ALLOCATIVE EFFICIENCY OF A SPECULATIVE FUTURES MARKET WITH ZERO-PATIENCE

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ABSTRACT

This paper extends the results of zero-intelligence (ZI) in double auction markets for goods to trade in futures under an illiquid regime of speculative traders. Despite this more volatile setting of risk-neutral speculators, short selling, margin trading, and multiple equilibria, the artificial futures exchange still converges to the stable theoretical equilibrium. To conform with the institutional structure of a futures market, there is no limit order book: the standard ZI model of constrained random limit orders is replaced with impatient or satisficing market orders. This modification maintains the essence of the ZI trader and builds on the insight of Gode and Sunder’s (1993) results that optimization is unnecessary to bring about allocative efficiency. The addition of scalpers or market makers creates a more liquid regime which increases trading activity and reduces the kurtosis in prices.

Keywords: Zero-Intelligence, Margins, Transaction Tax, Continuous Double Auction, Futures Market, Agent-based Model, Scalpers.

JEL Codes: C63, D44, D61

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1 INTRODUCTION

Price discovery is primarily attributed to new information and the competition between optimizing traders as they update their expectations and strategies. Market microstructure supplements this explanation, recognizing the trading mechanism to also play a crucial role in the equilibration of supply and demand. A third distinct ingredient in price formation, beyond human cognition and the trading mechanism, is that of liquidity – both at the market and individual trader level. This paper takes the last two of these categories, what Smith (1982) labeled as institutional structure and environment, and considers how efficient the continuous double auction (CDA) is with liquidity constrained zero-intelligent speculators. A particular form of liquidity is then added to the market through the addition of market makers.

The “normal” trading regime has been defined by Acharya and Schaefer (2006) as one where liquidity constraints are non-binding, traders are well capitalized and fundamentals dominate price movements. In contrast an “illiquidity” regime is one where traders are close to breaching their capital constraints such as an internal value-at-risk constraint; a capital adequacy requirement imposed by regulators; a collateral constraint imposed by lenders; or a margin requirement imposed by an exchange (Ibid). Illiquidity limits both arbitrage and speculation as trading becomes costly and collateral constraints become binding. This may lead to the forced liquidation of desired positions and sudden price movements.

In the two good model below, of cash and future contracts, agents are risk neutral and speculate on future price changes. Illiquidity is characterized by the degree to which speculators are leveraged and the enforcement of real-time gross settlement (RTGS) which requires speculators to mark to market their position after every trade. Transaction costs of 0.1 percent on one-way trades and a 25 percent margin requirement add to the characterization of illiquidity in the market. Speculators will go long or short in their derivative position based on their fixed expectation of future prices. Unlike most agent-based models (ABMs) (see summary of ABMs by Tesfatsion 2002) there is no evolutionary process in agent behavior, but there is an evolution wealth distribution. In this manner this 2 good model is closer to the general equilibrium ZI models of Gode and Sunder (2004) where wealth is explicitly modeled, as opposed to their original partial equilibrium ZI model of Gode and Sunder (1993).

Gode and Sunder “show how the elemental forces of want and scarcity cause Pareto-efficient outcomes even when agents do not maximize and when no evolutionary process [in agent behavior] exist” (2004, p2). By separating market rules from human cognitive behavior studies have shown that good market performance is not dependent on trader learning and rationality. This was previously put forward by Becker (1962) analytically with a Walrasian solution. Experimental economists who studied auction mechanisms found the CDA to be a design that yielded high allocative efficiency (Smith 1962). Computer simulations have proved very useful in distinguishing the contributions to price between market structure and agent behavior.

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1 Smith (1982) defines three categories that determine the performance of a micro system: the institutional structure (the rules that govern exchange), the environment (agents’ tastes, risk profile and endowments of information and resources), and agent behavior or learning and trading strategy.
In the ZI models of goods or stock trading, the CDA is able to drive price efficiency with outcomes that closely track the predictions of the Walrasian competitive equilibrium, without the individual optimization criteria usually assumed in the invisible hand analogy\(^2\). The ZI research program aims to aid market design and promote our understanding of the effect market rules have on efficiency, price volatility and liquidity.

Overall, the ZI traders are a tool to isolate and understand the effect of market rules on market outcomes. Understanding the effects of market rules and other social institutions is crucial because rules are observable and controllable, while individual strategies are inherently private and not directly controllable. Theories based on the effect of market rules are therefore easier to test. The ZI model provides a benchmark of the "structural” effect of market rules. The traditional strategic model in which traders respond fully to changes in market rules [and price outcomes] is another benchmark. The two benchmarks bracket the range in which human behavior lies (Gode and Sunder 2004, p.2).

This paper extends the model of the market to the trading of futures\(^3\), where there is short selling and margin trading, there is not limit book, and the presence of an exchange makes real-time gross settlement possible. [“Market discipline” is defined by Gode and Sunder as the restriction that traders are forbidden from making buys or sells at a loss because then they would not have been able to settle their accounts] footnote that this is not instituted yet]. Adding to this market structure the paper focuses on the trading of risk-neutral speculators rather than “fundamental” traders. While not relevantly different from Gode and Sunder’s (1993) heterogeneous buyers and sellers, they do not have the familiar smooth indifference curves of Gode and Sunder’s (2004) Edgeworth box presentation. By using risk-neutral speculators we hope to not only generalize the results but also add to the potential for market volatility which will aid in the testing of allocative efficiency of the CDA.

It is important to identify the similarities between this speculative futures model and that of Gode and Sunder’s (1993, 1994, & 2004) spot or stock market, of constrained random trading. The results here reinforce their same conclusion that the CDA is efficient in converging to the Pareto or Walrasian equilibrium without trader optimization and, in addition, even when potentially destabilizing leveraged speculators exist.

The Gode and Sunder ZI constrained model (1993) has traders divided into buyers and sellers who are given an equal allotment of shares to buy or sell, respectively. Each buyer (seller) has a given resale (cost) value remains fixed and creates the upper bound of their budget constraint. All traders maintain their desire to trade until they reach their pre-specified allotment of securities, which is the same for all traders. Their participation is not guided by optimization - minimization costs (maximize of profit) (which are calculated at the end of the trading period). Rather, all traders offer simultaneously a random bid (ask) that ranges between some nominated

\(^2\) Standard general equilibrium theory requires strong assumptions on agent optimization and information to assure the existence and stability of an equilibrium as in the Arrow-Debreu (1954) model.

\(^3\) The ZI model has been extended by various authors: Cliff and Bruten (1997) who criticized the price convergence properties and augmented minimal intelligence creating “ZI-Plus” or ZIP agents. Brewer et al (2002) and Duffy (2006).
floor (ceiling) and their budget constraint. Traders are selected and their bid (ask) for a single unit of the good is submitted to the limit order book where they are ranked, in accordance with CDA rules, such that the highest bid (lowest ask) is considered as the current bid (current ask). A trade occurs when the new bid equals or exceeds the current ask (the new ask equals or is less than the current bid). Following a transaction the limit order book is cleared and a new round of bids and asks are solicited and the process is repeated.

