The use of a premium-payment scheme in a supply chain involving capacity acquisition

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Abstract

We study a practice whereby a downstream firm makes to his supplier a premium-payment for a certain quantity of products. We show that the adoption of this practice can induce the supplier to build bigger capacity. The higher capacity level enables the supplier to satisfy a larger portion of demands from the downstream firm, and this leads to higher payoffs for both parties in the supply chain. With the assistance of an under-capacity penalty imposed on the supplier, this premium-payment scheme can help lure the parties into taking the channel-optimal actions. Our numerical examples help reveal various features of the scheme.

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1. Introduction

In some industries, especially in the telecommunications and information technology industries, products are often ephemeral and market conditions highly cyclical. At the same time, downstream firms (e.g., OEMs) in these industries rely heavily on outside suppliers (contract manufacturers) for the production of key components or even complete sales items, while they themselves keep minimal levels of inventory. The suppliers, on the other hand, need to make commitments in capacity acquisition (including raw material purchasing, work force training) prior to receiving firm orders from the downstream firms. An imminent problem in this environment is that, fearing being later burdened with wasteful excess capacities, suppliers are usually reluctant to commit to capacity levels that are most desired by the downstream firms, and therefore make the latter suffer from unnecessary lost sales opportunities.

This problem should be partially remedied if the downstream firms could demonstrate their willingness to share the downside risk early on. Indeed, there has emerged in practice a measure used by a downstream firm...
to secure enough capacity for himself at his supplier. Under this measure, the firm promises to pay or pays in advance to the supplier a higher wholesale price than the otherwise regular payment for a certain quantity of the involved product, and shows the willingness to pay a certain penalty when the realized demand level is below the quantity. The firm makes the premium-payment in hope of swaying the supplier’s capacity acquisition and capacity allocation decisions to his own advantage. Although the measure has been used in practice, its exact impact has not been fully understood, nor has the best way to implement it been determined (Dr. J. Ouyang, Supply Chain Network Director of Lucent Technologies (China) Co., Ltd., personal communication).

In this paper, we propose an exact implementation for the premium-payment scheme. Then, in a simple setting involving one firm (hereafter the manufacturer) and one supplier between whom there has been an existing linear payment scheme (fixed quantity-insensitive wholesale price), we show the relative advantage of employing this mechanism. When the mechanism is in place, as long as the loss of sales is costly enough to the manufacturer, the manufacturer will be willing to pay a premium for a positive quantity of products to the supplier, who will in turn set up her capacity to a level higher than she otherwise would without the payment. In so doing, the manufacturer will more than recover his higher payment to the supplier from the reduction in lost sales, while the supplier will more than recover her extra capacity acquisition cost from increased monetary transfer from the manufacturer.

The most salient advantage of this premium-payment type of contract over other types of contracts is its simplicity and ease of use: there is no extra cash or product transfer besides the necessary one-shot cash payment and product delivery, as opposed to what is required in reservation or other types of contracts that target the same problem of reducing the risk borne by the supplier in her capacity acquisition activity. Furthermore, in the simple and yet reasonable setting in which this contract is studied in the paper, the optimal premium-payment quantity promised by the manufacturer can be computed using a simple newsvendor-type formula, and the supplier’s optimal decisions on capacity commitment and product delivery are also extremely simple. Finally, the manufacturer should find our essentially concave payment schedule more acceptable since his “hunger” for additional product units diminishes over the quantity of product units he has already received.

Much research has been done on supply chain competition and coordination. Pasternack [12] introduced a simple buyback contract which allows a self-interested pair of supplier and manufacturer to behave optimally for the entire chain, thus achieving the so-called channel coordination. Bernstein and Federgruen [2] showed how channel coordination can be achieved in the face of price-sensitive demands and multiple manufacturers through the adoption of manufacturer-and-price-dependent price-discounting-and-buyback contracts. Cachon [3] and Lariviere [11] gave reviews on supplier-manufacturer coordination. The former focused on a multi-period setting where both parties employ base stock policies and the latter emphasized detailed analysis in the newsboy setting. Particularly, the latter presented a model involving a supplier’s different responses to a manufacturer’s payment offers; however, the model does not consider the supplier’s capacity acquisition decision. Anupindi and Bassok [1] reviewed work done on minimum quantity and dollar volume commitment contracts, mostly in a multi-period environment. Tsay et al. [15] offered a comprehensive survey of research on various types of supply chain contracts, which touched upon contracting under limited production capacity. In the face of price-sensitive demands, Corbett and Tang [8] conducted a comparative study on various contract types under different information availabilities.

There exists as well a body of literature on supply-chain competition involving limited production capacity. Serel et al. [13] examined a multi-period problem involving a manufacturer, a designated supplier, and a spot market. The manufacturer follows a certain inventory policy, and in every period, he pays a premium to receive a guaranteed level of delivery from the designated supplier, and at the same time, orders whatever amount he still needs as specified by his policy from the spot market. Assuming that the supplier follows an approximate base stock policy, the authors derived analytical results with regard to both the designated supplier’s and manufacturer’s decisions. Cachon and Lariviere [4] examined the situation where a supplier uses past sales data to allocate her limited capacity while manufacturers have to make ordering decisions to compete for the capacity. Cachon and Lariviere [5,6] studied problems where there are multiple manufacturers who have private demand information, the supplier’s capacity allocation rule is announced beforehand, and manufacturers may order more than what are demanded from them to compete for the limited capacity.
The emphasis of the studies was on finding capacity allocation rules that induce manufacturers to order truthfully.

