice of project are complementary. The agent demonstrates more initiative to acquire information when he chooses a project more in favor of the principal.

Dessin (2002) studies delegation as an alternative to communication. In his model, the central trade-off is between loss of control under delegation and loss of information under noisy communication. When the conflict of interest between the principal and the agent is too large relative to environmental uncertainty, loss of control overcomes loss of information, so communication dominates delegation. In our model, delegation increases both information benefit and loss of control; communication reduces both of them. When the conflict of interest between the principal and the agent is too large relative to environmental uncertainty, the loss in information benefit still overcomes the gain in control. Therefore, we reach the opposite conclusion: delegation dominates communication.

1.2 Outline

The rest of the paper is organized as follows. Section 2 introduces the delegation model. We formulate the contracting problem in section 3. In section 4, we characterize the solutions. The communication model is introduced in section 5. We characterize the solutions in section 6. An analysis of the general model is provided in section 7. Section 8 concludes. All calculations and proofs are relegated to the appendix.

2 Delegation Model

A risk-neutral principal and a risk-neutral agent can implement one and only one project from \( n \) potential candidates.\(^7\) The principal hires the agent to acquire information about the projects. Examples we have in mind are headquarters manager/branch manager, or

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\(^7\)Risk preference does not qualitatively change the results in our models. Due to the limited liability constraint and the binary distribution of project values, analysis of the risk-aversion case would follow in the same way as we present here.
CEO/divisional manager. In principle, the model could be applied to any hierarchical relationship involving monetary incentives.

Projects.—Ex ante, there are \( n \) identical projects. Each project will yield a high value \( v \) if it succeeds, but a low value \( 0 \) if it fails. Each project’s success probability \( x_i \) is independently and uniformly distributed on \( \left( \frac{1}{2} - a, \frac{1}{2} + a \right) \), \( 0 < a < \frac{1}{2} \). The parameter \( a \) measures the level of environmental uncertainty.

The agent has a personal preference over which project he would inherently most enjoy implementing. We assume there is a “favorite” project for the agent, which will give him a private benefit \( B > 0 \) if implemented. For example, this favorite project may help to build his private reputation (Hirshleifer, 1993; Holmstrom and Ricari, 1986), and so brings him better job opportunities, or strengthen his bargaining power to increase pay in the future. The value \( B \) is commonly known, but which project generates this benefit \( B \) is unknown to the principal. Ex ante, each project has probability \( \frac{1}{n} \) of being the agent’s favorite. This distribution is independent of all other distributions.

Information.—Each project’s probability of success is initially unknown to both parties. Only the agent can acquire this information. The principal selects \( k \) projects from the \( n \) projects for the agent to investigate. The parameter \( k \) measures the scope of delegation.

As soon as the \( k \) projects are assigned, the agent costlessly observes which one of the \( n \) projects gives him the private benefit \( B \). In other words, he observes whether his favorite is one of the \( k \) projects. Then the agent chooses unobservable effort \( e \) to learn the \( k \) projects’ success probabilities at a cost \( \frac{1}{2} ce^2 \). He perfectly learns the \( k \) projects’ success probabilities with probability \( e \); with probability \( 1 - e \), he learns nothing, and hence still views the \( k \) projects as identical in terms of success probability. Lastly, the agent chooses one of the \( k \) projects to implement.

Contracts.—The contract contains two parts. First, it specifies a contingent wage contract, \((u, u_0)\), which pays \( u \) to the agent if the implemented project succeeds and \( u_0 \) if it fails. Second, it specifies \( k \), the number of projects delegated to the agent.

We assume a limited liability constraint for the agent that requires payoffs to be non-negative: \( u_1, u_0 \geq 0 \). This prevents the principal from selling the enterprise to the agent. Under this constraint, it is obvious that the payoff should be zero if the implemented project

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\(^8\)Our results can be extended to general distributions. Detailed discussion of this assumption is provided in Appendix A.

\(^9\)Our results can be easily extended to the cost function \( c(e, k) = \frac{e^2}{k} \), so that the cost of effort depends on the number of projects to be studied, as long as the average cost is decreasing: \( \frac{d}{dk} \left( \frac{c(k)}{k} \right) < 0. \)
fails, i.e., \( u_0 = 0 \). The contract can thus be written as \((u, k)\). The parameter \( u \) measures the strength of incentives.

The timing is as follows. (i) The principal and the agent sign the contract \((u, k)\), and \( k \) of the \( n \) projects are assigned to the agent. (ii) After observing which project has \( B \), the agent exerts effort \( e \) to learn the success probabilities of the \( k \) projects. (iii) The agent chooses one project from the \( k \) projects to implement. (iv) Success or failure of the implemented project is realized, and \( 0 \) or \( u \) is paid accordingly. See Figure 1 for illustration.

3 Contracting in the Delegation Model

We solve the model backwards, starting with the agent’s choice of effort after the \( k \) projects have been assigned to him.

3.1 Agent’s Problem

Once the agent has observed which of the \( n \) projects would give him the private benefit \( B \), there are two possible cases. Either his favorite is one of the \( k \) projects, or it is not. We consider them in turn.

3.1.1 Conflicting Interests

In this case the agent’s favorite is one of the \( k \) projects. Denote as \( x \) the success probability of his favorite project. Denote as \( x_{k-1} \) the highest success probability of the other \( k - 1 \) projects.\(^{10} \) To simplify notations, let \( y = x_{k-1} \) in the delegation model.

If the agent learns the \( k \) projects’ success probabilities, he chooses between his favorite \( x \) and the best of the other \( k - 1 \) projects \( y \). These two projects give him expected payoffs \( ux + B \) and \( uy \), respectively. Notice that the agent does not always choose the principal’s favorite project, i.e., the one with highest success probability of the \( k \) projects \( \max \{ x, y \} = x_k \). The private benefit \( B \) thus generates a conflict of interest.

If the agent learns nothing about the success probabilities, he chooses his favorite one and obtains the payoff \( ux + B \).

\(^{10}\) \( x_{k-1} \) is the maximum order statistic of the \( k - 1 \) projects’ success probabilities. \( x_{n}^{(1)} \leq x_{n}^{(2)} \leq \cdots \leq x_{n}^{(n)} \) are the order statistics obtained by arranging a random sample of size \( n \) in increasing order. Refer to \( x_{n}^{(i)} \) as the \( ith \) order statistic of a sample of size \( n \).
The success probability of the agent’s favorite project $x$ is uniformly distributed on $(\frac{1}{2} - a, \frac{1}{2} + a)$, and so its probability density function is $\frac{1}{2a}$. Since the highest success probability of the other $k - 1$ projects $y$ is the maximum order statistic of them, its probability density function is $(k - 1) \frac{1}{2a} \left[ \frac{y - (\frac{1}{2} - a)}{2a} \right]^{k-2}$. Because $x$ and $y$ are independent, their joint probability density function $g(x, y)$ is as follows.

\[
g(x, y) = \frac{1}{2a} (k - 1) \frac{1}{2a} \left[ \frac{y - (\frac{1}{2} - a)}{2a} \right]^{k-2}
\]

We denote as $G(x, y)$ the joint cumulative distribution function of $x$ and $y$.

The entire set of possible probability pairs $(x, y)$ is

\[
\Omega \equiv \left\{ (x, y) \mid \frac{1}{2} - a < x, y < \frac{1}{2} + a \right\}.
\]

The set of probability pairs for which the agent will choose the best of the other $k - 1$ projects is

\[
\Omega_1 \equiv \{(x, y) \in \Omega \mid ux + B < uy \}.
\]

Let $\Omega_1^c = \Omega / \Omega_1$.

Given a contract $(u, k)$, the agent’s optimal effort, $e_1^*(u, k)$, maximizes

\[
e_1 \left[ \int_{\Omega_1^c} (ux + B)dG (x, y) + \int_{\Omega_1} uy dG (x, y) \right] + (1 - e_1) \int_{\Omega} (ux + B)dG (x, y) - \frac{ce_1^2}{2}.
\]

(1)

The first term in (1) represents the agent’s expected payoff conditional on a successful information acquisition, weighted by the probability $e$ of such a successful acquisition. It takes into account his choice between his favorite $x$ and the best of the others. The second term is the analogue for the case in which he learns nothing and therefore chooses his favorite project. The first order condition yields

\[
e_1^*(u, k) = \frac{1}{c} \int_{\Omega_1^c} (uy - ux - B)dG (x, y).
\]

(2)
3.1.2 Congruent Interests

This is the case in which the agent’s favorite is not one of the \( k \) projects, and so there is no conflict of interest. If the agent learns the \( k \) projects’ success probabilities, he chooses the one with the highest success probability, \( x^{(k)}_k \). Otherwise, the agent randomly chooses one of the \( k \) projects. The analysis is the same as the previous case, but with \( B = 0 \). Since every project’s success probability has the same ex ante probability distribution as \( x \), we can equivalently take any project’s success probability in the \( k \) projects as \( x \). Then we take the highest success probability of the other \( k-1 \) projects as \( y \). Therefore, in this case the agent chooses between \( x \) and \( y \), which give him \( ux \) and \( uy \) respectively. Accordingly, we define the set of probability pairs for which the agent will choose \( y \) as

\[
\Omega_2 \equiv \{(x, y) \in \Omega \mid x < y\}.
\]

In light of (2), the agent’s optimal effort \( e^*_2 (u, k) \) is given by

\[
e^*_2 (u, k) = \frac{1}{c} \int_{\Omega_2} (uy - ux) dG(x, y).
\]

3.2 Principal’s Problem

With probability \( \frac{k}{n} \), the agent’s favorite is one of the \( k \) projects. In this case the principal’s conditional expected payoff is\(^{11}\)

\[
(v - u) \left\{ e^*_1 \left[ \int_{\Omega_1} x dG(x, y) + \int_{\Omega_1} y dG(x, y) \right] + (1 - e^*_1) \int_{\Omega} x dG(x, y) \right\}
\]

\[
= (v - u) \left[ \int_{\Omega} x dG(x, y) + e^*_1 \int_{\Omega_1} (y - x) dG(x, y) \right]
\]

\[
= (v - u) \left[ \frac{1}{2} + e^*_1 \int_{\Omega_1} (y - x) dG(x, y) \right].
\]

The first term inside the final bracket, \( \frac{1}{2} \), is the expected success probability if the principal were to randomly choose a project to implement. The effort \( e^*_1 \) represents the agent’s initiative to acquire information. The term \( \int_{\Omega_1} (y - x) dG(x, y) \) is the increase in the success

\(^{11}\int_{\Omega} x dG(x, y) = Ex = \frac{1}{2} \).
probability due to the agent acquiring information, which represents the information value he generates. So, the second term inside the bracket is the total information gain created by the agent’s effort, the product of his initiative and the information value.