In contrast, the model presented here simulates liquidity constrained speculators who trade futures contracts and have fixed uniformly distributed expectations on the future spot price. The zero-intelligence characteristics are thought to be preserved, but the stock model is now applicable to a derivative model where promises rather than goods are traded for cash. In the simulations below the futures market is for a non-storable commodity and the spot market is independent of the futures trading, and is ignored in this trading period.

All speculators start with an equal endowment of cash to buy or sell futures contracts. The speculators’ risk-neutrality and marked to market wealth makes the decision – buy, sell, or hold. A speculator expects to make a profit by buying low and selling high, relative to their spot price expectation. Speculators will maintain their desire to buy or sell until constrained by their wealth, transaction costs and margin requirement. As in all futures markets, all long plus short positions sum to zero. Open outcry, an oral CDA, is used to find the current bid and ask, which is technically equivalent to a limit order book of length 1. Speculator participation in the market is guided by an immediacy or impatience to transact using market orders, rather than the Gode and Sunder random limit orders that are kept in the limit order book. It is argued here that both strategies are equivalent, and that both omit intelligent optimization strategies. As with Gode and Sunder’s emphasis on market discipline, speculators in this model are constrained from accumulating losses by obeying margin calls settled in real-time.

Both models outlined above incorporate zero intelligence and budget constrained traders in an attempt to determine whether it is intelligence or market discipline that is most important for the process of price discovery to the Pareto optimum. Gode and Sunder test 3 CDA markets: human constrained, ZI-constrained, and ZI-unconstrained; finding that the first two markets are almost identical in their allocative efficiency, thereby concluding that is the constraints within a CDA that was responsible for the allocative efficiency of the market and not intelligence. This paper compares the multi-lateral Walrasian equilibrium (super intelligence) to the ZI-constrained CDA price. While the traders here are ZI-C, they are renamed as zero-patience (ZP) traders to distinguish their trading with market orders rather than limit orders.

4 Gode and Sunder’s (1993) only has one good with no wealth restrictions, and Gode and Sunder (2004) have two goods: green and red chips. This model has two goods, cash and futures contracts, but the derivative has a zero-sum in aggregate. Transaction costs are a leakage from the system.

5 In Gode and Sunder, traders are selected to be either buyers or sellers hence producing an aggregate demand curve and an aggregate supply curve. Here, speculators produce just an aggregate demand curve that has both positive and negative sides. This is technically a “total demand” curve, but it looks similar to the “excess demand” curve of general equilibrium analysis.

6 Just as the resale or cost valuations were fixed in Gode and Sunder (1993), the expectations of the underlying commodity valuation in the future spot market is fixed, and it is initially drawn from a uniform distribution.

7 Zero-intelligence/patience (ZIP) traders may have been an alternative, but this anachronism has already been used by Cliff and Bruten (1997) for their zero-intelligence-plus traders.
ZP speculators desire *immediacy*, trading primarily with market orders when there is an expected gain, or the need to meet real-time margin requirements. ZP speculators are satisficing rather than optimizing, and in this sense are void of learning and intelligence. The impatient trading behavior specified here for *speculative* traders mirrors the random behavior of *fundamental* traders in the Gode and Sunder (1993) ZI stock market model. Both are subject to budget constraints and market rules but both typically trade at prices that do not optimize their profit. The probability of placing a market order at a certain quoted price in this model is similar to having a limit order accepted in the Gode and Sunder model due to the common CDA architecture in both models.

ZP speculators are randomly selected to trade in rounds and will submit a market order to buy at the current ask (sell at the current bid) if their end-price expectation is higher (lower) than this value. Because expectations are randomly distributed the probability of trading at the current bid or ask should be similar to the probability that God and Sunder’s agents trade following their random quotes. If the speculator’s end-price expectation is within the current bid-ask spread then they will not transact, but instead better one of these quotes, replacing either the current bid or ask with their own and thus narrowing the spread, as in Gode and Sunder. The details of this model will be elaborated on in section 3.

The second major contribution of this paper is that it considers market liquidity not just in terms of trading costs and individual trader liquidity constraints, but it builds on the theory that liquidity is provided through limit orders and removed with market orders (Schwartz 1988) and extends the market environment with the provision of market makers.

In the next section we describe the literature that underlies and motivates the model. There is a description of the CDA on the floor of a futures exchange and a discussion on the definition of liquidity and how it determines the bid-ask spread. Following this, section 3 explains the model and derives a ZP speculator demand curve with real-time gross settlement (RTGS). Risk-neutral speculative traders who trade on margin create the illiquidity regime and impose market discipline. ZPs are endowed with a common amount of wealth (cash), the security is infinitely divisible, and there is multiple bidding rounds. ZPs are constantly retrading (buying and selling) and quoting bid and ask prices. The algorithm used to simulate the CDA is taken from Chan, et al (1998) and modified for a futures exchange with transaction costs and margin trading. Speculator expectations are kept constant, mimicking Gode and Sunder’s (1993) individual reserve prices remaining constant, and the trading order is randomized, which mirrors the random limit orders in Gode and Sunder. Section 4 presents simulations of the ZP market model with and without scalpers. We find that the mid-point between the bid and ask converges to the Walrasian equilibrium price and the bid-ask spread narrows to the cost of transacting. The trading activity of ZP speculators increases with the introduction of market makers and price efficiency becomes more resilient.

In all cases the bid-ask spread narrows to the transaction tax when liquidity is available. Adding tax exempt scalpers increases the standard deviation of prices but reduces kurtosis. The presence of scalpers increases the turbulence or trading activity along with an increase in liquidity (depth of market. While pro-liquidity policies stimulate turbulence in prices, they still remain close to the competitive Walrasian equilibrium.
In general, despite the potential for individual instability among the ZP speculators and their lack of optimizing behavior, the CDA futures market is proven to be a relatively efficient allocator. This paper supports the view of Gode and Sunder that individuals may be impulsive or lack rationality and yet markets can remain orderly and efficient, if the right market institutions are in place.

2 FUTURES MARKET TRADING

2.1 Open outcry

In an open-outcry futures market, as described by Silber (1984), all bids and offers must be announced publicly to the pit through the outcry of buy or sell orders. In particular, no prearranged trades are permitted on futures exchanges. Strict priority is kept, where the highest bid price and the lowest offer take precedence, and this is known as the inside spread. Lower bidders must keep silent when a higher bid is called out, and higher offers are silenced when a lower offer is announced, although simultaneous offers and simultaneous bids at the same price can occur. To increase the probability of execution, a trader can raise his bid or lower his offer, and then other traders must remain silent. This rule is designed to insure best execution, in the sense that sales occur at the highest bid price and purchases occur at the lowest offering, and all bids or offers do not live longer than the moment needed to make a transaction.