Van Mieghem and Dada [16] considered a situation similar to ours, where the supplier needs to acquire capacity prior to the observation of the random demand level. However, their emphasis was on a comparative analysis of different production and pricing postponement strategies. Erkoc and Wu [9] considered the problem of a buyer using advanced reservation to secure capacity from a supplier with a convex capacity cost. The fee used in the reservation is deductible in the sense that the buyer only needs to pay the difference between the wholesale price and the unit reservation fee for any unit covered by the reservation. When the realized demand level is less than the reserved amount, the overpaid reservation fee is either not refunded or partially refunded. For both cases, the authors found that, under reasonable conditions, entering the reservation contract can benefit both the supplier and buyer. Certain partially refundable reservation contracts can reach complete channel coordination. Jin and Wu [10] studied similar deductible reservation contracts in which the supplier decides not only the unit reservation fee but also a certain (guaranteed) capacity level that is unaffected by the manufacturers’ capacity reservation decisions. They showed that channel coordination can be achieved through this type of contracts, and that under such a contract, a unique Nash equilibrium exists for manufacturers’ reservation quantities when there are two competing manufacturers.

Cachon and Lariviere [7] also studied a problem in which the supplier has to decide the amount of capacity to build and the manufacturer offers the supplier a contract. The type of contract being studied there consists of both firm commitments and options. The authors explored cases with and without demand forecast sharing, and cases of forced and voluntary supplier compliance. Our paper is close to Tomlin [14] as well, since it was also concerned with the situation where a manufacturer takes the initiative by offering a premium-payment to a supplier with the purpose of securing more capacity from that supplier. In Tomlin’s paper, the main premium-payment scheme under scrutiny is opposite to ours in that the marginal payment increases with the manufacturer’s order quantity.

The rest of this paper is organized as follows. We present the mathematical model of the problem in Section 2, carry out in Section 3 the analysis of the problem under determined degrees of premium-payment and an under-order penalty to be introduced later, shed light on the determination of the aforementioned degrees in Section 4, show the way to reach channel optimality in Section 5, discuss general concave payment schemes in Section 6, and make concluding remarks in Section 7.

2. Model formulation

This problem involves one supplier, one manufacturer, and one product type. The manufacturer performs no production and carries no inventory. He buys from the supplier and satisfies customer demands in the make-to-order fashion. The following are a few relevant parameters:

\( \tilde{c} \in (0, +\infty) \): the regular payment the manufacturer would make to the supplier for a unit item (determined by involved parties \textit{ex ante});
\( \tilde{z} \in (0, 1) \): the fraction of \( \tilde{c} \) made up by the cost incurred to the supplier for the production of a unit item;
\( \tilde{\gamma} \in (0, 1 - \tilde{z}) \): the fraction of \( \tilde{c} \) made up by the cost incurred to the supplier of a unit-capacity acquisition;
\( \tilde{\psi} \in (0, +\infty) \): the profit margin desired by the manufacturer—he asks \((1 + \tilde{\psi})\tilde{c}\) for a unit item from his customers;
\( \tilde{\lambda} \in (0, +\infty) \): the fraction of \( \tilde{c} \) made up by the cost incurred to the manufacturer of loss of goodwill due to every unit of unsatisfied customer demand.

For convenience, we define \( \tilde{\nu} = \tilde{\psi} + \tilde{\lambda} \) to be the manufacturer’s profit-margin-plus-goodwill factor. Also, we suppose that the supplier and the manufacturer have agreed \textit{ex ante} upon two positive constants, the degrees of premium-payment \( \theta \) and under-order penalty \( \omega \). In essence, \( \theta \tilde{c} \) is the extra unit premium paid by the manufacturer to the supplier for a certain quantity of products to be specified by him, while \( \omega \tilde{c} \) is the penalty for every unit less than this quantity that the manufacturer eventually orders from the supplier. The supplier has no production capability in the beginning.
The supplier and the manufacturer play a two-stage game. In stage 1, the demand level for the manufacturer is merely a random variable \( U \). The probability density and cumulative distribution of \( U \) follow functions \( f(u) \) and \( F(u) \), respectively, where in the range where \( 0 < F(u) < 1 \), \( f(u) \) is strictly positive and continuous and \( F(u) \) is strictly increasing and continuously differentiable. In this stage, the manufacturer decides the premium-payment level \( x \), the quantity for which he is willing to pay the higher-than-regular cost, and pays the supplier a premium-payment \((1 + \theta)\tilde{c} \cdot x \). The supplier then builds up her capacity to a level \( w \) at the cost of \( \gamma \tilde{c} \cdot w \). In stage 2, the demand level to the manufacturer is realized as \( u \). This information is immediately known to every party. Having learned the manufacturer’s demand level, the supplier decides the level of delivery \( y \) to the manufacturer, which costs her \( \tilde{c} \cdot y \) in production. The delivery level \( y \) is never above the demanded level \( u \). If the delivery level \( y \) is higher than the equivalent premium-payment level \( x \), the manufacturer will pay the regular cost of \( \tilde{c} \cdot (y - x) \) on top of the premium-payment; while in the opposite case, the supplier will refund the manufacturer a \((1 + \theta)\tilde{c} \cdot (x - y) - \omega \tilde{c} \cdot (x - u)^+ \) amount. In total, the manufacturer ends up paying the supplier \( t_{MS}(u,y,x) \), which is given by

\[
t_{MS}(u,y,x) = \theta \tilde{c} \cdot \min\{x,y\} + \tilde{c} \cdot y + \omega \tilde{c} \cdot (x - u)^+.
\]