With probability $1 - \frac{k}{n}$, the agent’s favorite is not one of the $k$ projects. In this case the principal’s conditional expected payoff is

$$
(v - u) \left[ \frac{1}{2} + e_2^* \int_{\Omega_2} (y - x)dG(x, y) \right].
$$

Putting these terms together, the principal’s expected payoff is

$$
(v - u) \left\{ \frac{1}{2} + e_2^* \int_{\Omega_2} (y - x)dG(x, y) - \frac{k}{n} \left[ e_2^* \int_{\Omega_2} (y - x)dG(x, y) - e_1^* \int_{\Omega_1} (y - x)dG(x, y) \right] \right\}.
$$

The principal’s optimal contract $(u^*, k^*)$ maximizes (4), subject to the constraints

$$
e_1^* = \frac{1}{c} \int_{\Omega_1} (uy - ux - B)dG(x, y),
$$

$$
e_2^* = \frac{1}{c} \int_{\Omega_2} (uy - ux)dG(x, y),
$$

$$
u \geq 0, k \leq n.$$

4 The Optimal Delegation Contract

4.1 Preliminary Analysis

Proposition 1 When the agent’s favorite is one of the $k$ projects, he exerts less effort and generates less information value than when it is not. That is, $e_1^* \leq e_2^*$, and

$$
\int_{\Omega_1} (y - x)dG(x, y) \leq \int_{\Omega_2} (y - x)dG(x, y).
$$

Proposition 1 is straightforward. When the agent’s favorite is one of the $k$ projects, he does not always choose the project with the highest success probability. So he exerts less effort to learn which of them has the highest success probability and hence generates less information value.

With the help of Proposition 1, we have a better understanding of the terms inside the principal’s profit function, (4). The first term inside the brace is $\frac{1}{2}$, the reservation value for
the principal. The principal can always achieve this expected success probability without
the agent’s information acquisition. The second term inside the brace,
\[ e_2^* \int_{\Omega_2} (y - x) dG(x,y), \]
we refer to as the information benefit that represents the information gain brought by the
agent’s effort when there is no conflict of interest. The third and final term inside the brace,
\[ \frac{k}{n} \left[ e_2^* \int_{\Omega_2} (y - x) dG(x,y) - e_1^* \int_{\Omega_1} (y - x) dG(x,y) \right], \]
we refer to as the agency cost due to the subsequent agency problem in project selection.
By Proposition 1, this agency cost is nonnegative; it is zero if there is no conflict of interest
\((B = 0)\). The agency cost is a product of two terms:

1. the probability of conflict, \( \frac{k}{n} \). This is the probability that the agent’s favorite is one of
   the \( k \) projects, in which case he exerts less effort and generates less information value.

2. the combined loss due to the conflict of interest,
\[ e_2^* \int_{\Omega_2} (y - x) dG(x,y) - e_1^* \int_{\Omega_1} (y - x) dG(x,y). \]
This loss is caused by both the loss in effort and the loss in information value (Proposition
1). The agency cost is the expected value of this combined loss from the conflict of interest.

We thus see that the trade-off between the information benefit and the agency cost
determines the optimal design of incentives and delegation, \((u^*, k^*)\).

To simplify the analysis, we now normalize the random variables to the standard uniform
distribution, \( U(0,1) \). The normalization of \( x \) is \( x' = \frac{x - (\frac{1}{2} - a)}{2a} \), which has the distribution
\( U(0,1) \). The corresponding order statistic is \( y' = \frac{y - (\frac{1}{2} - a)}{2a} \). Let
\[ t \equiv \frac{B}{2au}. \]
Then our three sets of probability pairs \((x, y)\) become
\[ \Omega = \{(x', y') \mid 0 < x', y' < 1\}, \]
\[ \Omega_1 = \{(x', y') \in \Omega \mid x' + t < y'\}, \]
\[ \Omega_2 = \{(x', y') \in \Omega \mid x' < y'\}. \]

There are three types of solutions to the principal’s problem, depending on the agent’s response to the incentives.

**Case A** When \( t = 0 \), i.e., \( B = 0 \), there is no conflict of interest, and so the agent always chooses the principal’s favorite regardless of the incentives.

**Case B** When \( t \geq 1 \), the conflict of interest is so large that the agent always chooses his own favorite regardless of the incentives.

**Case C** When \( 0 < t < 1 \), the agent’s choice of project is affected by the incentives. He sometimes chooses his favorite and sometimes the principal’s.

The parameter \( t \) measures the amount of conflict of interest between the principal and the agent. It also shows to what extent incentives can align the interests of the principal and the agent. In Case A, their interests are automatically and completely aligned. In Case B, incentives can partially align their interests. In Case C, incentives can never align their interests.

### 4.2 Case A. Zero Conflict of Interest: \( t = 0 \)

This is the case in which there is no conflict of interest, i.e., \( B = 0 \). The agent always selects the project with the highest success probability when he acquires information. Since \( B = 0 \) implies \( \Omega_1 = \Omega_2 \), in light of (2), the agent’s effort and the information value are as follows:\footnote{Derivations are relegated to Appendix B.}

\[ e_1^* = e_2^* = \frac{ak - 1}{ck + 1}u, \]
\[ \int_{\Omega_2} (y - x)dG(x, y) = \frac{ak - 1}{k + 1}. \]
These expressions tell us that an increase in either the uncertainty $a$, or the delegation $k$, increases both the effort and the information value generated by the agent. Stronger incentives also induce more effort. In light of (4), the principal’s problem is now

$$\max_{u,k \leq n} (v - u) \left[ \frac{a^2(k - 1)^2}{c(k + 1)^2} u + \frac{1}{2} \right].$$

(5)

This is a sequential two-step optimization problem. First, we choose the optimal scope of delegation $k^*$ to maximize $\left( \frac{k-1}{k+1} \right)^2$. Second, given $k^*$, we choose the optimal strength of incentives $u^*$ to maximize the principal’s profit. Since $\left( \frac{k-1}{k+1} \right)^2$ is strictly increasing with $k$, then $k^* = n$, i.e., full delegation. Now we have a quadratic profit function of $u$, so the optimal strength of incentives $u^*$, is

$$u^* = \frac{v}{2} - \frac{c (n + 1)^2}{4a^2 (n - 1)^2}.$$

(6)

We then see that in this case the optimal strength of incentives $u^*$ increases with the level of uncertainty $a$. Since $k^* = n$, the optimal scope of delegation is independent of uncertainty and incentives.

In this case of $B = 0$, the agent’s effort only generates the information benefit, since there is no agency cost at the project selection stage. The information benefit here is $\frac{a^2 u}{c} \left( \frac{k-1}{k+1} \right)^2$. So the information benefit strictly increases with the strength of incentives $u$, the scope of delegation $k$, and the level of uncertainty $a$.

### 4.3 Case B. Large Conflict of Interest: $t \geq 1$

This is the case in which the conflict of interest is so large that the agent always chooses his own favorite project. For example, this must be the case when $B \geq av$.\(^{13}\)

In this case $\Omega_1$ is empty, but $\Omega_2$ is the same as when $B = 0$. So, both $e_2^*$ and $\int_{\Omega_2} (y - x) dG(x, y)$ are the same as in the $B = 0$ case, but now

$$e_1^* = 0, \text{ and } \int_{\Omega_1} (y - x) dG(x, y) = 0.$$

\(^{13}\)The optimal strength $u^*$ is necessarily smaller than $\frac{v}{2}$, as otherwise the principal’s profit would be strictly lower than $\frac{v}{2}$, his reservation value. Therefore, $B \geq av$ implies $t = \frac{B}{2av} \geq 1$ and the agent always chooses his own favorite.
In light of (4), the principal’s problem now is

$$\max_{u,k \leq n} (v - u) \left[ \frac{a^2(k - 1)^2}{c(k + 1)^2} \left( 1 - \frac{k}{n} \right) u + \frac{1}{2} \right]. \quad (7)$$

The optimal \( k^* \) is thus

$$k^* = \arg \max_{1 \leq k \leq n} \left[ \frac{(k - 1)^2}{(k + 1)^2} \left( 1 - \frac{k}{n} \right) \right] = \sqrt{4n + 5} - 2. \quad (8)$$

Note that \( k^* = \sqrt{4n + 5} - 2 < n \) for all \( n > 1 \). The optimal scope of delegation \( k^* \) is smaller now than when there is no conflict, \( B = 0 \). But it is still independent of uncertainty and incentives.

Substituting the \( k^* \) into (7) and optimizing with respect to \( u \) yields

$$u^* = \frac{v}{2} - \frac{c}{4a^2} \left( \frac{\sqrt{4n + 5} - 1}{\sqrt{4n + 5} - 3} \right)^2 \frac{n}{n + 2 - \sqrt{4n + 5}} \quad (9)$$

The optimal strength of incentives \( u^* \) is also smaller now than when there is no conflict, \( B = 0 \). But it still strictly increases with the level of uncertainty \( a \).

In this case the information benefit is the same as before, \( \frac{a^2}{c} \left( \frac{k}{k+1} \right)^2 \). This term is also the combined loss. The probability of conflict is \( \frac{k}{n} \), and so the agency cost is \( \frac{k}{n} \cdot \frac{a^2}{c} \left( \frac{k-1}{k+1} \right)^2 \). Thus an increase in the delegation \( k \) increases both the information benefit and the agency cost. The optimal scope of delegation \( k^* \) balances these two effects.

Notice that incentives \( u \) and uncertainty \( a \) affect the information benefit and the agency cost in the same way. Since they offset each other, the optimal delegation is independent of incentives and uncertainty. In fact, when the agent’s choice of project is not responsive to the incentives as it is in both Case A and Case B, the ratio of the agency cost to the information benefit is independent of incentives and uncertainty. This ratio is 0 (or \( \frac{k}{n} \)) when the agent always chooses the principal’s (or the agent’s) favorite. Furthermore, in both Case A and Case B, incentives can only affect the agent’s effort, having no impact on the agent’s choice of project. Since the agent can generate higher information value in Case A than in Case B, stronger incentives and more delegation should be provided in Case A.

Summarizing the above results, we have Theorem 1.

**Theorem 1** When the agent’s choice of project is not responsive to the incentives, the optimal strength of incentives \( u^* \) strictly increases with the level of uncertainty \( a \), and the optimal
scope of delegation $k^*$ is independent of uncertainty and incentives. Moreover, both $u^*$ and $k^*$ are larger when there is no conflict of interest than when the conflict is sufficiently large.