Scalpers, also known as locals because of their exchange membership, are floor traders who trade on their own account and have low transaction costs and more flexible margin requirements than speculators. Like dealers, in bond or foreign exchange markets, scalpers regularly quote a bid price at which to buy and an ask price at which to sell, making a market and thereby offering to complete orders quickly, typically at a price close to the last price, for those anxious to trade. By inserting this spread between the buy and sell, the scaler thereby receives a profit for providing the service of immediacy, which is just one dimension of liquidity. Scalpers may also provide depth commensurate with the quantity they are willing to buy or sell. While scalpers typically provide liquidity, it is important to note that they can also “consume” liquidity when they liquidate or offset positions, by selling at the bid price or buying at the ask price. This reduction in liquidity may cause temporary instability (Schwartz 1988).

An ordinary trader (non-scalper) can either tender his own ask or bid quote that competes with the scalper, called a limit order, or accept the price currently quoted in the market, called a market order. When a market participant accepts the market bid, he is said to hit the bid. When he accepts the market ask, he is said to lift the ask. The following example highlighting the choices of a non-scalper who wants to buy contracts is taken from Silber (1984, p. 940). A commercial hedger can instruct his broker (on the floor) to buy 50 contracts at the market, in which case the broker lifts the asks of others in the pit. Alternatively, the commercial hedger can try to buy more cheaply by instructing the floor broker to bid for 50 contracts at the prevailing bid price in the pit. In the first case, the market order uses the immediate execution services provided by the offerers in the pit (from scalpers or whomever) consuming liquidity. In the second case, the bid represented by the floor broker can be used by others to sell into, thereby providing liquidity.
This study first considers how effectively a financial market with asynchronous trading operates without scalpers. Often the mismatch between buyers and sellers that typically exists at any given instant is resolved by some agents who are willing to play the role of market maker and provide liquidity.

2.3 What is Liquidity?

Liquidity is defined in many different ways. A market is commonly thought of as perfectly liquid if trades can be executed with no cost (O’Hara 1997; Engle and Lange 1997). The academic literature on market microstructure recognizes that the arrival of random traders to buy or sell is asynchronous, and market activities are temporally discrete. Rarely is there a single price and research into liquidity in a CDA typically focus on the bid-ask spread. A narrower spread means a more liquid market. Modeling the spread is an extremely complex matter, given that the markets are composed of numerous limit traders (which include dealers and ordinary traders) embedded in a dynamic, interactive environment. Such a system may best be modeled with an agent-based methodology.

The analytical bid-ask spread literature (Stoll 1978; Ho and Stoll 1981) explains the demand for immediacy from the asynchronous arrival of random traders to buy or sell. It is often assumed that dealers participate in every trade, known as a quote-driven market. The behavior of the market maker or dealer is typically described as a trader who inserts a spread between the buy and sell and thereby receives a profit for providing the service of immediacy in what might otherwise be a fragmented market. This view of the market maker, as a provider of predictable immediacy, was first formalized by Demsetz (1968) and then elaborated on by Garman (1976) and many others.8

It is generally accepted that the bid-ask spread is representative of the risks faced by the dealer as a result of inventory control and asymmetric information. When scalpers fill market orders, they profit from impatient traders but lose to traders more informed. It is usually concluded that with competition, the spread is reduced to the dealer’s trading costs. This theory has formalized the idea of dealers as the providers of liquidity and controllers of the size of the spread. The size of the premium charged by immediacy providers to cover these expected costs determines the size of the spread and thereby the extent of illiquidity in the market. Inventory control costs are assumed to be reasonably constant over time, while risks of asymmetric information are not (Engle and Lange 1997, p. 4). Since this paper excludes information then by this argument suggests that the bid-ask spread would remain constant.

The bid-ask spread measure of liquidity has gained popularity (see Flemming 2003), although many other definitions have long been debated.9 Schwartz (1988) argues that too much emphasis has been made of market makers and their spread. More attention should be paid to the manner in which ordinary traders supply immediacy to each other and compete to reduce market spreads with the scalpers (Cohen et al. 1979, p. 814). Any trader that makes market orders should be seen as removing liquidity and those that offer limit orders can be characterized as providing

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8 See Stoll (1985) and Schwartz (1988) for further discussion and references on alternative views of dealers and scalpers.

9 See Bernstein (1987), Black (1986), and Harris (2003) for alternative descriptions and measures of liquidity.
liquidity. While market makers may be needed in illiquid markets, they are not a necessity for liquidity (Schwartz 1988).

Schwartz emphasizes the resiliency dimension of liquidity, rather than costs of transacting. Resiliency refers to how quickly prices revert to former levels after they change in response to large order flow imbalances (Bernstein 1987, and Harris 2003, pp. 398–405). For market makers to stabilize a market, they must commit capital or inventory risk, and this may become substantial. Schwartz warns us that market makers injecting liquidity into a system to stabilize prices might also be just as quickly withdrawn at a later date if shortages are incurred or if the market makers seek to rebalance their portfolios. This issue of scalpers balancing their inventory positions is directly incorporated into the model below.

3 A MODEL FUTURES MARKET WITH SPECULATORS, SCALPERS AND HEDGERS

3.1 Trader Population

This futures market model has up to 3 different traders: speculators, scalpers and hedgers; all contribute to the market microstructure and price formation in different ways. In all cases trading behavior focuses on quantity and wealth constraints rather than strategies that optimize capital gains.

We present a model of a futures trading pit with open-outcry and a continuous double auction trading mechanism. The model has two markets: a speculative futures market for an underlying non-storable commodity (this allows us to ignore the market) and a residual money market. The price of money is normalized to 1, one-way transaction costs and margin requirements, both as a percentage of each trade or position, are imposed by the exchange on all traders except locals (scalpers). There is no restriction on short selling, the futures contract size is perfectly divisible, and prices are always non-negative. Near to real time gross settlement RTGS is approached as all traders try to settle with each other before each trade takes place through variation margins. Each type of trader has their own quantity constraints and rules for trading.

Zero-patient (ZP) speculators will trade with market orders before considering the placement of a limit order. All speculators are risk neutral and differ only in their expectation of what the futures price should be and their cumulative wealth positions. Expectations of the next-period

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10 Instead of using the close-of-day settlement price to calculate margin calls, settlement is adjusted continuously throughout the day and the settlement price used to calculate margin calls is the average of the bid and ask price, or mid-price. This means that profits and losses transfer hands via the exchange, between traders continuously, removing the risk of accumulated losses and trader default. The amount paid is known as the variation margin. As a simplification, initial margin and maintenance margin are considered the same and the margin requirement is specified as a fixed percentage of the contract value rather than an absolute dollar value per contract. By using a margin requirement that changes with the percentage change in prices, we get closer to the essence of what the exchange considers in setting the margin.
futures-contract price stay constant during the trading period. Being risk neutral, speculators typically end up at the corner solutions of their budget constraint, trading on margin and maximizing their futures position (long or short) at every chance they get to trade.