(1)

The supplier’s second-stage profit \( \pi_S(u,y,x) \), when the manufacturer has offered a premium-payment level \( x \) and the supplier herself makes a delivery level \( y \) under the demand level \( u \), satisfies

\[
\pi_S(u,y,x) = t_{MS}(u,y,x) - \tilde{c} \cdot y,
\]

(2)

since the supplier is to receive \( t_{MS}(u,y,x) \) from the manufacturer and also is to be charged a \( \tilde{c} \cdot y \) production cost. In stage 2, having already known the premium-payment level \( x \) and the realized demand level \( u \), the supplier’s problem of deciding the optimal delivery level \( y^*_S(w,u,x) \) and achieving the optimal second-stage profit \( \pi^*_S(w,u,x) \) is

\[
\pi^*_S(w,u,x) = \max\{\pi_S(u,y,x) | 0 \leq y \leq \min\{w,u\}\}.
\]

(3)

In stage 1, when already given the premium-payment level \( x \), the supplier’s problem of deciding the optimal capacity level \( w^*_S(x) \) and achieving the optimal overall profit \( \pi^*_S(x) \) is

\[
\pi^*_S(x) = \max \{E[\pi^*_S(w,u,x)] - \gamma \tilde{c} \cdot w | w \geq 0\},
\]

(4)

since the capacity acquisition cost incurred to the supplier is \( \gamma \tilde{c} \cdot w \). In the same stage, knowing the optimal supplier decisions \( y^*_S(w,u,x) \) and \( w^*_S(x) \) in stages 2 and 1, respectively, the manufacturer can earn an average revenue \( r_M(x) \), which is given by

\[
r_M(x) = (1 + \psi)\tilde{c} \cdot E[\min\{U,y^*_S(w^*_S(x),U,x)\}]
\]

(5)

and has to pay on average the cost \( c_M(x) \), which can be expressed as

\[
c_M(x) = E[t_{MS}(U,y^*_S(w^*_S(x),U,x),x)] + \lambda \tilde{c} \cdot E[(U - y^*_S(w^*_S(x),U,x))^+],
\]

(6)

where in the above, the first term is the total payment the manufacturer has to make to the supplier and the second term captures his loss of goodwill. Therefore, the problem of deciding the manufacturer’s optimal premium-payment level \( x^*_M \) that achieves his optimal profit \( \pi^*_M \) can be written as

\[
\pi^*_M = \max\{r_M(x) - c_M(x) | x \geq 0\} = (1 + \tilde{\psi})\tilde{c} \cdot E[U] - \min\{z_M(x) | x \geq 0\}.
\]

(7)

In the above, we have let

\[
z_M(x) = E[t_{MS}(U,y^*_S(w^*_S(x),U,x),x)] + E[z_{MD}(U,y^*_S(w^*_S(x),U,x))],
\]

(8)

where \( z_{MD}(u,y) \) is the manufacturer’s effectual cost incurred by the demand and it is given by

\[
z_{MD}(u,y) = (1 + \bar{\psi})\tilde{c} \cdot (u - y)^+.
\]

(9)

3. Analysis at fixed \( \theta \) and \( \omega \)

After regrouping of terms, we have

\[
\pi_S(u,y,x) = (1 + 0 - \tilde{x})\tilde{c} \cdot y - \theta \tilde{c} \cdot (y - x)^+ + \omega \tilde{c} \cdot (x - u)^+.
\]

(10)
As a function of \( y, \) \( \pi_S(u, y, x) \) is positive, increasing, and two-piece concave, where the first piece is \([0, x]\) with an ascending rate of \((1 + \theta - \bar{z}) \bar{c}\) and the second piece is \([x, +\infty)\) with an ascending rate of \((1 - \bar{z}) \bar{c}\). Therefore, we have

\[
y_S^u(w, u, x) = \min \{w, u\} \tag{11}
\]

and

\[
\pi_S^u(w, u, x) = \begin{cases}
(1 + \theta - \bar{z}) \bar{c} \cdot w & \text{if } w \leq x \leq u, \\
\omega \bar{c} \cdot x + (1 + \theta - \bar{z}) \bar{c} \cdot w - \omega \bar{c} \cdot u & \text{if } w \leq u \leq x, \\
\theta \bar{c} \cdot x + (1 - \bar{z}) \bar{c} \cdot w & \text{if } x \leq w \leq u, \\
\theta \bar{c} \cdot x + (1 - \bar{z}) \bar{c} \cdot u & \text{if } x \leq u \leq w, \\
\omega \bar{c} \cdot x + (1 + \theta - \bar{z} - \omega) \bar{c} \cdot u & \text{if } u \leq \min \{x, w\}.
\end{cases} \tag{12}
\]