4.4 Case C. Aligned Interest: $0 < t < 1$

In this case incentives can affect the agent’s choice of project and align the interests of the principal and the agent. The previous analysis suggests that what really counts in the optimal design is the relative balance between the information benefit and the agency cost. In this case incentives and uncertainty affect the ratio of the agency cost to the information benefit, and thus the optimal delegation will be related to uncertainty and incentives. We first remove the common factors of the agency cost and the information benefit in order to define some important relative terms.

Define $I(u, k, a)$ as the relative loss in information value:

$$I(u, k, a) = \frac{1}{a} \left[ \int_{\Omega_2} (y-x)dG(y, x) - \int_{\Omega_1} (y-x)dG(y, x) \right].$$

Define $J(u, k, a)$ as the relative loss in effort:

$$J(u, k, a) = \frac{c}{au}(e^*_2 - e^*_1).$$

We have the following proposition regarding the properties of $I(u, k, a)$, and $J(u, k, a)$.

Proposition 2 When the agent’s choice of project is responsive to the incentives, an increase in the level of uncertainty or the strength of incentives reduces both the relative loss in information value and the relative loss in effort. That is, $I_a < 0$, $I_u < 0$, $J_a < 0$, $J_u < 0$. Moreover, an increase in the level of uncertainty or the strength of incentives reduces the marginal relative losses from delegation. That is, $I_{ak} < 0$, $I_{uk} < 0$, $J_{ak} < 0$, $J_{uk} < 0$.

Proof. See Appendix B.

Stronger incentives induce the agent to choose a project more in the principal’s interest. Greater uncertainty brings higher information value to the principal’s favorite project, which also induces the agent to choose a project more in the principal’s interest. In both cases, the information value of the chosen project is higher, and the marginal benefit of effort is higher. Hence the relative losses in effort and information value are reduced. The
results on marginal effects are more subtle. They suggest that delegation is complementary to incentives and uncertainty. Either stronger incentives or greater uncertainty aligns the interests of the principal and the agent, by inducing the agent to choose a project more in the principal’s interest. Once their interests are aligned, the agency cost is reduced relative to the information benefit. Therefore, delegation becomes a more effective tool in information acquisition.

Furthermore, we define $L(u, k, a)$ as the relative combined loss:

$$L(u, k, a) = \frac{k-1}{k+1} [(I(u, k, a) + J(u, k, a)) − I(u, k, a) J(u, k, a)]$$

Then the relative information benefit is $\left(\frac{k-1}{k+1}\right)^2$, and the relative agency cost is $\frac{k}{n}L(u, k, a)$. So the principal’s expected payoff function is

$$f(u, k) \equiv (v − u) \left[ R(u, k, a) u + \frac{1}{2} \right],$$

where $R(u, k, a) \equiv \frac{a^2}{c} \left[ \left(\frac{k-1}{k+1}\right)^2 - \frac{k}{n}L(u, k, a) \right]$ (10)

We thus see that the optimal contract $(u^*, k^*)$ critically depends on the properties of the relative agency cost, $\frac{k}{n}L(u, k, a)$. On the other hand, the relative benefit $\left(\frac{k-1}{k+1}\right)^2$, becomes less important. After complicated derivations, we reach the main conclusion in this section.

**Theorem 2** When the agent’s choice of project is responsive to the incentives, the optimal scope of delegation $k^*$, the optimal strength of incentives $u^*$, and the level of uncertainty $a$ are complementary with each other: $f_{uk} > 0$, $f_{ua} > 0$, and $f_{ka} > 0$. The optimal scope of delegation, $k^*$, and the optimal strength of incentives, $u^*$, both increase with the level of uncertainty $a$: $\frac{du^*}{da} > 0$, $\frac{dk^*}{da} > 0$.

**Proof.** See Appendix B.

Intuitively, these complementarities arise from the complementarity between the information acquisition problem and the subsequent agency problem. The more favorable choice made in the principal’s interest, the higher is the information value of the chosen project, and the greater is the induced initiative to acquire the information. Proposition 2 more or
less reveals the complementarities between delegation and the other two factors. The complementarity between uncertainty and incentives is more subtle. Increases in either induce the agent to exert more effort and choose a more favorable project. But greater incentive payments also reduce the principal’s surplus. When a higher level of uncertainty increases the information value, the sum of the two positive effects of incentives (inducing more effort, choosing a more favorable project) overcome the negative effect (reducing surplus). The stronger incentives and the higher level of uncertainty make the interests of the principal and the agent more aligned. This lowering of the conflict of interest makes it relatively more important to increase the information benefit. Therefore, more delegation and stronger incentives arise from an increase in uncertainty.

5 Communication Model

Now we extend our analysis to study the principal’s optimal design of ex post communication and incentives. In the standard revelation game, the principal can fully commit to all possible decision rules and outcomes. A state is characterized by the agent’s effort, his knowledge of the success probabilities of all projects and the distribution of his favorite project. Then the principal’s decision rules depend on the agent’s report of state, and the agent’s payoff depends on both his report and realized outcome. But this scheme is quite unrealistic. More often, the principal may not commit to these complicated decision rules. The reasons are as follows. First, the state space is complicated, and so it is impossible or too costly to describe all contingencies. Second, it is difficult to verify the decision rules to a third party. Then the principal can take advantage of the revealed information for those decisions that are hard to verify. Anticipating this, the agent will not truthfully report his information.

In this paper we model the ex post communication by taking the imperfect commitment contracting approach (e.g. Bester and Strausz, 2001; Krishna and Morgan, 2004). Specifically, we assume the principal commits to the monetary payoff but need not commit to the project choice. This form of contracting is common in many settings. For example, branch managers in retail banks give advice on promotion plans to the headquarter managers. The headquarter manager retains the ultimate authority to select promotion plan, and the branch manager’s compensation only depends on the sales of the selected plan.

Since the principal need not commit to the project choice, we model communication between the principal and the agent as a discrete cheap-talk game. The framework is close to that of the delegation model we have studied before. The timing of the model is as
follows. (i) The principal and the agent sign the contract \((u, m)\), and all \(n\) projects are assigned to the agent. (ii) The agent exerts effort \(e\) to learn the success probabilities of the \(n\) projects with cost \(\frac{1}{2}Ce^2\). (iii) He has to report \(m\) projects to the principal from the \(n\) projects. (iv) The principal randomly implements one of the \(m\) reported projects. (v) Success or failure of the implemented project is realized, and 0 or \(u\) is paid accordingly. The contract \((u, m)\) specifies the strength of incentives \(u\) and the amount of reports \(m\). See Figure 1 for illustration. In Section (6.1) we will show that this communication contract \((u, m)\) is self-enforcing. Although the principal is allowed to implement a project outside the \(m\) projects reported by the agent, he has no incentive to do so. Therefore, the principal actually commits himself to the communication contract \((u, m)\).

5.1 Agent’s Problem

If the agent learns the projects’ success probabilities, he reports a set of \(m\) projects from his favorite and the other \(n-1\) projects. We still denote the success probability of the agent’s favorite as \(x\). Denote as \(x^{(1)}_{n-1}, x^{(2)}_{n-1}, \ldots, x^{(n-1)}_{n-1}\) as the order statistics of the other \(n-1\) projects.

If \(ux + B > ux^{(n-m)}_{n-1}\), the agent reports his favorite \(x\) and \(x^{(n-m+1)}_{n-1}, x^{(n-m+2)}_{n-1}, \ldots, x^{(n-1)}_{n-1}\). Otherwise he reports \(x^{(n-m)}_{n-1}, x^{(n-m+1)}_{n-1}, x^{(n-m+2)}_{n-1}, \ldots, x^{(n-1)}_{n-1}\). So the \(m-1\) largest order statistics will always be reported. In this case the agent’s real choice is between his favorite \(x\) and the \((n-m)th\) order statistic of the other \(n-1\) projects \(x^{(n-m)}_{n-1}\). This reminds us of the situation in the delegation model where the agent’s choice is between \(x\) and \(x^{(k-1)}_{k-1}\).

If the agent learns nothing about the \(n\) projects’ success probabilities, he includes his favorite \(x\) in the report, and randomly chooses \(m-1\) projects from the other \(n-1\) projects to report.

The set of all possible probability pair \((x, x^{(n-m)}_{n-1})\) is

\[
\Theta \equiv \left\{ \left( x, x^{(n-m)}_{n-1} \right) \mid \frac{1}{2} - a < x, x^{(n-m)}_{n-1} < \frac{1}{2} + a \right\},
\]

The set of probability pairs for which the agent will report \(x^{(n-m)}_{n-1}\) is

\[
\Theta_1 \equiv \left\{ \left( x, x^{(n-m)}_{n-1} \right) \in \Theta \mid ux + B < ux^{(n-m)}_{n-1} \right\}.
\]

As before, we define
\[ \Theta_2 \equiv \left\{ \left( x, x_{n-1}^{(n-m)} \right) \in \Theta \mid x < x_{n-1}^{(n-m)} \right\}. \]

Given the contract \((u, m)\), the optimal effort \(e^*(u, m)\) maximizes

\[
e \left\{ \frac{1}{m} \sum_{i=1}^{m-1} u E x_{n-1}^{(n-i)} + \frac{1}{m} \left[ \int_{\Theta_1} (ux + B)dG \left( x, x_{n-1}^{(n-m)} \right) \right] \right\} \\
+ (1 - e) \left\{ \frac{1}{m} \sum_{i=1}^{m-1} uE x + \frac{1}{m} \int_{\Theta} (ux + B)dG \left( x, x_{n-1}^{(n-m)} \right) \right\} - ce^2/2 \tag{11}
\]

This payoff formula is quite complex. The first term represents the agent’s expected payoff conditional on a successful information acquisition, weighted by the probability of success \(e\). The second term is the analogue for the case in which he learns nothing. The first term inside the first brace, \(\frac{1}{m} \sum_{i=1}^{m-1} u E x_{n-1}^{(n-i)}\), reflects the fact that the agent always reports the largest \(m-1\) order statistics from the \(n-1\) projects. The second item takes into account the choice made between \(x\) and \(x_{n-1}^{(n-m)}\).