*Scalpers* are members of the exchange and operate on the floor of the exchange without paying a trading fee. They do not have an opinion on the fundamental price and instead try to buy as low and sell as high as they can. They want to maximize the turnover of buys and sells while minimizing their inventory holding. Scalpers prefer to place limit orders (quotes) and to buy at their bid quote and sell at their ask quote. Scalper activity assists in balancing order flow over the long run, which does promote price efficiency, but it could create price instability in the bilateral CDA, when they offer liquidity to “smart” traders and are then forced to liquidate their own inventory holdings with market orders.

*Hedgers* play a limited but important role in setting up the fundamental demand and supply of contracts in the market. There are only two representative hedgers — one going long (to cover the expected purchase of the underlying commodity in the future) and the other going short (to cover the expected sale of the underlying commodity in the future). The quantity desired is fixed above or below a certain reserve price regardless of their wealth position. The difference between the long and short hedge is the *net hedge*, or the net desired contract position of the hedgers. Hedgers only place market orders until they fill their exogenous desired contract position. Once their futures position is attained, they stop trading. Since the sum of all futures contracts sum to zero, the net hedge will determine the long-run net position of the remaining trader population.

The net hedge is used here to counter the Cliff and Bruten (1997) criticism of the Gode and Sunder (1993) ZI model. They replicated the ZI model and found that the mean trading price for the ZI-C traders was only close to the to the theoretical equilibrium price when supply and demand curves were symmetric. In general, the trading price for the ZI-C traders “approached the expected price E(P) from the probability density (PDF) function given by the intersection of sellers’ offer-price PDF and the buyers’ bid-price PDF” (Ibid 1997, p.19). However for a comparison to a Walrasian price the economy needs to be closed and wealth needs to be specified. Once the ZI model became a two good model, as in Gode and Sunder 2004 and this paper, a result based on expected price and probabilities no-longer holds. To show this, we have simulated 2 of the 6 markets’ below with a positive net hedge of 5000 contracts.

Cliff and Bruten (1997), by considering only reservation prices (as presented in the original Gode and Sunder (1993) model), showed that some intelligence was required and they created ZI-plus traders which incorporated an adaptive behaviour based on the behavior of other traders. However, this is an unnecessary addition of intelligence. Rather, imposing market discipline in a general equilibrium context is enough to bring about a convergence to the theoretical Walrasian equilibrium price. This was shown by Gode and Sunder (2004) and it is supported in the results below.

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11 Keynes argued that the net hedge is typically negative and justifies the backwardation of futures prices.
3.2 Speculator’s Demand Function

In our model with leveraged speculation, $\kappa$ represents the limit on how much larger a speculator’s futures position — price multiplied by the number of contracts ($p_{t_\xi}$) can be than a trader’s wealth $m_t$. All simulations in this paper use $\kappa = 4$, which means that a trader can have up to 4 times his wealth dedicated to a long or short futures position. In other words the margin requirement is 25%, $1/\kappa = 0.25$. The collateral kept in the margin account by speculator $i$ is held as Treasury bills or money, represented here as $m^i_t$. Collateral held must be greater than the margin requirement, $m^i_t \geq p_{t_\xi} x^i_t / \kappa$ for the current futures position, at all times (to the extent that trading allows). There will be several transaction prices throughout the day which represent a trade at either a quoted bid $p^b_t$ or a quoted ask $p^a_t$. If there is not enough collateral in the margin account to meet the margin requirement then speculator $i$ will have to liquidate their position with an offset purchase or sale at their next turn to trade.

The futures position $x_t$ at price $p_t$ is taken on by the speculator as a contract at time $t$ to sell or buy $x$ units of the underlying commodity at price $p_t$ on the spot or maturity date of the futures contract. Since the speculator does not intend on making delivery on this contract, the purpose of holding this position is to flip the position and profit on price changes. On the basis of a constant price expectations $p^{\theta \omega}$ about the next transaction price $p_{t+1}$, speculator $i$ will decide to go either long or short in futures. Transaction costs are incurred for each one-way trade as a percentage of the trade value. In the simulations below this fee 0.1 percent and is represented by $\varpi = 0.001$. If the expected short-term gain does not compensate the cost of trading over the next period:

$$(p^\theta - p_t) x_t \leq \varpi p_t \mid (x_t - x_{t-1})$$

then the speculator will hold his current position instead of trading. The trader is myopic and on opening a position there is no consideration of costs incurred for reversing the position. Each speculator $i$ is risk neutral and simply maximizes expected wealth $\pi$ over the period $t$ to $t+1$:

$$\pi^{\theta \omega}_{t+1} = (p^{\theta \omega} - p_t)x^i_t + m^i_t$$

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12 There is no opportunity cost in holding cash since it would receive an interest on the T-bills, however interest in this version of our model is zero.
The speculator’s demand curve is derived in Appendix 1 via linear programming. In summary, speculator $i$’s demand for futures in each period $t$ is a slightly simplified version from Ussher (2004):

$$x_i^t(p_t, x_{i-1}^t, m_{i-1}^t, p_{i-1}^t, p_i^\theta, \kappa, \varpi, \sigma)$$

where:

- $p_t$ = Intra day futures market transaction price at time $t$,
- $x_{i-1}^t$ = Previous contract position,
- $m_{i-1}^t$ = Previous cash position in margin account following last transaction,
- $p_i^\theta$ = Price expectation $p_i^\theta$ of the next futures price $p_{i+1}$,
- $1/\kappa$ = Margin requirement as a percentage of futures position value, and
- $\sigma$ = Percentage transaction tax on a one-way trade (paid each way).

A futures demand curve is usually represented as a smooth downward sloping line from the top of quadrant II to the bottom of quadrant I in the two dimensional $R^2$ space in Figure 1. This model produces a non-linear demand function with inherent corner solutions from the risk neutral speculators’ wealth constraints and the regulatory setting of margin limits, transaction costs and RTGS.

Each risk-neutral speculator maximizes the next period’s expected wealth by holding money as collateral and buying or selling (going long or short in) futures. The decision to buy or sell futures depends on whether the speculator expects prices to rise or fall, respectively. There is no restriction or disincentive to short selling (i.e., selling commodities that one doesn’t own). A trader will trade only when price expectations $p_i^\theta$ are far enough away from actual prices $p_t$ to pay for the one-way transaction costs. Figure 1 has a zero contract position held over from last period. If a speculator currently has a futures position, then margin calls can lead to forced liquidation of the position when prices move against expectations. The possibility of a backward bending demand function, as in Figure 2, is a result of the collateral $px$, which underlies demand for $x$, being priced in the same market.
The speculator will sell (buy) futures if he expects the price to fall (rise) when the slope of the demand function is positive. The demand function has a negative slope when purchasing power is declining from higher futures prices or when collateral is devalued and the speculator must liquidate part of his position to maintain the margin requirement.