Hence,

\[
E[\pi_S^u(w, U, x)] = \begin{cases}
(1 + \theta - \bar{z}) \bar{c} \cdot E[\min \{w, U\}] + \omega \bar{c} \cdot E[(x - U)^+] & \text{if } 0 \leq w \leq x, \\
(1 + \theta - \bar{z}) \bar{c} \cdot \int_0^w (1 - F(u)) \, du + \omega \bar{c} \cdot \int_0^w F(u) \, du & \text{if } x \leq w \leq U, \\
\theta \bar{c} \cdot \int_0^w (1 - F(u)) \, du + (1 - \bar{z}) \bar{c} \cdot \int_0^w (1 - F(u)) \, du + \omega \bar{c} \cdot \int_0^w F(u) \, du & \text{if } w \geq U.
\end{cases} \tag{13}
\]

In the above and later, we shall use the fact that for any \( z \geq 0, \) \( E[\min \{z, U\}] = \int_0^z (1 - F(u)) \, du, \) \( E[(z - U)^+] = \int_z^\infty F(u) \, du, \) and \( E[(U - z)^+] = \int_z^\infty (1 - F(u)) \, du. \)

Since \( \pi_S(u, y, x) \) is increasing in \( x, \) \( \pi_S^u(w, u, x) \) is increasing in \( x \) by the nature of (3). So \( \pi_S^u(w, x) = E[\pi_S^u(w, U, x)] - \bar{c} \cdot \bar{w} \) is increasing in \( x. \) On the other hand, an optimal solution \( w_S^u(x) \) for (4) is also a feasible, but not necessarily optimal, solution for the same problem with \( x \) being replaced by \( x + \delta x \) for some positive \( \delta x. \) Therefore, we have

\[
\pi_S^u(x + \delta x) = \pi_S^u(x + \delta x, x + \delta x) \geq \pi_S^u(w_S^u(x), x + \delta x) \geq \pi_S^u(w_S^u(x), x) = \pi_S^u(x),
\]

where in the first inequality is due to the maximization nature of (4) and the second is due to the monotonicity of \( \pi_S^u(w, x) \) in \( x. \) Hence, \( \pi_S^u(x) \) is an increasing function of \( x. \)

**Proposition 1.** The supplier will welcome a higher premium-payment level from the manufacturer, even though she is potentially to be charged with a higher capacity acquisition cost.

Now, we have

\[
\partial E[\pi_S^u(w, U, x)] / \partial w = \begin{cases}
(1 + \theta - \bar{z}) \bar{c} \cdot (1 - F(w)) & \text{if } 0 < w < x, \\
(1 - \bar{z}) \bar{c} \cdot (1 - F(w)) & \text{if } w > x.
\end{cases} \tag{15}
\]

It is easy to see that \( E[\pi_S^u(w, u, x)] \) is concave in \( w. \) By the nature of (4), we see that \( w_S^u(x) = 0 \) when \( \partial^+ E[\pi_S^u(w, U, x)] / \partial w < \bar{c}, \) and otherwise it can be one of the solutions for \( \partial^+ E[\pi_S^u(w, U, x)] / \partial w \geq \bar{c} \geq \partial^+ E[\pi_S^u(w, U, x)] / \partial w. \)

Let

\[
\begin{align*}
\bar{x}^L &= F^{-1}((1 - \bar{z} - \bar{\gamma})/(1 - \bar{z})), \\
\bar{x}^U &= F^{-1}((1 + \theta - \bar{z} - \bar{\gamma})/(1 + \theta - \bar{z})).
\end{align*} \tag{16}
\]

We note that \( \bar{x}^U > \bar{x}^L > 0 \) just because \( \theta > 0 \) and \( 0 < \bar{\gamma} < 1 - \bar{z}. \) When \( 0 \leq x < \bar{x}^L, \) we have

\[
\partial E[\pi_S^u(w, U, x)] / \partial w > \bar{c} \text{ when } 0 < w < x \text{ or } x < w < \bar{x}^L, \text{ while}
\]

\[
\partial E[\pi_S^u(w, U, x)] / \partial w < \bar{c} \text{ when } w > \bar{x}^L.
\]
when $\bar{x}^L \leq x \leq \bar{x}^U$, we have
\[
\frac{\partial E[\pi_S^*(w, U, x)]}{\partial w} > \frac{\gamma c}{\gamma c} \text{ when } 0 < w < x, \text{ while }
\frac{\partial E[\pi_S^*(w, U, x)]}{\partial w} < \frac{\gamma c}{\gamma c} \text{ when } w > x;
\]
and when $x > \bar{x}^U$, we have
\[
\frac{\partial E[\pi_S^*(w, U, x)]}{\partial w} > \frac{\gamma c}{\gamma c} \text{ when } 0 < w < \bar{x}^U, \text{ while }
\frac{\partial E[\pi_S^*(w, U, x)]}{\partial w} < \frac{\gamma c}{\gamma c} \text{ when } \bar{x}^U < w < x \text{ or } w > x.
\]
Therefore, we should set
\[
w_S^*(x) = \begin{cases} 
\bar{x}^L & \text{if } 0 \leq x < \bar{x}^L, \\
x & \text{if } \bar{x}^L \leq x \leq \bar{x}^U, \\
\bar{x}^U & \text{if } x > \bar{x}^U. 
\end{cases} \tag{17}
\]
By plugging (17) into (13), we obtain that
\[
E[\pi_S^*(w_S^*(x), U, x)] = \begin{cases} 
0\bar{c} \cdot \int_0^{\bar{x}} (1 - F(u)) du + (1 - \bar{x})\bar{c} \cdot \int_{\bar{x}}^x (1 - F(u)) du & \text{if } 0 \leq x < \bar{x}^L, \\
(1 + \theta - \bar{x})\bar{c} \cdot \int_{\bar{x}}^\bar{x} F(u) du + \omega \bar{c} \cdot \int_0^{\bar{x}} F(u) du & \text{if } \bar{x}^L \leq x \leq \bar{x}^U, \\
(1 + \theta - \bar{x})\bar{c} \cdot \int_{\bar{x}}^\bar{x} (1 - F(u)) du + \omega \bar{c} \cdot \int_0^{\bar{x}} F(u) du & \text{if } x > \bar{x}^U. 
\end{cases} \tag{18}
\]
Therefore, according to (4), we have
\[
\pi_S^*(x) = \begin{cases} 
0\bar{c} \cdot \int_0^{\bar{x}} (1 - F(u)) du + (1 - \bar{x})\bar{c} \cdot \int_{\bar{x}}^x (1 - F(u)) du & \text{if } 0 \leq x < \bar{x}^L, \\
(1 + \theta - \bar{x})\bar{c} \cdot \int_{\bar{x}}^\bar{x} F(u) du + \omega \bar{c} \cdot \int_0^{\bar{x}} F(u) du - \frac{\gamma c}{\gamma c} \cdot \bar{x}^L & \text{if } \bar{x}^L \leq x \leq \bar{x}^U, \\
(1 + \theta - \bar{x})\bar{c} \cdot \int_{\bar{x}}^\bar{x} (1 - F(u)) du + \omega \bar{c} \cdot \int_0^{\bar{x}} F(u) du - \frac{\gamma c}{\gamma c} \cdot \bar{x}^U & \text{if } x > \bar{x}^U. 
\end{cases} \tag{19}
\]
That is,