The first order condition yields

\[
e^* (u, m) = \frac{1}{cm} \left[ \sum_{i=1}^{m-1} u (E x_{n-1}^{(n-i)} - E x) + \int_{\Theta_1} (ux_{n-1}^{(n-m)} - ux - B)dG \left( x, x_{n-1}^{(n-m)} \right) \right]
\]

### 5.2 Principal’s Problem

The principal’s expected profit is

\[
(v - u) \left\{ e^* \left[ \frac{1}{m} \sum_{i=1}^{m-1} E x_{n-1}^{(n-i)} \right] + \frac{1}{m} \int_{\Theta_1} x dG \left( x, x_{n-1}^{(n-m)} \right) \right\} \\
+ (1 - e^*) \int_{\Theta} x dG \left( x, x_{n-1}^{(n-m)} \right)
\]

\[
= (v - u) \left\{ \frac{1}{2} + e^* \left[ \frac{1}{m} \sum_{i=1}^{m-1} (E x_{n-1}^{(n-i)} - E x) \right] + \frac{1}{m} \int_{\Theta_1} (x_{n-1}^{(n-m)} - x) dG \left( x, x_{n-1}^{(n-m)} \right) \right\}
\]

Notice that the first term inside the brace is the principal’s reservation value \(\frac{1}{2}\), i.e., the
expected success probability without the agent’s information acquisition. The effort $e^*$ represents the agent’s initiative to acquire information. The first term inside the square bracket represents the average information value of the $m - 1$ largest order statistics, which will always be included in the report. The second term inside the square bracket represents the information value of the agent’s real choice, i.e., the choice between $x$ and $x_{n-m}$.

Given $e^*(u, m)$, the principal’s optimal contract $(u^*, m^*)$ solves

$$\max_{u, m} (v - u) \left\{ \frac{1}{2} + \frac{e^*}{m} \left[ \sum_{i=1}^{m-1} (Ex_{n-1}^{(n-i)} - Ex) + \int_{\Theta_1} (x_{n-1}^{(n-m)} - x) dG(x, x_{n-1}^{(n-m)}) \right] \right\}$$

(12)

s.t. $e^* = \frac{1}{cm} \left[ \sum_{i=1}^{m-1} u(Ex_{n-1}^{(n-i)} - Ex) + \int_{\Theta_1} (ux_{n-1}^{(n-m)} - ux - B) dG(x, x_{n-1}^{(n-m)}) \right]$. \hspace{1cm} (13)

6 The Optimal Communication Contract

6.1 Preliminary Analysis

First, we show that our communication contract is self-enforcing. Since we allow the agent to freely report $m$ projects, he will always select the $m$ best projects in his own interest. The proposition below shows that the principal will commit to implementing a project from the $m$ projects reported by the agent.

**Proposition 3** The principal has no incentive to implement a project outside the $m$ projects reported by the agent. Therefore, our communication contract is self-enforcing.

**Proof.** See Appendix C.

If the agent learns nothing, implementing any project gives the principal the same expected success probability $\frac{1}{2}$. If the agent successfully acquires information, the $m - 1$ largest

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$^{14} \int_{\Theta} xdG(x, x_{n-1}^{(n-m)}) = Ex = \frac{1}{2}.$

$^{15}$Actually, we can allow the agent to freely report any number of projects. If the agent reports $l$ projects ($l > m$), the principal randomly implements one from the $l$ reported projects. If the agent reports $l$ projects ($l < m$), the principal randomly adds $m - l$ projects from the rest $n - l$ projects, and then randomly implements one from the augmented $m$ projects. Under this scheme, the agent will always be induced to report $m$ projects.
order statistics of the other \( n - 1 \) projects will always be reported. Most of the unreported projects are lower order statistics, and so implementing a project from the unreported projects gives the principal a lower expected payoff.

We now analyze the information benefit and the agency cost in this new framework. As before, the information benefit is defined as the total information gain created by the agent when there is no conflict of interest, i.e., \( B = 0 \). In this case the agent always reports the \( m \) largest statistics from the \( n \) projects if he successfully acquires the information. Since in this situation \( \Theta_1 = \Theta_2 \), in light of (12), (13), then the information value and the effort are as follows.\(^{16}\)

\[
\frac{1}{m} \left[ \sum_{i=0}^{m-1} (E x_{n-i}^{(n)}) - E x \right] = a \frac{n - m}{n + 1}, \\
e = \frac{a n - m}{c n + 1} u.
\]

The information benefit is the product of effort and information value, i.e., \( \frac{a^2}{c} \left( \frac{n-m}{n+1} \right)^2 u \). So the strength of incentives \( u \) and the level of uncertainty \( a \) increase the information benefit. But the information benefit is reduced by the amount of reports \( m \). The reason is as follows. Given the sample size \( n \), the more projects reported, the more order statistics included, the lower is the average expectation of them.

The agent will not always report \( \max \{ x, x_{n-1}^{(n-m)} \} \), the one in the principal’s interest. So there is a conflict of interest in this case. The probability of this case is \( \frac{1}{m} \), since the principal puts probability \( \frac{1}{m} \) on each reported project.

The above analysis shows the trade-off caused by communication. The more reports \( m \), the lower is the information benefit, but the probability of conflict is also lower. So an increase in the amount of reports \( m \) reduces both the information benefit and the agency cost. Then the optimal communication should balance these two effects. Compared with what happen in the delegation model, the contrast is interesting. Delegation increases the information benefit and the agency cost, while communication decreases both of them.

As before, we normalize all random variables and denote \( t = \frac{B}{2mu} \). Depending on how the agent choose between \( x \) and \( x_{n-1}^{(n-m)} \), we have three types of solutions. We still start from the two extreme cases when the agent’s choice is not responsive to the incentives.

\(^{16}\) Derivations are relegated to Appendix C.
6.2 Two Extreme cases

6.2.1 Zero Conflict of Interest: \( t = 0 \)

This is the case where there is no conflict of interest, i.e., \( B = 0 \). So the agent always reports in the principal’s interest. It is obvious that everything here should be the same as the \( B = 0 \) case in the delegation model. So

\[
m^* = 1, \text{ and } u^* = \frac{v}{2} - \frac{c(n + 1)^2}{4a^2(n - 1)^2}. \tag{16}
\]

6.2.2 Large Conflict of Interest: \( t \geq 1 \)

This is the case in which the conflict of interest is so large that the agent always reports his own favorite. For example, \( B \geq av \). In this case \( x \) and \( x^{(n-m+1)}, ..., x^{(n-1)} \) are reported when the agent learns the success probabilities. Otherwise he reports his favorite \( x \) and \( m - 1 \) randomly chosen projects.

Since \( \Theta_1 \) is empty, in light of (13), then the agent’s effort is

\[
e^* = \frac{u}{cm} \sum_{i=1}^{m-1} (Ex \cdot (n-i) - Ex) = \frac{a n - m m - 1}{c n m} u. \tag{17}
\]

In light of (12), (13), the principal’s optimal contract \((u^*, m^*)\) solves

\[
\max_{u,m \leq n} (v - u) \left\{ \frac{a^2}{c} \left[ \frac{(m-1)(n-m)}{mn} \right]^2 u + \frac{1}{2} \right\}. \tag{18}
\]

We first choose the optimal amount of reports \( m^* \) to solve

\[
\max_{1 \leq m \leq n} \frac{n - m m - 1}{n m} \]

The first order condition yields

\[
m^* = \sqrt{n} > 1 \text{ if } n > 1. \tag{19}
\]

The optimal amount of reports is larger now than when there is no conflict, but still independent of incentives and uncertainty. Then \( u^* \) is chosen to maximize the quadratic payoff function. The first order condition yields
The optimal strength of incentives $u^*$ is smaller now than when there is no conflict. But it still strictly increases with the level of uncertainty $a$.

This case highlights the trade-off between the information benefit and agency cost. Since the agent always includes his favorite in the report, the principal cannot extract any information value from this project. To reduce the probability of this event, the principal would like to increase the amount of reports. But an increase in the amount of reports reduces the average information value for the other $m-1$ reported projects. So the optimal reports $m^*$ should balance these two effects. As before, in these two extreme cases the optimal amount of reports is independent of incentives and uncertainty. Summarizing our findings above, we have Theorem 3 below.

**Theorem 3** When the agent’s report of projects is not responsive to the incentives, the optimal strength of incentives $u^*$ strictly increases with the level of uncertainty $a$, and the optimal amount of reports $m^*$ is independent of uncertainty and incentives. Moreover, the optimal strength of incentives $u^*$ is larger and the optimal amount of reports $m^*$ is smaller when there is no conflict of interest than when the conflict is sufficiently large.

### 6.3 Aligned Interest: $0 < t < 1$

In this case incentives can affect the agent’s report of projects and align the interests of the principal and the agent. As before, we know only the relative values matter. So we first remove the common factors and define the following relative terms.

Define $I(u,m,a)$ as the relative loss in information value

$$I(u,m,a) = \frac{1}{am} \left[ \int_{\Theta_2} (x_{n-1}^{(n-m)} - x) dG(x,x_{n-1}^{(n-m)}) - \int_{\Theta_1} (x_{n-1}^{(n-m)} - x) dG(x,x_{n-1}^{(n-m)}) \right]. \tag{21}$$

Define $J(u,m,a)$ as the relative loss in effort.

$$J(u,m,a) = \frac{c}{aum} \left[ \int_{\Theta_2} (ux_{n-1}^{(n-m)} - ux) dG(x,x_{n-1}^{(n-m)}) - \int_{\Theta_1} (ux_{n-1}^{(n-m)} - ux - B) dG(x,x_{n-1}^{(n-m)}) \right]. \tag{22}$$
We have the following proposition regarding the properties of \( I(u, m, a) \) and \( J(u, m, a) \).

**Proposition 4** When the agent’s report of projects is responsive to the incentives, an increase in the level of uncertainty or the strength of incentives reduces both the relative loss in information value and the relative loss in effort. That is, \( I_a < 0, I_u < 0, J_a < 0, J_u < 0 \). But an increase in the level of uncertainty or the strength of incentives increases the marginal relative losses from the reports. That is, \( I_a m > 0, I_u m > 0, J_a m > 0, J_u m > 0 \).

**Proof.** See Appendix C.

The first part of Proposition 3 follows the same logic as in Proposition 2. Either stronger incentives or greater uncertainty can induce the agent to report a project more in the principal’s interest. In both cases, the information value of the reported project is higher, and the marginal benefit of effort is higher. Hence the relative losses in effort and information value are reduced. The results on marginal effects are more subtle. They suggest that communication is a substitute for both incentives and uncertainty.

Define \( L(u, m, a) \) as the relative agency cost

\[
L(u, m, a) = \frac{n - m}{n + 1} [I(u, m, a) + J(u, m, a)] - I(u, m, a) J(u, m, a)
\] (23)

The relative information benefit is \( \left( \frac{a - m}{n+1} \right)^2 \), i.e., the total information benefit divided by the common factor \( \frac{a^2}{c} \). So the principal’s expected payoff function is

\[
f(u, m) \equiv (v - u) \left[ R(u, m, a) u + \frac{1}{2} \right]
\]

where \( R(u, m, a) \equiv \frac{a^2}{c} \left[ \left( \frac{n - m}{n + 1} \right)^2 - L(u, m, a) \right] \).