At each $t$, the variation margin is calculated and net wealth is adjusted. The mid-price $p^m$ is the average of the bid quote $p^b$ and ask quote $p^a$:

$$p^m = (p^a + p^b) / 2$$

The profit or loss is calculated with price changes of the mid-price and paid from the losing agent to the winning agent via the exchange clearing house, equivalent to

$$(p_t^m - p_{t-1}^m)x_t^i$$

Each speculator estimates his net wealth at each $t$ given prices $(p^a, p^b, p^m)$ which determines their decision on how many futures contracts to buy or sell to maximize expected wealth, while at the same time meeting his margin requirement — a position, $p_t^i x_t$, that is less not more than net wealth multiplied by $\kappa$. The mid-price is used in accounting for net wealth every period, as long as a position is held.\(^\text{13}\)

\(^\text{13}\) After the initial purchase of a market order the trader must pay a variation margin of $(p_t^m - p_{t-1}^m)(x_t^i - x_{t-1}^i)$. Important in this calculation of variation margin is that we keep the distinction between those that profit by buying at the bid or selling at the ask, versus those who are considered impatient and sell at the ask or buy at the bid. When a contract is bought and $(x_t^i - x_{t-1}^i) > 0$, if it is bought at the bid with a limit order then the variation margin is positive $(p_t^m - p_t^a) > 0$. If however it is bought at the ask with a market order then the variation margin is negative $(p_t^m - p_t^b) < 0$. This results in a transfer of wealth from the trader who is willing to pay for immediacy to the trader who gets paid for providing liquidity and making the market. The maximization
Speculator is short futures $x_{t-1} = -60$

Speculator is long futures $x_{t-1} = +60$

FIGURE 2 A speculator’s demand curve with either a short or long starting position: $m_{t-1} = 5000$, $p^\theta = 150$, and $\kappa = 2$ for each graph

3.3 The Bidding and Trading Process

Within the CDA, speculators and scalpers (if included) are selected randomly for a sequence of bilateral trading with non-replacement in each round, so that each trader has an equal chance of trading and trades every round. The hedgers are placed last in this sequence, which represents one round. The intraday period of futures trading has several rounds of quoting or transacting, at the bid or ask price. Quantities traded and their transaction prices are registered at each time $t$.

Central to the trading process is the auction that simulates the open-outcry on the floor of an exchange, leading to transactions and thus transaction prices. It is a groping mechanism where both bid and ask prices adjust and where out-of-equilibrium trades take place when an agent agrees to sell contracts (hit the bid) to another agent who is bidding for them, or when another agent decides to buy contracts (lift the ask) from the agent who is asking for them. This process of quoting and trading is repeated many times, giving each market participant the chance to quote and trade several times and fill his orders. No new information is brought into this process; expectations remain constant.

The competitive bidding algorithm presented here for the ZP speculators is drawn from several sources. The manner in which speculators compete and how their price expectations interact with the bid-ask spread during the bidding process comes from Chan et al. (1998) and Yang (2002). An important modification to their model, apart from keeping expectations constant, is the presence of risk-neutral speculators with collateral constraints and transaction costs. Hedgers act similarly to speculators but only place market orders to fill expectations and do have constraining margin requirements. Hence, hedgers do not compete in the bid-ask spread. Another algorithm, derived from Silber (1984), is presented for our scalpers, emphasizing inventory control and noncompetitive behavior.

The calculation of expected wealth by the speculator only takes into account the expected change in the trade price $\left( p^\theta - p_t \right)$ without anticipating whether the transaction is by market order or limit order.
This asynchronous bilateral bidding process allows two or three traders to participate at any one time: offering, or bettering, limit order quotations or carrying out market order trades. Agents take turns in entering into the inter-dealer market to quote price and quantity, to transact, or to exit. A round is completed when all agents have participated once, with the hedgers coming last. This is repeated for a different random sequence of scalpers and speculators for more than 50 rounds. The repetition or trading rounds represents competition within the price mechanism and helps the convergence to equilibrium of market demand and supply. This bidding and bilateral trading process is detailed next.

3.4 Auction Algorithm for a ZP Speculator

ZP speculator \(i\)’s reserve price is his expected price, \(p_{i}^{\theta}\), plus the one-way transaction tax \(\pi p_{i}\). Half of the bid-ask spread is often thought of as a measure of the cost of executing a market order (the difference between the mid-point price and the payment price). We shall represent this price difference by the lowercase letter \(s\). The size of this half spread is actually endogenous to the bilateral trading process.

At times when there is no bid or ask, a speculator will announce his own noncompetitive limit order and increase the half spread on the basis of expectations \((1 \pm S \pi) p_{i}^{\theta}\). In this case, \(S\) is a percentage of the transaction fee. If \(S\) is greater than 100%, then the new limit order will guarantee that a new hit or bid occurs with a demand different from zero.

We present the trading algorithm with three traders: inter-dealers \(k\) and \(j\) (which could be a speculator or scalper) and a new ZP speculator, entrant \(i\). In this presentation, agent \(i\) represents a speculator who demands immediacy and will always prefer to trade with a market order rather than a limit order if possible. ZP speculator \(i\) enters the market and witnesses the current bid:ask \((p_{a}^{b}: p_{a}^{b})\) spread and makes a trade choice under the following four scenarios.

- **Scenario 1.** (Figure 3a) The ask, \(p_{i}^{i,a}\) and bid, \(p_{i}^{i,b}\), currently exist with non-zero offers, at time \(t\).

  1. If \(p_{i}^{i,a} > p_{i}^{i,a}\) speculator \(i\) will post a market order and buy at this ask price — lift the ask quote.

  2. If \(p_{i}^{i,a} < p_{i}^{i,b}\), speculator \(i\) will post a market order and sell at this bid price — hit the bid quote.

  3. If \(p_{i}^{i,b} \leq p_{i}^{i,a} \leq p_{i}^{i,a}\) and \(<\left(p_{i}^{i,b} + p_{i}^{i,a}\right)/2\), speculator \(i\) will post a sell limit order at a price of \((1 + S \pi) p_{i}^{\theta}\) and thus quote his own ask, replacing agent \(j\)

  4. If \(p_{i}^{i,b} \leq p_{i}^{i,a} \leq p_{i}^{i,a}\) and \(\geq\left(p_{i}^{i,b} + p_{i}^{i,a}\right)/2\), speculator \(i\) will post a buy limit order at a price of \((1 + S \pi) p_{i}^{\theta}\), and thus quote his own bid, replacing agent \(k\)
 FIGURE 3a  Scenario 1, in which both competitive quotes — bid and ask — exist in the marketplace prior to new entrant

- Scenario 2. (Figure 3b) Only the best ask, $p_i^{j,a}$, exists; that is, at $p_i^{k,b}$ demand to go long is zero as $(x_i^k - x_i^{k-1}) \leq 0$.