**Proposition 2.** The manufacturer’s premium-payment level $x$ impacts the supplier’s capacity acquisition decision $w_S^*(x)$ in such a way that the capacity level is, respectively, flat at $\bar{x}^L$ when the premium-payment level is less than $\bar{x}^L$, the same as the premium-payment level when the latter is between $\bar{x}^L$ and $\bar{x}^U$, and flat at $\bar{x}^U$ when the latter is more than $\bar{x}^U$. (The $\bar{x}^L$ and $\bar{x}^U$ levels are defined in (16).)

By plugging (17) into (8) and taking into consideration that $y_S^*(w, U, x) = \min\{w, u\}$, we can learn more about the manufacturer’s cost term $z_M(x)$ defined in (8):
\[
z_M(x) = \begin{cases} 
(1 + \theta)\bar{c} \cdot \int_0^{\bar{x}} (1 - F(u)) du + \bar{c} \cdot \int_{\bar{x}}^x (1 - F(u)) du & \text{if } 0 \leq x < \bar{x}^L, \\
(1 + \theta - \bar{x})\bar{c} \cdot \int_{\bar{x}}^\bar{x} (1 - F(u)) du + (1 + \bar{v})\bar{c} \cdot \int_{\bar{x}}^\bar{x} (1 - F(u)) du + \omega \bar{c} \cdot \int_0^{\bar{x}} F(u) du & \text{if } \bar{x}^L \leq x \leq \bar{x}^U, \\
(1 + \theta)\bar{c} \cdot \int_0^{\bar{x}} (1 - F(u)) du + (1 + \bar{v})\bar{c} \cdot \int_{\bar{x}}^\bar{x} (1 - F(u)) du + \omega \bar{c} \cdot \int_0^{\bar{x}} F(u) du & \text{if } x > \bar{x}^U. 
\end{cases} \tag{20}
\]
Taking the derivative on $x$, we obtain
\[
\frac{dz_M(x)}{dx} = \begin{cases} 
0 - (\theta - \omega)\bar{c} \cdot F(x) & \text{if } 0 < x < \bar{x}^L, \\
(\bar{v} - \theta + \omega)\bar{c} \cdot F(x) - (\bar{v} - \theta)\bar{c} & \text{if } \bar{x}^L < x < \bar{x}^U, \\
\omega \bar{c} \cdot F(x) & \text{if } x > \bar{x}^U.
\end{cases}
\] (21)

Since $0 \leq F(x) \leq 1$, $\frac{dz_M(x)}{dx} \geq 0$ and hence $z_M(x)$ is increasing when $0 < x < \bar{x}^L$ and $x > \bar{x}^U$. In this study, we concentrate on the case where neither $\theta$ nor $\omega$ is too large. More specifically, we suppose the supplier and manufacturer would choose the parameters so that
\[
\begin{cases} 
\theta \leq \bar{v}, \\
\omega \leq \bar{\gamma} \cdot (\bar{v} - \theta)/(1 + \theta - \bar{\gamma} - \bar{\gamma}).
\end{cases}
\] (22)

Then, by the monotonicity of $F(x)$, we know that $\frac{dz_M(x)}{dx}$ increases when $\bar{x}^L < x < \bar{x}^U$ and yet by (16), we have $\frac{dz_M(x)}{dx} \leq 0$. So $z_M(x)$ is convex and decreasing in the interval $[\bar{x}^L, \bar{x}^U]$, and hence achieves its minimum at $\bar{x}^U$ for $x \in [\bar{x}^L, \bar{x}^U]$. The optimal $x^*_M$ that minimizes $z_M(x)$ in (7) is therefore either $\theta$ or $\bar{x}^U$, depending on whether or not $z_M(0) \leq z_M(\bar{x}^U)$.