After complicated derivations, we reach our main conclusion in this section.

**Theorem 4** When the agent’s report of projects is responsive to the incentives, the optimal amount of reports \( m^* \) is a substitute for the optimal strength of incentives \( u^* \) and the level of uncertainty \( a \), but the optimal strength of incentives \( u^* \) is still complementary to the level of uncertainty \( a \). That is, \( f_{am} < 0, f_{am} < 0, f_{ua} > 0 \), where \( f \) is the principal’s payoff function. So the optimal strength of incentives \( u^* \) strictly increases with the level of uncertainty \( a \), but
the optimal amount of reports $m^*$ strictly decreases with the level of uncertainty $a$. That is, 
\[ \frac{da^*}{da} > 0, \quad \frac{dm^*}{da} < 0. \]

**Proof.** See Appendix C.

The reason for the complementarity between incentives and uncertainty is similar to that in Theorem 2. The reason that communication is a substitute for incentives and uncertainty is as follows. When the conflict of interest has been already mitigated by incentives or uncertainty, a further reduction in the agency cost by more reports is limited. Moreover, more reports reduce the information benefit. So the principal’s marginal profit of incentives or uncertainty is reduced by more reports.

## 7 General model

Now we consider the general model in which the principal can use all three instruments: communication, delegation, and incentive pay. The contract $(u, k, m)$ specifies: (1) the strength of incentives $u$; (2) the scope of delegation $k$; (3) the amount of reports $m$. The timing of the model is as follows. (i) The principal and the agent sign the contract $(u, k, m)$, and $k$ of the $n$ projects are assigned to the agent. (ii) After observing which project has $B$, the agent exerts effort $e$ to learn the success probabilities of the $k$ projects with cost $\frac{1}{2}cc^2$. (iii) He has to report $m$ projects to the principal from the $k$ projects. (iv) The principal randomly implements one from the $m$ reported projects. (v) Success or failure of the implemented project is realized, and 0 or $u$ is paid accordingly. See Figure 1 for illustration.

Let us start from the extreme cases. If there is no conflict of interest, i.e., $B = 0$, it is obvious that everything should be the same as the delegation model. So $k^* = n, m^* = 1, u^* = \frac{v}{2} - \frac{c(n+1)^2}{4a^2(n-1)^2}$. In this case, the principal will not include communication in his design.

When the conflict of interest is sufficiently large (e.g. $B \geq av$), there are two possible cases facing the agent.

**Case 1. With probability $\frac{k}{n}$, the agent’s favorite is one of the $k$ projects.**

This is similar to the case in (6.2.2). We only need to replace $n$ with $k$.

If the agent learns the success probabilities, he reports his favorite $x$ and $x_{k-1}^{(k-m+1)}, x_{k-1}^{(k-m+2)}, \ldots, x_{k-1}^{(k-1)}$. Otherwise he reports his favorite $x$ and $m - 1$ randomly chosen projects. In light of (18), the principal’s conditional expected profit is
\[(v - u) \left\{ \frac{a^2}{c} \left[ \frac{(m - 1)(k - m)}{mk} \right]^2 u + \frac{1}{2} \right\} \].

Case 2. With probability \(1 - \frac{k}{n}\), the agent’s favorite is not one of the \(k\) projects.

This is similar to the case in (6.1). Given successful information acquisition, the agent reports the largest \(m\) order statistics from the \(k\) projects. In light of (14), (15), the principal’s conditional expected profit is

\[ (v - u) \left( \frac{a^2}{c} \left( \frac{k - m}{k + 1} \right)^2 u + \frac{1}{2} \right) \]

The principal’s optimal contract \((u^*, k^*, m^*)\) solves

\[
\max_{1 \leq m \leq k \leq n} \left( v - u \right) \left\{ \frac{a^2}{c} \left[ \frac{k}{n} \frac{(m - 1)^2 (k - m)^2}{m^2 k^2} + \left( 1 - \frac{k}{n} \right) \left( \frac{k - m}{k + 1} \right)^2 \right] u + \frac{1}{2} \right\}
\]

As before, this is a sequential two-step optimization problem. The optimal scope of delegation \(k^*\) and the optimal amount of reports \(m^*\) solve

\[
\max_{1 \leq m \leq k \leq n} H(k, m) = \frac{k}{n} \frac{(m - 1)^2 (k - m)^2}{m^2 k^2} + \left( 1 - \frac{k}{n} \right) \left( \frac{k - m}{k + 1} \right)^2
\]

(24)

**Numerical Solution.** We solve the above problem by numerical optimizations. We have tested \(n\) from 1 to 1000, and the results show that \(m^* = 1\) always.\(^{17}\)

Figure 3 shows the distribution of function \(H(k, m)\) when \(n\) takes value 100. Interestingly, the two local maximizers in the picture correspond to the delegation model \((m = 1)\) and the communication model \((k = n)\) respectively. From (8), (19), the two local maximizers are \((k_1 = 18, m_1 = 1)\) and \((k_2 = 100, m_2 = 10)\). Since \(H(18, 1) = 0.65645 > 0.65610 = H(100, 10)\), we know that the global maximizer is \((k_1 = 18, m_1 = 1)\), and communication

\(^{17}\)Although analytical proof is difficult, the programming of this optimization problem is easy because the values of \(m\) and \(k\) are natural numbers.
will not be included. The picture also illustrates the difficulties of analytical solution. First, the shape of function $H(k,m)$ is very irregular. Second, there are two distinct local maximizers, and both of them are boundary solutions. Furthermore, their function values are very close to each other. These observations suggest that the usual first order approach does not help here.

Summarizing the above results, we have the following proposition.

**Proposition 5** The principal will not include communication in his design if the amount of conflict of interest between him and the agent is either zero or, alternatively, sufficiently large.

Although formal proof is difficult, the intuition behind Proposition 5 is simple. Notice that the case when the agent’s favorite is one of the $k$ projects is the same as case (6.2.2) in the communication model. The information benefit is reduced by the amount of communication in this case. Moreover, there is no conflict of interest when the agent’s favorite is not one of the $k$ projects. In this situation communication further reduces the information benefit, and has no effect on the agency cost. Thus the negative effect of communication on the information benefit is magnified relative to its positive effect on the agency cost. Therefore, the principal will not include communication in his design.

Following this line of logic, we conjecture that the principal will never include communication in his design. Our intuition is as follows. The only positive effect of communication is to reduce the agency cost. This effect becomes the most pronounced when the conflict of interest reaches maximum. But even in this situation, it cannot overcome the negative effect on the information benefit. When the conflict of interest is not that severe, this positive effect is further reduced. So the principal will never include communication in his design. However, due to the technical difficulty, we have not analytically substantiated our argument. Further study on this issue is needed.

### 8 Conclusion

In this paper we have argued that optimal organizational design must simultaneously motivate economic agents to acquire costly information and to use that information in the principal’s interest. A complete understanding of organizational structure requires a thorough analysis of both the information acquisition problem and the subsequent agency problem. Overlooking either one of these two problems will lead to partial or misleading conclusions.
Incorporating these two considerations, our model successfully accounts for the recent empirical findings that standard models of agency and delegation cannot adequately explain.

Our models differ from the standard models in several significant ways. The standard agency models focus on the negative effect of uncertainty on performance measures. In this paper we emphasize the positive contribution that uncertainty makes to the value of information, and thus information acquisition is of central importance in our models. Unlike the standard delegation models, ours explicitly include monetary incentives. Our delegation model is thus able to explain why delegation is positively related to uncertainty and incentives.

Although there is an abundant theoretical literature on incentives and decision rights separately, theoretical work on the simultaneous design of optimal incentives and optimal decision rights is relatively scant. But within any organizational hierarchy, these two designs are closely related. Milgrom and Roberts argue:

“In the language of economics, incentives and delegated authority are complements: each makes the other more valuable. Evaluating complementarities—how the pieces of a successful organization fit together and how they fit with the company’s strategy—is one of the most challenging and rewarding parts of organizational analysis.” Milgrom and Roberts (1992) P.15.

In this paper we try to address these issues by investigating the relationships among delegation, communication, incentives, and uncertainty. Our findings confirm their claim that delegation is complementary with incentives. Moreover, we find communication is a substitute to incentives. Their relationships with environmental uncertainty are also identified. In doing so, we close the gap between optimal incentives theory and the optimal decision rights theory, and enhance our understanding of organizational design.
Appendix A: Order Statistics

We first introduce some important properties of order statistics of uniform distribution on \((0, 1)\). Suppose that \(x_1, x_2, \ldots, x_n\) are \(n\) independent random variables with uniform distribution on \((0, 1)\). Let \(x_n^{(1)} \leq x_n^{(2)} \leq \cdots \leq x_n^{(n)}\) be the order statistics obtained by arranging the random sample in increasing order of magnitude. So \(x_n^{(i)}\) is the \(i\)th order statistic of a sample of size \(n\).

**Claim 1** \(x_n^{(i)}\) has Beta\((i, n - i + 1)\) distribution. That is to say, \(x_n^{(i)}\) has probability density function \(f_{i,n}(y)\),

\[
f_{i,n}(y) = \begin{cases} 
\frac{y^{i-1}(1-y)^{n-i}}{B(i,n-i+1)}, & y \in (0, 1) \\
0, & y \notin (0, 1)
\end{cases}
\]

where \(B(i, n - i + 1) = \int_0^1 y^{i-1} (1-y)^{n-i} \, dy = \frac{(i-1)! (n-i)!}{n!}\).

**Claim 2**

\[Ex_n^{(i)} = \frac{i}{n+1}\]

**Claim 3** The cumulative distribution function \(F_{x:n}(y) = y^n\) of \(x_{n:n}\) is decreasing with the sample size \(n\). That is to say, the maximum order statistics of a larger sample size first-order stochastically dominates the maximum order statistics of a smaller size.

**Claim 4** The cumulative distribution function \(F_{i,n}(y)\) of \(x_n^{(i)}\) is decreasing with the rank \(i\): \(\forall y \in (0, 1), F_{i-1,n}(y) > F_{i,n}(y)\), because the higher rank order statistics first-order stochastically dominates the lower rank order statistics of the same sample.