1. If, $p_i^{j,a} > p_i^{j,a}$, speculator $i$ will post a market order, buy at this ask price.

2. If $p_i^{j,a} \leq p_i^{j,a}$, speculator $i$ will post a buy limit order $p_i^{j,b}$ at a price of $(1 - S \varpi) p_i^{j,a}$, but only if excess demand at this price is $(x_i^j - x_i^{j-1}) > 0$

- Scenario 3. (Figure 3b) Only the best bid, $p_i^{k,b}$, exists; that is, at $p_i^{j,a}$ demand to go short is zero as $(x_i^j - x_i^{j-1}) \geq 0$

1. If $p_i^{j,a} < p_i^{k,b}$, speculator $i$ will post a market order and sell at this bid price;

2. If $p_i^{j,a} \geq p_i^{k,b}$, speculator $i$ will post a sell limit order $p_i^{j,a}$ at a price of $(1 + S \varpi) p_i^{j,a}$, but only if excess demand at this price is $(x_i^j - x_i^{j-1}) < 0$
i will post market order
to buy - lift ask quote

i will post limit order
Bid (1- Sσ)p^{i,θ}

j ask p^a

k buy p^b
but x^k = 0

Scenario 2

Scenario 3

i will post market order
to sell - hit bid quote

j ask p^a
but x^j = 0

k buy p^b

Inside Quotes

FIGURE 3b Scenario 2 in which an ask but no bid exists prior to new entrant and Scenario 3, in which a bid but no ask exists prior to new entrant

- Scenario 4. If no bid or ask effectively exists; that is at the ask quote p^{i,θ}_{t,a}, (x^j_t - x^j_{t-1}) ≥ 0, and at the bid quote p^{k,θ}_{t,b}, (x^k_t - x^k_{t-1}) ≤ 0

1. The new entrant speculator will post both a buy and a sell limit order at (1- Sσ)p^{i,θ}_{t,a} and/or (1+ Sσ)p^{k,θ}_{t,b} respectively, as long as his bid is quoted for a buy of greater-than-zero contracts, and the ask is to sell greater-than-zero contracts. If this is not the case then the current bid-ask remains, even though both traders have zero demand, and entrant i exits to join the queue to trade again later.

In this model, under Scenario 2 (Scenario 3) the speculator tendering the best bid (ask) might have had prices move against him; for example, if he is long (short) and prices fell (rose). They may remain offering a bid (ask) price to buy (sell), but at a quantity of zero. Now he wants to offset his position and sell (buy) so that excess demand is less (greater) than zero.

Scenario 2: (x^j_t [p^{k,b}_{t,b} - x^k_{t-1}] ≤ 0 where x^j_t is a function of p^{k,b}_{t,b}

Scenario 3: (x^j_t [p^{i,a}_{t,a} - x^j_{t-1}] ≥ 0 where x^j_t is a function of p^{i,a}_{t,a}

Effectively under Scenario 2 (Scenario 3) agent k (agent j) falls silent and will eventually be replaced by a new entrant, as long as the new entrant has p^{i,θ} < p^{i,θ}_{t,a} (has p^{i,θ} > p^{i,θ}_{t,b}) and as long as [x^j_t (1 + Sσ)p^{i,θ}_{t,a} - x^j_{t-1}] > 0 (as long as [x^j_t (1 - Sσ)p^{i,θ}_{t,b} - x^j_{t-1}] < 0) otherwise agent k (agent j) will remain. Only when agent k (agent j) is replaced and exits the market will he be given the chance to satisfy margin requirements by liquidating their position with a market order, in turn, in the random trading round.
This model considerably changes the Chan et al (Ibid) rules, which emphasize the manner in which price formation feeds back into the market by agents updating their expectations, to one where price formation feeds back into the market via quantity constraints, margin requirements, and inventory control. The model allows for leveraged trading and short selling and makes the method of settlement a central variable of the model.

3.5 Auction Algorithm for a Scalper

Scalpers will try to charge as high a price as possible when selling and as low a price as possible when buying, while still competing with other traders to make a sale or purchase. Only the highest bid and lowest ask are heard in the trading pit. All other noncompetitive quotes must remain silent. Since speculators must compete on price; only speculators willing to narrow the inside-market bid-ask spread can quote such limit orders. Scalpers balance market order flow by using the inter-dealer market to offset their own inventory excesses. Taking a loss in order to liquidate an unbalanced inventory position may force other inter-dealer scalpers to also liquidate, and this dries up liquidity in the market until prices are modified to equate demand and supply.

The scalper algorithm we use is a simplified version of one stated in Smidt (1985). The objective is to buy low at the bid and sell high at the ask, maximizing a profit equal to the turnaround of inventory multiplied by the spread. At the same time, the scalper will minimize inventory risk with a very simple inventory control mechanism. There is a maximum net inventory ceiling $K$ for each scalper. Netting out the long and short trades by a single agent consolidates the inventory $x_t$. Scalper inventory is kept below a position limit $K$:

$$-K \leq x^n_t \leq K \quad \text{for scalper } n$$

In actual markets $K$ maybe as small as one contract and could be different for different scalpers. In this model all scalpers have $K=10$. When a scalper enters the trading floor from the random sequence, if his inventory is less than his maximum limit $K$, he always has the right to replace any agent in the inter-dealer market by simply matching the agent’s quoted bid and ask. This is in contrast to speculators who must offer a better price to replace the agents in the inter-dealer market. If, however, the scalper’s inventory is on his limit, then the scalper will place a market order to offload all inventory, if possible. The scalper algorithm is one of simple inventory control:

- New entrant scalper $n$:
  1. If $-K < x^n_t < K$, replace current market makers and quote both bid and ask at the current quotations $p_{t}^{k,b}$ and $p_{t}^{i,a}$.
  2. If short and $x^n_t \leq -K$ hit the market bid for a maximum $-x^n_t$, and post no quotes
  3. If long and $x^n_t \geq K$ lift the market ask for a maximum $-x^n_t$, and post no quotes
The dealer inventory control model outlined here, where a scalper will choose to make a market order rather than change his limit order prices, is in contrast to the more commonly accepted inventory control models such as Garman (1976) and Amihud and Mendelson (1980). These authors present dealers as changing their bid and ask quotations to induce an imbalance of incoming orders, in order to reduce inventory. Hasbrouck (2003) questions this latter model and claims that as a general rule, most empirical analyses of inventory control refute this method of changing the quote for inventory control. He argues that a dealer who would pursue this price adjustment mechanism would be signaling to the world at large his desire to buy or sell. This would put him at a competitive disadvantage (Ibid 2003, p. 78). This simplified mechanism does not touch on information signaling, yet it does avoid this specific criticism.