Plugging the two numbers into (20), we have
\[
\begin{align*}
  z_M(0) &= \bar{c} \cdot \int_0^{\bar{x}^L} (1 - F(u)) \, du + (1 + \bar{v})\bar{c} \cdot \int_{\bar{x}^L}^{\bar{x}^U} (1 - F(u)) \, du, \\
  z_M(\bar{x}^U) &= (1 + \theta)\bar{c} \cdot \int_0^{\bar{x}^U} (1 - F(u)) \, du + (1 + \bar{v})\bar{c} \cdot \int_{\bar{x}^L}^{\bar{x}^U} (1 - F(u)) \, du + \omega \bar{c} \cdot \int_0^{\bar{x}^U} F(u) \, du.
\end{align*}
\] (23)

Therefore,
\[
\Delta \pi_M = z_M(0) - z_M(\bar{x}^U) = (\bar{v} - \theta)\bar{c} \cdot \int_0^{\bar{x}^L} (1 - F(u)) \, du - \bar{c} \cdot \int_0^{\bar{x}^U} (1 - F(u)) \, du - \omega \bar{c} \cdot \int_0^{\bar{x}^U} F(u) \, du.
\] (24)

On the other hand, by plugging the two numbers into (19), we can have
\[
\Delta \pi_S = \pi^*_S(\bar{x}^U) - \pi^*_S(0) = \bar{c} \cdot \int_0^{\bar{x}^L} (1 - F(u)) \, du + (1 + \theta - \bar{\gamma})\bar{c} \cdot \int_{\bar{x}^L}^{\bar{x}^U} (1 - F(u)) \, du - \bar{\gamma} \bar{c} \cdot (\bar{x}^U - \bar{x}^L) + \bar{c} \cdot \int_0^{\bar{x}^U} F(u) \, du.
\] (25)

This is the amount to be gained by the supplier if the manufacturer decides to make a premium-payment of $(1 + \theta)\bar{c} \cdot \bar{x}^U$ instead of making no premium-payment. By Proposition 1, we note that $\Delta \pi_S \geq 0$. Grouping the above, we have the following result:

**Proposition 3.** The manufacturer will either promise a premium-payment level of $\bar{x}^U$ or make no commitment at all, depending on whether the term
\[
\Delta \pi_M = (\bar{v} - \theta)\bar{c} \cdot \int_0^{\bar{x}^L} (1 - F(u)) \, du - \bar{c} \cdot \int_0^{\bar{x}^U} (1 - F(u)) \, du - \omega \bar{c} \cdot \int_0^{\bar{x}^U} F(u) \, du
\]
is positive or not. When the former is realized, the supplier will gain a positive sum
\[
\Delta \pi_S = \bar{c} \cdot \int_0^{\bar{x}^L} (1 - F(u)) \, du + (1 + \theta - \bar{\gamma})\bar{c} \cdot \int_{\bar{x}^L}^{\bar{x}^U} (1 - F(u)) \, du - \bar{\gamma} \bar{c} \cdot (\bar{x}^U - \bar{x}^L) + \bar{c} \cdot \int_0^{\bar{x}^U} F(u) \, du
\]
more than she otherwise would have gotten when no premium-payment has been promised. In total, the system can gain
\[
\Delta \pi_{SM} = (1 + \bar{v} - \bar{\gamma})\bar{c} \cdot \int_{\bar{x}^L}^{\bar{x}^U} (1 - F(u)) \, du - \bar{\gamma} \bar{c} \cdot (\bar{x}^U - \bar{x}^L),
\]
when the manufacturer decides to go ahead with the premium-payment.

**Numerical Example 1.** We suppose $\bar{c} = 1.0$, $\bar{\gamma} = 20\%$, $\bar{\gamma} = 60\%$, $\psi = 50\%$, and $\bar{\gamma} = 30\%$, and demand is uniformly distributed between $[0,1000]$: $f(u) = 0.001$ and $F(u) = 0.001u$ for $u \in [0,1000]$. The $\bar{v}$ for this example is $80\%$. Suppose it has also been decided that $\theta = 20\%$ and $\omega = 5\%$. Then from (16), we have
\[
\begin{align*}
\bar{x}^U &= F^{-1}((1 - 0.2 - 0.6)/(1 - 0.2)) = F^{-1}(0.25) = 250, \\
\bar{x}^L &= F^{-1}((1 + 0.2 - 0.2 - 0.6)/(1 + 0.2 - 0.2)) = F^{-1}(0.4) = 400.
\end{align*}
\]

(26)

Plugging these into (24) and (25), we find that

\[
\begin{align*}
\Delta \pi_M &= (0.8 - 0.2) \cdot \int_{0}^{250} (1 - 0.001u) \, du - 0.2 \cdot \int_{0}^{250} (1 - 0.001u) \, du \\
&\quad - 0.05 \cdot \int_{0}^{400} 0.001u \, du = $13, \\
\Delta \pi_S &= 0.2 \cdot \int_{0}^{250} (1 - 0.001u) \, du + (1 + 0.2 - 0.2) \cdot \int_{0}^{400} (1 - 0.001u) \, du \\
&\quad - 0.6 \cdot (400 - 250) + 0.05 \cdot \int_{0}^{400} 0.001u \, du = $59.
\end{align*}
\]

Therefore, the manufacturer will be willing to pay a 20% premium and a 5% under-order penalty for 400 items and expect to earn $13 more on average, while by accepting the premium-payment, the supplier will acquire a 400-item capacity instead of a 250-item capacity, and stand to gain $59 on average. The entire system will gain $72 when the manufacturer decides to make the premium-payment.