Most of our results only depend on the properties of the standard uniform distribution. This is a canonical and general distribution. First, any random sample can be transformed into a sample of standard uniform distribution. Second, it can used to approximate any random sample. Moreover its order statistics have the Beta distribution. When it comes to describing a random variable on a closed interval, the Beta distribution is the most commonly used one. There are several ways to transform and approximate any random sample. We use the method developed by Scheffe and Turkey (1945) and David and Johnson (1954).
Let \( U_1, U_2, \ldots, U_n \) be a random sample from the standard uniform distribution and \( X_1, X_2, \ldots, X_n \) be a random sample with continuous cdf \( F(x) \). Furthermore, let \( U_n^{(1)} \leq U_n^{(2)} \leq \ldots \leq U_n^{(n)} \) and \( X_n^{(1)} \leq X_n^{(2)} \leq \ldots \leq X_n^{(n)} \) be the order statistics obtained from these samples.

\[
F \left( X_n^{(i)} \right) = U_n^{(i)}
\]

Define the inverse function as \( F^{-1}(y) = \sup \{ x : F(x) \leq y \} \), then,

\[
X_n^{(i)} = F^{-1}(U_n^{(i)})
\]

Expand \( F^{-1}(U_n^{(i)}) \) at \( E(U_n^{(i)}) = \frac{i}{n+1} = p_i \), first order approximation gives us

\[
X_n^{(i)} = F^{-1}(p_i) + F^{-1} (p_i) \left( U_n^{(i)} - p_i \right) + o \left( \frac{1}{n^2} \right)
\]

So, if the sample size is large enough, we can approximate any order statistic by a linear transformation of the same rank order statistic of the standard uniform distribution. In this way our results can be extended to more general distributions.

Appendix B: Delegation Model

**Proposition 1.** \( \int_{\Omega_1} (y - x) dG(x,y) \leq \int_{\Omega_2} (y - x) dG(x,y) \), \( \epsilon_1^* \leq \epsilon_2^* \).

**Proof.**

\[
\begin{align*}
\int_{\Omega_2} (y - x) dG(x,y) - \int_{\Omega_1} (y - x) dG(x,y) &= \int_{\{ y \geq x \}} (y - x) dG(x,y) - \int_{\{ y \geq x + \frac{n}{2}\}} (y - x) dG(x,y) \\
&= \int_{\{ x \leq y \leq x + \frac{n}{2} \}} (y - x) dG(x,y) \geq 0
\end{align*}
\]
\[ e_2^* - e_1^* = \frac{1}{c} \left[ \int_{\Omega_2} (uy - ux)dG(x, y) - \int_{\Omega_1} (uy - ux - B)dG(x, y) \right] \]
\[ = \frac{1}{c} \left[ \int_{\{y \geq x\}} (uy - ux)dG(x, y) - \int_{\{y \geq x + \frac{B}{2}\}} (uy - ux - B)dG(x, y) \right] \]
\[ = \frac{1}{c} \left[ \int_{\{x \leq y \leq x + \frac{B}{2}\}} (y - x)dG(x, y) + \int_{\{y \geq x + \frac{B}{2}\}} BdG(x, y) \right] \geq 0 \]

Preliminary Calculation

After normalization of \( x, y \), let us first calculate \( \int_{\Omega_1} (y - x)dG(x, y) \), \( \int_{\Omega_2} (y - x)dG(x, y) \), \( e_1^* \), and \( e_2^* \).

\[ \int_{\Omega_1} (y - x)dG(x, y) = \int_{\{0 < x' + t < y' < 1\}} 2a(y' - x')f_{k-1:k-1}(y') dx'dy' \]
\[ = a \int_t^1 f_{k-1:k-1}(y') \int_0^{y'-t} 2(y' - x')dx'dy' = a \int_t^1 f_{k-1:k-1}(y')(y'^2 - t^2)dy' \]
\[ = a \int_t^1 f_{k-1:k-1}(y')(y'^2 - t^2)dy' \]

\[ \int_{\Omega_2} (y - x)dG(x, y) = a \int_0^1 f_{k-1:k-1}(y') y'^2dy' \]
\[ = a \int_0^1 (k - 1) y'^{k-1}y'^2dy' = a \frac{k - 1}{k + 1} \]

\[ \int_{\Omega_1} (y - x - \frac{B}{u})dG(x, y) = \int_{\{0 < x' + t < y' < 1\}} 2a(y' - x' - t)f_{k-1:k-1}(y') dx'dy' \]
\[ = a \int_t^1 f_{k-1:k-1}(y') \int_0^{y'-t} 2(y' - x' - t)dx'dy' \]
\[ = a \int_t^1 f_{k-1:k-1}(y')(y' - t)^2dy' \]

\[ e_1^* = \frac{1}{c} \int_{\Omega_1} (uy - ux - B)dG(x, y) = \frac{a}{c} u \int_t^1 f_{k-1:k-1}(y')(y' - t)^2dy' \]
\[ e_2^* = \frac{1}{c} \int_{\Omega_2} (uy - ux)dG(x, y) = \frac{a(k - 1)}{c(k + 1)} u \]

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Notice that \( u, a \) enter \( I(u, k, a), J(u, k, a), L(u, k, a) \) from \( t = \frac{R}{2au} \). So we can rewrite them as \( I(t, k), J(t, k), L(t, k) \), and focus on \( t \).

To prove Proposition 2, we first prove a lemma.

**Lemma 1.** If \( 0 < t < 1 \), then \( I_t > 0, I_{tk} > 0, J_t > 0, J_{tk} > 0 \).

**Proof.**

(1)

\[
I(t, k) = \frac{1}{a} \int_{\Omega_2} (y - x) dG(x, y) - \int_{\Omega_1} (y - x) dG(x, y) \\
= \frac{k - 1}{k + 1} - \int t f_{k-1:k-1}(y')(y'^2 - t^2) dy'
\]

\[
I_t = 2t \int_{t}^{1} f_{k-1:k-1}(y') dy' = 2t(1 - F_{k-1:k-1}(t)) > 0
\]

Claim 3 tells us \( \frac{d}{dk} (F_{k-1:k-1}(t)) < 0 \), then \( I_{tk} = -2t \cdot \frac{d}{dk} (F_{k-1:k-1}(t)) > 0 \)

(2)

\[
J(t, k) = \frac{c}{au}(e_2^* - e_1^*) \\
= \frac{k - 1}{k + 1} - \int t f_{k-1:k-1}(y')(y'^2 - t^2) dy' \\
J_t = 2 \int_{t}^{1} f_{k-1:k-1}(y')(y'^2 - t^2) dy' > 0
\]

\[
J_t = 2 \int_{t}^{1} (y' - t) dF_{k-1:k-1}(y') \\
= 2 (y' - t) F_{k-1:k-1}(y') \bigg|_{t}^{1} - 2 \int_{t}^{1} F_{k-1:k-1}(y') dy' \\
= 2 (1 - t) - 2 \int_{t}^{1} F_{k-1:k-1}(y') dy' \\
\frac{d}{dk} (F_{k-1:k-1}(t)) < 0 \rightarrow J_{tk} = -2 \int_{t}^{1} \frac{d}{dk} (F_{k-1:k-1}(y')) dy' > 0
\]

**Proposition 2.** If \( 0 < t < 1 \), then \( I_a < 0, I_u < 0, J_a < 0, J_u < 0, I_{ak} < 0, I_{uk} < 0, J_{ak} < 0, J_{ak} < 0 \).

**Proof.** The proof easily follows from Lemma 1.
\[ I_a = -I_t \cdot \frac{B}{2a^2 u} < 0, \quad I_u = -I_t \cdot \frac{B}{2au^2} < 0. \]

\[ I_{ak} = \frac{\partial}{\partial k} \left( -I_t \cdot \frac{B}{2a^2 u} \right) = -I_{tk} \cdot \frac{B}{2a^2 u} < 0, \quad I_{uk} = \frac{\partial}{\partial k} \left( -I_t \cdot \frac{B}{2au^2} \right) = -I_{tk} \cdot \frac{B}{2au^2} < 0. \]

Similarly, \( J_a < 0, J_u < 0, J_{ak} < 0, J_{uk} < 0. \)

To prove Theorem 2, we first prove a lemma.

**Lemma 2.** If \( 0 < t < 1, \) then \( L_t > 0, L_{tk} > 0, L_{tt} < 0 \)

**Proof.**

\[ L(t, k) = \frac{k - 1}{k + 1} \left( I(t, k) + J(t, k) \right) - I(t, k)J(t, k) \]

**Step 1:** \( 0 < I < J < \frac{k - 1}{k + 1} \)

\[ J - I = \int_t^1 f_{k-1:k-1}(y') (y'^2 - t^2) dy' - \int_t^1 f_{k-1:k-1}(y') (y' - t)^2 dy' \]

\[ = 2 \int_t^1 f_{k-1:k-1}(y') (y' - t) y' dy' > 0 \]

\[ I(t = 0) = \frac{k - 1}{k + 1} - \int_0^1 f_{k-1:k-1}(y') y'^2 dy' = \frac{k - 1}{k + 1} - \frac{k - 1}{k + 1} = 0 \]

\[ I(t = 1) = \frac{k - 1}{k + 1} - \int_1^k f_{k-1:k-1}(y') (y'^2 - 1) dy' = \frac{k - 1}{k + 1} \]

Then \( 0 < I < \frac{k - 1}{k + 1}. \) Follow the same way, we have \( 0 < J < \frac{k - 1}{k + 1}. \) So \( 0 < I < J < \frac{k - 1}{k + 1}. \)

**Step 2:**

\[ L_t = (I_t + J_t) \frac{k - 1}{k + 1} - I_t J - I J_t \]

\[ = I_t \left( \frac{k - 1}{k + 1} - J \right) + J_t \left( \frac{k - 1}{k + 1} - I \right) > 0 \]

**Step 3:**

\[ J_{tk} > 0 \rightarrow J_k < J_k(t = 1) = \frac{d (\frac{k - 1}{k + 1} - J)}{dk} \rightarrow \frac{d (\frac{k - 1}{k + 1} - J)}{dk} > 0 \]

\[ I_{tk} > 0 \rightarrow I_k < I_k(t = 1) = \frac{d (\frac{k - 1}{k + 1} - I)}{dk} \rightarrow \frac{d (\frac{k - 1}{k + 1} - I)}{dk} > 0 \]
\[ L_{tk} = I_t \frac{d}{dk} \left( \frac{(k-1)}{k+1} - J \right) + J_t \frac{d}{dk} \left( \frac{(k-1)}{k+1} - I \right) + I_{tk} \left( \frac{k-1}{k+1} - J \right) + J_{tk} \left( \frac{k-1}{k+1} - I \right) > 0 \]

**Step 4:**

\[
J_t = 2 \int_0^1 (y' - t) \, dF_{k-1:k-1}(y') = 2 (1 - t) - 2 \int_0^1 \, F_{k-1:k-1}(y') \, dy' \\
J_{tt} = -2 + 2F_{k-1:k-1}(t) < 0 \\
I_t = 2t(1 - F_{k-1:k-1}(t)) \rightarrow I_{tt} = 2(1 - F_{k-1:k-1}(t)) - 2tf_{k-1:k-1}(t) \\
J_{tt} + I_{tt} = -2tf_{k-1:k-1}(t) < 0 \\
L_{tt} = (I_{tt} + J_{tt}) \frac{k-1}{k+1} - I_{tt}J - J_{tt} = 2tJ_t \\
\]

Since \(0 < I < J < \frac{k-1}{k+1}\) and \(J_{tt} < 0\), \(-IJ_{tt} < -JJ_{tt}\). So,

\[
L_{tt} < (I_{tt} + J_{tt}) \frac{k-1}{k+1} - I_{tt}J - J_{tt} = 2tJ_t < 0 \\
\]

**Theorem 2.** If \(0 < t < 1\), then \(f_{uk} > 0\), \(f_{ua} > 0\), \(f_{ka} > 0\), \(\frac{du^*}{da} > 0\), \(\frac{dk^*}{da} > 0\).