3.6 Auction Algorithm for a Representative Hedger

Hedgers are only concerned about filling their expected sales or purchases at the spot date via market orders in futures. They always come last in each round of the random sequence of speculators and scalpers.

- **Hedger Scenario:**
  1. The future purchaser of the commodity at spot, agent \( q \), will lift the ask, \( p_t^{ja} \), for maximum ask quote quantity, in each round until market buy order is filled \( x_t^q = x^{aq} \)
  2. The futures seller of the commodity at spot, agent \( r \), will hit the bid, \( p_t^{kb} \), for the maximum bid quote quantity in each round until market sell order is filled \( x_t^r = x^{kr} \)

Since speculators and scalpers do not usually offer large size limit order contract lots, it may take several rounds for each hedger to finalize their purchases or sales. The hedgers contribute so called *fundamentals* to the speculative market.

3.7 The Trading Sequence

In the simulations which follow, trading begins with a random ordering of 20 speculative agents all with $10,000 cash and, when included, 10 scalpers with zero cash. The two representative hedgers come last in this sequence, which, once completed, is called a trading round. Speculators have equal endowments and the heterogeneous expectations are taken from a uniform distribution \( p^q \sim U(100,130) \). Speculators come together, along with hedgers and scalpers, for a length of 500 bilateral trades and prices. The quantity traded is the lesser of the market and limit order demands that are crossed in the CDA.

Two randomly selected traders (speculator or scalper) begin with initial random market quotes, set 10 points apart, e.g \( p_0^b = 120: p_0^a = 130 \). A new entrant, randomly selected from the remaining traders (but not a hedger), enters the floor to either accept or better the prices quoted.
If a bid or ask is accepted, a trade is done and a transaction price $p_i$ occurs for the market order by the new entrant. If, instead, the entrant replaces a bid or ask or both, then a new set of quotations $p_i^b : p_i^a$ (bid:ask) is created, with no transaction price. A sequence of quotes, and transaction prices, is generated as each agent enters the market during the trading round, with only transaction prices and volumes registered. Repeating the round, drawing a new random sequence of speculators and scalpers each time, creates an inter-day trading session. This trading sequence is summarized here:

1. Speculators are initialized with initial wealth and random price expectations. Two randomly selected speculators or scalpers begin with initial quotes of $p_0^b : p_0^a$ and their respective buy and sell quantities (which may be zero), given their expectations.

2. The random sequence of speculators and scalpers to enter the market with non-replacement is determined, with hedgers coming last.

3. With one or two agents quoting a bid-ask spread, the new entrant can either submit a new bid or ask, accept the existing bid or ask, or hold (pass).

4. A transaction occurs when the existing bid or ask orders are accepted and the transaction price is recorded accordingly. The transaction is the minimum of the quantities proposed for exchange by each bilateral trader.

5. At each point, mid-point prices are used to calculate speculator budget constraints in real time. On the basis of the past transaction price, each agent’s wealth is updated, taking account of all margin calls (profits and losses).

6. Steps 3 through 5 are repeated for $n$ times, $n =$ number of traders (one round).

7. Steps 2 through 6 are repeated for $N$ times, $N =$ number of rounds.

8. Final market price is recorded as the 500th transaction price for this trading session.
4 SIMULATIONS CDA TRADING

Six markets are simulated: the first 3 are without scalpers (markets 1, 2 and 3). All markets have 20 ZP speculative agents starting with $10,000 cash, and two representative hedgers. In 4 markets the ex ante net hedge is set to zero, and in the other 2 markets it is set to 5000 which means that the hedgers will be 5000 futures long by the time they stop trading.

Trading begins with a random ordering of speculators (and scalpers if included) in each trading round, the hedgers enter with market orders at the end of this round. Each round has an unspecified number of quotes and continues until $t$ bilateral trades and transaction prices are produced at which time trading ends. In all markets $t$ is set to 500, and there are anywhere from 1000 to 2000 trading rounds. The quantity traded is the lesser of the market and limit orders that are crossed (hit or lifted) in the CDA.

The 20 ZP speculators have expectations taken from a uniform distribution $p^\theta \sim U(100,130)$ in markets 1 and 4, and a binomial distribution in markets 2, 3, 5 and 6, $p^\theta \sim \{U(100,110), U(120,130)\}$. The frequency distributions of these two ZP speculator expectation sets are presented in Figure 4. By comparing two different sets of expectation dispersion we hope to see what impact this may have on the bid-ask spread.

![Figure 4](image)

**FIGURE 4** Two different distributions of the ZP speculator expectations.

The aggregate demand curves and transaction and equilibrium price series for a single run, and the average of several runs and their standard deviation for several different ordering sequences, of the same traders and their same expectations, into the CDA are presented in figures 5 and 6. Markets 1, 2 and 3 are in the respective columns of Table 1, and have no scalpers. Where as Markets 4, 5 and 6 are in the respective columns of Table 2, and have 10 scalpers included in the trading with an inventory capacity of $K=\pm 10$. The last two columns of each figure has a binomial distribution of expectations and the very last column also has a net hedge of 5000 futures contracts.

The order of ZP entry into the market makes quite a big difference to the price path. The Walrasian equilibrium, is defined as the dark (blue) flattest line in the tables in row 3. Where the Walrasian equilibrium does move about, it is almost always due to the presence of multiple equilibrium, and the jumping from one equilibrium to the other.
Continuing price volatility in these markets is due to liquidity constraints and it is significantly different from volatility that comes from exogenous information shocks or adaptive expectations. There are considerable price and quantity feedbacks. While for an individual high-risk speculative trader, marking to market is a cautionary act and reduces counterparty risk, it can also result in a volatile market price which may be greater the higher the settlement frequency is (Farmer et al. 2004). If traders are on their budget constraints, then they will liquidate some of their position when prices move against them in order to stay within their margin requirements. This creates backward-bending demand functions, as introduced in Section 3.2 and can lead to spikes or large shifts in the equilibrium price series.

First we consider the visual price convergence of the CDA trades to a theoretical Walrasian equilibrium price for 6 different markets. The uniform distributions of reservation prices for the ZP speculators led to a unique equilibrium which was stable over the period in the market without scalpers (market 1) and had a tendency to drift over the market with scalpers (market 3). In the case where reservation prices were binomial the theoretical Walrasian price was quite often a multiple equilibrium. Dominated by 2 stable equilibrium points either side of the unstable equilibrium when the net hedge was zero, and a often a single stable and unstable fixed point when the net hedge was equal to 5000. Despite the multiple equilibriums, in both cases the CDA converged to the stable point. It took longer to converge in the case of the 5000 net hedge.