4. The determination of \( \theta \)

To signify the importance of the choice of \( \theta \), we denote \( \bar{x}^U \) by \( \bar{x}^U(\theta) \). Note that \( \lim_{\theta \to 0^+} \bar{x}^U(\theta) = \bar{x}^L \), and
d\( \bar{x}^U(\theta)/d\theta = \gamma/(1 + \theta - \bar{x})^2 \cdot f(\bar{x}^U(\theta)) \).

(28)

From (24), we see that \( x^U_M \) will be strictly positive if \( \omega \) is small enough and \( \bar{v} > \theta \cdot (1 + h(\theta)) \), where

\[
h(\theta) = \left( \int_{0}^{x^L} (1 - F(u)) \, du \right) / \left( \int_{\bar{x}^L}^{\bar{x}^U(\theta)} (1 - F(u)) \, du \right).
\]

(29)

Using L'Hospital’s rule, we can reach that

\[
\lim_{\theta \to 0^+} (\theta \cdot (1 + h(\theta))) = \lim_{\theta \to 0^+} \theta \cdot h(\theta) = 1 / \left( \lim_{\theta \to 0^+} ((1/h(\theta))/\theta) \right) = 1 / \left( \lim_{\theta \to 0^+} (-d(1/h(\theta))/d\theta) \right)
\]

\[
= \left( \int_{0}^{x^L} (1 - F(u)) \, du \right) / \left( \int_{0}^{x^L} (1 - F(u)) \, du \right) / \left( \lim_{\theta \to 0^+} \left( \int_{0}^{x^L} (1 - F(u)) \, du \right) / \left( \gamma \cdot (1 - F(x^L)) \right) \right)
\]

\[
= ((1 - \bar{x})^2 \cdot f(\bar{x}^L) \cdot \int_{0}^{x^L} (1 - F(u)) \, du) / (\gamma \cdot (1 - F(x^L))
\]

\[
= ((1 - \bar{x})^3 \cdot f(\bar{x}^L) \cdot \int_{0}^{x^L} (1 - F(u)) \, du) / (\gamma^2).
\]

(30)

That is, \( v^0 = \lim_{\theta \to 0^+} (\theta \cdot (1 + h(\theta))) \) is a finite number. Let \( H(\epsilon) = \min(\theta \cdot (1 + h(\theta))|\theta \geq \epsilon) \) for any \( \epsilon > 0 \). Apparently, \( H(\epsilon) \) decreases as \( \epsilon \) decreases. Therefore, \( v^\ell = \lim_{\epsilon \to 0^+} H(\epsilon) \) exists and is in \([0, v^0]\): either \( v^\ell = v^0 \), or there is some \( \theta_0 > 0 \) such that \( v^\ell = \theta_0 (1 + h(\theta_0)) \) while for any \( \theta > 0, \theta (1 + h(\theta)) \geq v^\ell \). Now, as long as \( v > v^\ell \), we can select a \( \theta \) such that \( z_M(\bar{x}^U(\theta)) < z_M(0) \). The following should re-capture the above observations:

**Proposition 4.** There exists a nonnegative \( v^\ell \) level, such that when the manufacturer’s profit-margin-plus-goodwill factor \( v \) is more than \( v^\ell \), there will exist some premium factor \( \theta \) (with \( \theta \cdot (1 + h(\theta)) < v \)) and under-order penalty \( \omega \) (satisfying \( \omega \leq \min \{ \gamma \cdot (v - \theta)/(1 + \theta - \bar{x} - \bar{v}), (v - \theta \cdot (1 + h(\theta)))/(\int_{0}^{x^L} (1 - F(u)) \, du) \}) under which the manufacturer will be willing to commit to the positive premium-payment level \( \bar{x}^U(\theta) \).
When such $\theta$ and $\omega$ are selected, the manufacturer can make a premium-payment for $x^U(\theta) = F^{-1}(1 + \theta - \bar{x} - \bar{\gamma})/(1 + \theta - \bar{x})$ items to the supplier. By (17), the supplier can acquire an exact capacity level of $x^U(\theta)$. On the other hand, if the manufacturer makes no premium-payment to the supplier, then according again to (17), the supplier will only acquire a lower capacity level of $x^L$. Both parties will suffer.

From (24) and (25), it is clear that the manufacturer would prefer a small $\omega$ while the supplier would most prefer a large $\omega$. When $\omega$ has been agreed upon, we may view $\Delta\pi_M$, $\Delta\pi_S$, and $\Delta\pi_{SM}$ as functions of $\theta$.