**Proof.**

\[
\max_{u,k} f(u,k) = (v - u)(R(t,k)u + \frac{1}{2}) \\
R(t,k) = \frac{a^2}{c} \left[ \left( \frac{k-1}{k+1} \right)^2 - \frac{k}{n} L(t,k) \right] \\
\text{where } t = \frac{B}{2au} \\
\]

(1) Since \(L_{tk} > 0\), \(L_t > 0\), then

\[
R_{uk} = \frac{\partial}{\partial k} \left( \frac{\partial R}{\partial u} \right) = \frac{a^2}{c} \frac{\partial}{\partial k} \left( \frac{k}{n} L_t \frac{B}{2au^2} \right) \\
= \frac{a^2}{c} \frac{B}{2na^2} \left( kL_{tk} + L_t \right) > 0 \\
\]

At the optimal delegation \(k\), the boundary and first order condition tells us \(R_k \geq 0\). As we have showed before, the optimal incentive \(u < \frac{v}{2}\). So,

\[
f_{uk} = \frac{\partial}{\partial u} [(v - u)uR_k] \\
= (v - 2u) R_k + (v - u)uR_{uk} > 0 \\
\]
(2) Since $L_t < 0$, $L_t > 0$, then

$$R_{ua} = \frac{\partial}{\partial a} \left( \frac{a^2 k}{c n} L_t \frac{B}{2au^2} \right)$$

$$= \frac{k B}{nc 2u^2} \left[ aL_{tt} \left( -\frac{B}{2a^2u} \right) + L_t \right] > 0,$$

$$R_a = \frac{2R}{a} + \frac{a^2}{c} \left[ -\frac{k}{n} L_t \left( -\frac{B}{2a^2u} \right) \right] > 0.$$

$$f_{ua} = \frac{\partial}{\partial u} [(v-u)uR_a]$$

$$= (v-2u) R_a + (v-u) uR_{ua} > 0$$

(3) Since $R_k \geq 0$, $L_{tk} > 0$, $L_t > 0$, then

$$R_{ka} = \frac{\partial}{\partial k} \left\{ \frac{2R}{a} + \frac{a^2}{c} \left[ -\frac{k}{n} L_t \left( -\frac{B}{2a^2u} \right) \right] \right\}$$

$$= \frac{2R_k}{a} + \frac{B}{2cnu} [k L_{tk} + L_t] > 0$$

$$f_{ka} = \frac{\partial}{\partial k} [(v-u)uR_a]$$

$$= (v-u) uR_{ka} > 0$$

Given (1), (2), (3) and the second order conditions, we know $\frac{du^*}{da} > 0$, $\frac{dk^*}{da} > 0$.

Appendix C: Communication Model

For notational convenience, we replace $x_{n-1}^{(n-m)}$ with $y$ in the communication model analysis.

Preliminary Calculation.

After normalization, let us first calculate $\frac{1}{m} \sum_{i=1}^{m-1} (Ex_{n-1}^{(n-i)} - Ex), \frac{1}{m} \sum_{i=0}^{m-1} (Ex_{n}^{(n-i)} - Ex)$, $\int_{\Omega_1} (y-x) dG(x,y)$, and $e^*$.

$$\frac{1}{m} \sum_{i=1}^{m-1} (Ex_{n-1}^{(n-i)} - Ex) = \frac{2a}{m} \sum_{i=1}^{m-1} (Ex_{n-1}^{(n-i)} - Ex')$$

$$= a \left( \frac{1}{m} \sum_{i=1}^{m-1} \frac{2n-i}{m} - \frac{m-1}{m} \right) = \frac{a}{m} \frac{m-1}{m} n-m$$
\[
\frac{1}{m} \sum_{i=0}^{m-1} (Ex_{n-i} - Ex) = \frac{2a}{m} \sum_{i=0}^{m-1} (Ex_{n-i} - Ex') = \frac{a}{m} \left( \frac{2n - m + 1}{n + 1} - 1 \right) = \frac{n - m}{n + 1}
\]

\[
\int_{\Theta_1} (y - x) dG(x, y) = \int_{\{0 < x' + t < y' < 1\}} 2a(y' - x') f_{n-m:n-1}(y') dx' dy' = a \int_t^1 f_{n-m:n-1}(y') \left( y'^2 - t^2 \right) dy'
\]

\[
\int_{\Theta_1} (y - x - B) dG(x, y) = \int_{\{0 < x' + t < y' < 1\}} 2a(y' - x' - t) f_{n-m:n-1}(y') dx' dy' = a \int_t^1 f_{n-m:n-1}(y') \left( (y' - t)^2 \right) dy'
\]

\[
e^* = \frac{1}{cm} \sum_{i=1}^{m-1} u(Ex_{n-i} - Ex) + \int_{\Theta_1} (ux_{n-1}^{(n-m)} - ux - B) dG(x, x_{n-1}^{(n-m)})
\]

\[
= \frac{a}{c} u \left( \left( \frac{m - 1}{m} \right) \left( \frac{n - m}{m} \right) + \int_t^1 f_{n-m:n-1}(y') (y' - t)^2 dy' \right)
\]

**Proposition 3.** The principal has no incentive to implement a project outside the \( m \) reported projects. Therefore, our communication contract is self-enforcing.

**Proof.**

(1) If the agent learns nothing, then all projects are identical to the principal and the agent in terms of success probability. In this case, implementing any project gives the principal the same expected success probability \( \frac{1}{2} \), and so the principal has no incentive to choose a project outside the \( m \) projects reported by the agent.

(2) If the agent successfully acquires information, randomly implementing one project from the \( m \) reported projects will give the principal the conditional expected payoff, \( E_1 \),
\[ E_1 = \sum_{i=1}^{m-1} \mathbb{E}(x^{(n-i)}_{n-1}) + \frac{1}{m} \int_{\Theta_i} x dG (x, x^{(n-m)}_{n-1}) + \frac{1}{m} \int_{\Theta_1} x^{(n-m)} dG (x, x^{(n-m)}_{n-1}) \]

\[ = \frac{1}{m} \sum_{i=1}^{m-1} \mathbb{E}(x^{(n-i)}_{n-1}) + \frac{1}{m} \mathbb{E}x + \frac{1}{m} \int_{\Theta_1} (x^{(n-m)}_{n-1} - x) dG (x, x^{(n-m)}_{n-1}) \]

\[ = \left[ \frac{1}{2} - a + \frac{2n - m}{n} a \right] \left( \frac{m - 1}{m} \right) + \left( \frac{1}{2} \right) \left( \frac{1}{m} \right) + \frac{1}{m} a \int_{t}^{1} f_{n-m:n-1} (y') (y'^2 - t^2) dy'. \]

If in this case the principal reneges on the original contract and implements a project outside the \( m \) reported projects, his conditional expected payoff \( E_2 \) is

\[ E_2 = \sum_{i=m+1}^{n-1} \mathbb{E}(x^{(n-i)}_{n-1}) + \frac{1}{n-m} \int_{\Theta_i} x^{(n-m)}_{n-1} dG (x, x^{(n-m)}_{n-1}) + \frac{1}{n-m} \int_{\Theta_1} x dG (x, x^{(n-m)}_{n-1}) \]

\[ = \frac{1}{n-m} \sum_{i=m}^{n-1} \mathbb{E}(x^{(n-i)}_{n-1}) - \frac{1}{n-m} \int_{\Theta_1} (x^{(n-m)}_{n-1} - x) dG (x, x^{(n-m)}_{n-1}) \]

\[ = \frac{1}{2} - a + a \frac{n - m + 1}{n} - \frac{1}{n - m} a \int_{t}^{1} f_{n-m:n-1} (y') (y'^2 - t^2) dy'. \]

\[ a \frac{n - m + 1}{n} \leq a \rightarrow \frac{1}{2} - a + a \frac{n - m + 1}{n} \leq \frac{1}{2} \] \hspace{1cm} (25)

\[ \frac{2n - m}{n} a \geq a \rightarrow \frac{1}{2} - a + \frac{2n - m}{n} a \geq \frac{1}{2} \]

\[ \rightarrow \left[ \frac{1}{2} - a + \frac{2n - m}{n} a \right] \left( \frac{m - 1}{m} \right) + \left( \frac{1}{2} \right) \left( \frac{1}{m} \right) \geq \frac{1}{2} \] \hspace{1cm} (26)

\[ a \int_{t}^{1} f_{n-m:n-1} (y') (y'^2 - t^2) dy' \geq 0 \] \hspace{1cm} (27)

From (25), (26), (27), we know \( E_1 \geq E_2 \). So the principal has no incentive to choose a project outside the \( m \) reported projects, when the agent successfully acquires information.

Therefore, the principal always commits to implementing a project from the \( m \) reported projects. Our communication contract is self-enforcing.

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As before, focus on $t$ can simplify our analysis. To prove Proposition 4, we first prove a lemma.