When scalpers were added to these 3 markets, the CDA produced a price series that had a larger standard deviation, but a smaller amount of kurtosis. The theoretical prices were more stable with the presence of scalpers in the binomial case (market 2 versus market 4). In the market with a net hedge of 5000 the prices converged at a much quicker rate to the much higher price. While this price was not typically sustained, often moving back down closer to speculator expectations, it was a constant gravitation point for the CDA prices. This shows that the Cliff and Bruten critique (1997) that prices in the ZI model will approach expected prices rather than the Walrasian equilibrium is inaccurate. It is true that expectations and their distribution do impact prices, but gravitation is to the Walrasian equilibrium even when there is zero intelligence.

The extent that bilateral prices converge to the Walrasian equilibrium or pareto optimum can also be a measure of the efficient allocation of our market. Drawing from Smith (1962)\textsuperscript{14} we take our \( t \) transactions prices \( p_j : j = 1 \ldots t \), and formulate a price convergence measure
\[
\alpha = \frac{100\sigma}{\bar{P}}
\]
where
\[
\sigma = \sqrt{\frac{1}{t} \sum_{j=1}^{t} (p_j - \bar{p})^2}
\]

We calculate \( \alpha \) for each individual run, presented in table 3. The closer \( \alpha \) is to zero the closer the CDA price series is to the price that equates supply and demand for each point on average over

\textsuperscript{14} Smith (1962) created a price convergence measure, \( \alpha = 100 \sigma_0 / P_0 \) where \( P_0 \) is the static theoretical equilibrium price given by the intersection of the aggregate supply and demand curves, and \( \sigma_0 = \sqrt{\frac{1}{t} \sum_{j=1}^{t} (p_j - p_0)^2} \). Since the model here has a dynamic concept of wealth the theoretical Walrasian equilibrium is also evolving over time.
the period $t=500$. This average root mean squared difference between the CDA actual and equilibrium prices might also be thought of as a measure of price resiliency in the market.

<table>
<thead>
<tr>
<th>A</th>
<th>Market 1: U</th>
<th>Market 2: Bi</th>
<th>Market: 3 Bi, NH5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>No scalpers</td>
<td>5.56</td>
<td>6.44</td>
<td>6.89</td>
</tr>
<tr>
<td>10 scalpers</td>
<td>6.46</td>
<td>8.37</td>
<td>9.33</td>
</tr>
</tbody>
</table>

**TABLE 3** Average root mean squared difference between CDA actual and equilibrium prices

In all CDA simulations remain around the stable equilibrium point\(^{15}\), even though there are potentially destabilizing equilibrium points. While the CDA prices Prices do not always immediately jump to this equilibrium although they appear faster to converge under the binomial distribution with no scalpers and a zero sum position than the meandering of the uniform distribution of traders. However in table 3 the lower scores of average convergence are for the uniform distribution markets with no net hedge. In these markets, once speculators have used up their resources and are constrained by their budgets then the uniform distribution of reservation prices for the ZP traders leads to a smaller bid ask spread and smaller big jumps in a liquidity crisis. This spread is much smaller than the bid ask spread offered in the scalper market.

\(^{15}\) Which could not be said for the selection of the fixedpoint using Newton’s Method: `FindRoot` in the Mathematica software.
APPENDIX

The risk-neutral speculator maximizes next period's expected wealth (1). The first four boundary constraints represent the limit on a speculator's investment by the margin requirement when one is short in futures, (2) and (3), versus the extent to which futures can be bought long, (4) and (5). We have two each of these restrictions to take into account the one-way tax on both buys and sells $\sigma p_t \mid (x_t - x_{t-1}) \mid$ for speculator $i$. If the transaction tax is positive then this boundary constraint will be slack. This dual tax restriction also impacts the budget constraint, (6) and (7). The bankruptcy conditions, (8) through (10), stop money wealth from going below zero.

For ZP speculator $i$:

Maximize:

$$\pi_{i,t}^* = (p^d - p_t)x_t + m_t$$

Subject to:

$$p_t x_t \geq -\kappa[(p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} - \sigma p_t(x_t - x_{t-1})]$$

(2)

$$p_t x_t \geq -\kappa[(p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} + \sigma p_t(x_t - x_{t-1})]$$

(3)

$$p_t x_t \geq \kappa[(p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} - \sigma p_t(x_t - x_{t-1})]$$

(4)

$$p_t x_t \geq \kappa[(p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} + \sigma p_t(x_t - x_{t-1})]$$

(5)

$$m_t \leq (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} - \sigma p_t(x_t - x_{t-1})$$

(6)

$$m_t \leq (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} + \sigma p_t(x_t - x_{t-1})$$

(7)

$$0 \leq (p_t^m - p_{t-1}^m)x_{t-1} + m_{t-1} - \sigma p_t(x_t - x_{t-1})$$

(8)

$$0 \leq (p_t^m - p_{t-1}^m)x_{t-1} + (p_t^m - p_{t-1})(x_t - x_{t-1}) + m_{t-1} + \sigma p_t(x_t - x_{t-1})$$

(9)

$$m_t \geq 0$$

(10)
REFERENCES


Table 1:

<table>
<thead>
<tr>
<th>Uniform Distribution Price Expectations</th>
<th>Binomial distribution</th>
<th>Binomial distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 speculators pe from U~(110,150)</td>
<td>10 speculators pe from U~(100,110)</td>
<td>10 speculators pe from U~(100,110)</td>
</tr>
<tr>
<td>net hedge demand = zero</td>
<td>10 speculators pe from U~(120,130)</td>
<td>10 speculators pe from U~(120,130)</td>
</tr>
<tr>
<td></td>
<td>Net hedger demand = 0</td>
<td>Net hedger demand = +5000</td>
</tr>
</tbody>
</table>

Example Run (papb = 120:130)

```
transaction price $\kappa=4$
```

```
transaction price $\kappa=4$
```

```
transaction price $\kappa=4$
```
<table>
<thead>
<tr>
<th>Uniform Distribution Price Expectations</th>
<th>Binomial distribution</th>
<th>Binomial distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net hedge excess demand = zero</td>
<td>10 speculators pe from U~(100,110)</td>
<td>10 speculators pe from U~(100,110)</td>
</tr>
<tr>
<td>20 speculators pe from U~(110,150)</td>
<td>10 speculators pe from U~(120,130)</td>
<td>10 speculators pe from U~(120,130)</td>
</tr>
<tr>
<td>Plus 10 scalpers K = 4</td>
<td>Net hedger excess demand = 0</td>
<td>Net hedger excess demand +5000</td>
</tr>
</tbody>
</table>

**Example Run**: \(\frac{p_a}{p_b} = 120:130\)