**Lemma 3.** If $0 < t < 1$, then $I_t > 0$, $I_{tm} < 0$, $J_t > 0$, $J_{tm} < 0$.

**Proof.**

\[
\int_{\Theta_2} (y - x) dG(x, y) = a \int_0^1 f_{n-m-1} (y') y'^2 dy' = a \int_0^1 \frac{y'^{n-m+1} (1 - y')^{m-1}}{B(n - m, m)} dy' \\
= \frac{a}{B(n - m, m)} \left( \frac{(n - m + 1)(n - m)}{(n + 1)n} \right)
\]

(1)

\[
I(t, m) = \frac{1}{ma} \left[ \int_{\Theta_2} (y - x) dG(x, y) - \int_{\Theta_1} (y - x) dG(x, y) \right] \\
= \frac{(n - m + 1)(n - m)}{m(n + 1)n} - \frac{1}{m} \int_t^1 f_{n-m-1} (y') (y'^2 - t^2) dy'
\]

\[
I_t = \frac{2t}{m} \int_t^1 f_{n-m-1} (y') dy' = \frac{2t}{m} (1 - F_{n-m-1}(t)) > 0
\]

Claim 4 tells us $\frac{d}{dm} (F_{n-m-1}(t)) > 0$, and then $I_{tm} = -\frac{2t(1-F_{n-m-1}(t))}{m^2} - \frac{2t}{m^2} (F_{n-m-1}(t)) < 0$

(2)

\[
J(t, m) = \frac{c}{aum} \left[ \int_{\Theta_2} (uy - ux) dG(x, y) - \int_{\Theta_1} (uy - ux - B) dG(x, y) \right] \\
= \frac{(n - m + 1)(n - m)}{m(n + 1)n} - \frac{1}{m} \int_t^1 f_{n-m-1} (y') (y' - t)^2 dy'
\]

\[
J_t = \frac{2}{m} \int_t^1 f_{n-m-1} (y') (y' - t) dy' > 0
\]

\[
J_t = \frac{2}{m} \int_t^1 (y' - t) dF_{n-m-1} (y') \\
= \frac{2}{m} (y' - t) F_{n-m-1} (y') \big|_t^1 - 2 \int_t^1 F_{n-m-1} (y') dy' \\
= \frac{2}{m} \left[ (1 - t) - \int_t^1 F_{n-m-1} (y') dy' \right]
\]

Since $\frac{d}{dm} (F_{n-m-1}(t)) > 0$, then

\[
\frac{d}{dm} \left[ 2(1 - t) - 2 \int_t^1 F_{n-m-1}(y') dy' \right] = -2 \int_t^1 \frac{d}{dm} (F_{n-m-1}(y'))dy' < 0
\]

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Moreover, \(2 (1 - t) - 2 \int_t^1 F_{n-m-n-1} (y') dy' \geq 0\). So \(J_{tm} < 0\).

**Proposition 4.** If \(0 < t < 1, I_a < 0, I_u < 0, J_a < 0, J_u < 0, I_{am} > 0, I_{um} > 0, J_{am} > 0, J_{um} > 0\).

**Proof.** The proof easily follows from Lemma 3.

\[
I_a = -I_t \cdot \frac{B}{2a^2 u} < 0, \quad I_u = -I_t \cdot \frac{B}{2au^2} < 0.
\]

\[
I_{am} = \frac{\partial}{\partial m} \left(-I_t \cdot \frac{B}{2a^2 u}\right) = -I_{tm} \cdot \frac{B}{2a^2 u} > 0, \quad I_{um} = \frac{\partial}{\partial m} \left(-I_t \cdot \frac{B}{2au^2}\right) = -I_{tm} \cdot \frac{B}{2au^2} > 0.
\]

Similarly, \(J_a < 0, J_u < 0, J_{am} > 0, J_{um} > 0\).

To prove Theorem 4, we first prove a lemma.

**Lemma 4.** If \(0 < t < 1\), then \(L_t > 0, L_{tm} < 0, L_{tt} < 0\)

**Proof.**

\[
L(t, m) = \frac{n-m}{n+1} (I(t, m) + J(t, m)) - I(t, m) J(t, m)
\]

**Step 1:**

\[
0 < I < J < \frac{n-m}{n+1}
\]

\[
J - I = \frac{1}{m} \int_t^1 f_{n-m-n-1} (y') \left(y'^2 - t^2\right) dy' - \int_t^1 f_{n-m-n-1} (y') (y' - t)^2 dy' = 0
\]

\[
I(t = 0) = 0 \rightarrow I(t = 0) < I < I(t = 1)
\]

\[
I(t = 0) = \frac{1}{m} \left[\int_0^1 f_{n-m-n-1} (y') y'^2 dy' - \int_0^1 f_{n-m-n-1} (y') y'^2 dy'\right] = 0
\]

\[
I(t = 1) = \frac{(n-m+1)(n-m)}{m(n+1)n}
\]

\[
\frac{(n-m+1)(n-m)}{m(n+1)n} - \frac{n-m}{n+1} = \frac{n-m}{n+1} \left[\frac{(n-m+1)}{mn} - 1\right] < 0 \rightarrow
\]

\[
0 < I < I(t = 1) < \frac{n-m}{n+1}
\]
Following the same way, we have $0 < J < \frac{m-1}{m+1}$. So $0 < I < \frac{n-m}{n+1}$.

**Step 2:**

\[ L_t = (I_t + J_t) \frac{n - m}{n + 1} - I_t J - I J_t \]
\[ = I_t \left( \frac{n - m}{n + 1} - J \right) + J_t \left( \frac{n - m}{n + 1} - I \right) > 0 \]

**Step 3:**

\[ J_{tm} < 0 \rightarrow -J_m < J_m(t = 1) \rightarrow \]
\[ \frac{d \left( \frac{n-m}{n+1} - J \right)}{dm} < \frac{d \left( \frac{n-m}{n+1} - J \right)}{dm} \bigg|_{t=1} = 0 \]

Similarly, \( \frac{d \left( \frac{n-m}{n+1} - I \right)}{dm} < 0 \), so

\[ L_{tm} = I_t \frac{d \left( \frac{n-m}{n+1} - J \right)}{dm} + J_t \frac{d \left( \frac{n-m}{n+1} - I \right)}{dm} + I_{tm} \left( \frac{n-m}{n+1} - J \right) + J_{tm} \left( \frac{n-m}{n+1} - I \right) < 0 \]

**Step 4:**

\[ m J_t = m \int_t^1 (y' - t) dF_{n-m:n-1} (y') = 2 (1 - t) - 2 \int_t^1 F_{n-m:n-1} (y') dy' \]
\[ J_{tt} = \frac{-2 + 2F_{n-m:n-1} (t)}{m} < 0 \]

\[ m I_t = 2t (1 - F_{n-m:n-1} (t)) \rightarrow m I_{tt} = 2(1 - F_{n-m:n-1} (t)) - 2t F_{n-m:n-1} (t) \]
\[ J_{tt} + I_{tt} = \frac{-2t F_{n-m:n-1} (t)}{m} < 0 \]
\[ L_{tt} = (I_{tt} + J_{tt}) \frac{n - m}{n + 1} - I_{tt} J - J_{tt} - 2I_t J_t \]

Since $0 < I < J < \frac{n-m}{n+1}$ and $J_{tt} < 0$, then $-IJ_{tt} < -J_{tt}$. So,

\[ L_{tt} < (I_{tt} + J_{tt}) \frac{n - m}{n + 1} - I_{tt} J - J_{tt} - 2I_t J_t \]
\[ = (I_t + J_t) \left( \frac{n - m}{n + 1} - J \right) - 2I_t J_t < 0 \]

**Theorem 4.** If $0 < t < 1$, then $f_{um} < 0$, $f_{ua} > 0$, $f_{ma} < 0$, $\frac{da^*}{da} > 0$, $\frac{dm^*}{da} < 0$. 41
Proof.

\[
\max_{u,m} f(u,m) = (v-u)(R(t,m)u + \frac{1}{2})
\]

where \(R(t,m) = \frac{a^2}{c} \left[ \left( \frac{n-m}{n+1} \right)^2 - L(t,m) \right], \ t = \frac{B}{2au}.\)

(1) Since \(Ltm < 0, L_t > 0\), then

\[
R_{um} = \frac{\partial}{\partial m} \left( \frac{\partial R}{\partial u} \right) = \frac{a^2}{c} \frac{\partial}{\partial m} \left( \frac{L_t B}{2au^2} \right) \\
= \frac{a^2}{c} \frac{B}{2au^2} Ltm < 0.
\]

Notice in the communication model \(m^* < n. m^* = n\) can never be the optimal solution. The boundary and first order conditions tells us \(R_m \leq 0\). As we have showed before, the optimal incentive \(u < \frac{v}{2}\). So

\[
f_{um} = \frac{\partial}{\partial u} [(v-u)uR_m] \\
= (v-2u)R_m + (v-u)uR_{um} < 0.
\]

(2) Since \(Ltt < 0, L_t > 0\), then

\[
R_{ua} = \frac{\partial}{\partial a} \left( \frac{a^2}{c} \frac{L_t B}{2au^2} \right) \\
= \frac{1}{c} \frac{B}{2au^2} \left[ aLtt \left( -\frac{B}{2a^2u} \right) + L_t \right] > 0,
\]

\[
R_a = \frac{2M}{a} + \frac{a^2}{c} \left[ -L_t \left( -\frac{B}{2a^2u} \right) \right] > 0,
\]

\[
f_{ua} = \frac{\partial}{\partial u} [(v-u)uR_a] \\
= (v-2u)R_a + (v-u)uR_{ua} > 0.
\]

(3) Since \(R_m \leq 0, Ltm < 0\), then

\[
R_{ma} = \frac{\partial}{\partial m} \left\{ \frac{2R}{a} + \frac{a^2}{c} \left[ -L_t \left( -\frac{B}{2a^2u} \right) \right] \right\} \\
= \frac{2R_m}{a} + \frac{B}{2cu} Ltm < 0
\]
Given (1), (2), (3) and the second order conditions, we know $\frac{du^*}{da} > 0$, $\frac{dm^*}{da} < 0$.

References


Contract \((u, k)\) is signed, and \(k\) projects are delegated. The agent observes which project has \(B\). The agent exerts effort to acquire information. Success or failure is realized, and 0 or \(u\) is paid. The agent implements one project.

**Timing of Delegation Model**

Contract \((u, m)\) is signed, and \(n\) projects are delegated. The agent exerts effort to acquire information. The agent reports \(m\) projects to the principal. Success or failure is realized, and 0 or \(u\) is paid. The principal randomly implements one from the \(m\) reported projects.

**Timing of Communication Model**

Contract \((u, k, m)\) is signed, and \(k\) projects are delegated. The agent observes which project has \(B\). The agent exerts effort to acquire information. The principal randomly implements one from the \(m\) reported projects. The agent reports \(m\) projects to the principal. Success or failure is realized, and 0 or \(u\) is paid.

**Timing of General Model**

Figure 1.
The agent choice depends on $x', y'$ as follows:

- $\Omega_1$: the principal’s favorite, not the agent’s favorite
- $\Omega_2/\Omega_1$: not the principal’s favorite, the agent’s favorite
- $\Omega_2^c$: the principal’s favorite, the agent’s favorite

Figure 2.
Figure 3.