Taking derivatives, we obtain

$$d(\Delta\pi_M(\theta))/d\theta = (\bar{\gamma}(\bar{v} - \theta - \omega(1 + \theta - \bar{x} - \bar{\gamma}))\bar{c} \cdot (dx^U(\theta)/d\theta)/(1 + \theta - \bar{x})$$

$$- \bar{c} \cdot \int_0^{x^U(\theta)} (1 - F(u)) \, du,$$

$$d(\Delta\pi_S(\theta))/d\theta = \omega(1 + \theta - \bar{x} - \bar{\gamma})\bar{c} \cdot (dx^U(\theta)/d\theta)/(1 + \theta - \bar{x}) + \bar{c} \cdot \int_0^{x^U(\theta)} (1 - F(u)) \, du,$$

$$d(\Delta\pi_{SM}(\theta))/d\theta = \bar{\gamma}(\bar{v} - \theta)\bar{c} \cdot (dx^U(\theta)/d\theta)/(1 + \theta - \bar{x}).$$

By (28), we have $x^U(\theta)$ being increasing in $\theta$. Hence, both $\Delta\pi_S(\theta)$ and $\Delta\pi_{SM}(\theta)$ are increasing in $\theta$ while $\Delta\pi_M(\theta)$ depends on $\theta$ in a fairly complicated fashion. Since $\Delta\pi_S(\theta) \geq 0$ all the time, the manufacturer would most prefer $\theta^*_M$ which is the solution for

$$\Delta\pi_M^* = \max\{\Delta\pi_M(\theta) | 0 \leq \theta \leq \bar{v}\},$$

while the supplier would most prefer $\theta^*_S$ which is the solution for

$$\Delta\pi_S^* = \max\{\Delta\pi_S(\theta) | 0 \leq \theta \leq \bar{v} \text{ and } \Delta\pi_M(\theta) \geq 0\}. \quad (32)$$

In view of the above, it may take a complete search to find $\theta^*_M$, while when $\Delta\pi_M(\bar{v}) < 0$, $\theta^*_S$ will be the largest $\theta$ in $[0, \bar{v}]$ satisfying $\Delta\pi_M(\theta) = 0$. The final $\theta$ the two parties agree upon should result from their negotiations. To achieve the maximum total benefit, one should choose $\theta^*_{SM}$ which solves

$$\Delta\pi_{SM}^* = \max\{\Delta\pi_{SM}(\theta) | 0 \leq \theta \leq \bar{v} \text{ and } \Delta\pi_M(\theta) \geq 0\}. \quad (33)$$

When $\Delta\pi_M(\bar{v}) < 0$, $\theta^*_{SM}$ should agree with $\theta^*_S$.

In the following, we explore two functional examples where the demands are uniformly and exponentially distributed, respectively. In the exposition, we have let $\bar{e} = 1 - \bar{a} - \bar{\gamma}$, the ideal profit margin the supplier can enjoy when her capacity is fully utilized and the manufacturer only makes the regular payment.

**Functional Example 1.** When the demand level $U$ is uniformly distributed in $[0, 2\bar{D}]$ for some positive constant $\bar{D}$, we have $f(u) = 0$ and $F(u) = 0$ for $x < 0$, $f(u) = 1/(2\bar{D})$ and $F(u) = u/(2\bar{D})$ for $0 \leq x \leq 2\bar{D}$, and $f(u) = 0$ and $F(u) = 1$ for $x > 2\bar{D}$. So

$$\begin{cases} x^L = 2\bar{D} \cdot (1 - \bar{x} - \bar{\gamma})/(1 - \bar{x}) = 2\bar{D}/(1 + \bar{\gamma}/\bar{e}), \\ x^U(\theta) = 2\bar{D} \cdot (1 + \theta - \bar{x} - \bar{\gamma})/(1 + \theta - \bar{x}) = 2\bar{D}/(1 + \bar{\gamma}/(\bar{e} + \theta)). \end{cases} \quad (35)$$

We then have

$$\theta \cdot (1 + h(\theta)) = \theta \cdot \left(1 + \left(\int_0^{2\bar{D}/(1 + \bar{\gamma} - \bar{e})/(1 - \bar{x})} (1 - u/(2\bar{D})) \, du \right) \left(\int_0^{2\bar{D}/(1 + \bar{\gamma} - \bar{e})/(1 - \bar{x})} (1 - u/(2\bar{D})) \, du \right) \right)$$

$$= \theta + ((1 - \bar{x} + \bar{\gamma}) \cdot (1 - \bar{x} - \bar{\gamma}) \cdot (1 + \theta - \bar{x})^2)/(\bar{\gamma}^2 \cdot (2 + \theta - 2\bar{x})),$$

which is increasing in $\theta$ for $\theta > 0$. Therefore, we have

$$\bar{v}^L = \bar{v}^U = \bar{v}^0 = \frac{(1 - \bar{x} + \bar{\gamma}) \cdot (1 - \bar{x} - \bar{\gamma}) \cdot (1 - \bar{x})}{2\bar{\gamma}^2} = \frac{1}{\bar{e}} \cdot \left(1 + \frac{\bar{e}}{\bar{\gamma}} \right) \cdot \left(1 + \frac{\bar{e}}{\bar{\gamma}} \right) \cdot \left(1 + \frac{\bar{e}}{\bar{\gamma}} \right). \quad (37)$$

That is, whenever $\bar{v} > \bar{e} \cdot (1 + \bar{e}/\bar{\gamma} \cdot (1 + \bar{e}/(2\bar{\gamma}))$, there will exist a premium-payment level $\theta$ along with an under-order penalty level $\omega$ such that the manufacturer will have the incentive to make the premium payment for up to the quantity of $2\bar{D}/(1 + \bar{\gamma}/(\bar{e} + \theta))$ and the supplier will be willing to build her capacity to exactly this level.
Numerical Example 2. Here we continue with the previous numerical example. We have $\overline{D} = 500$ and $\bar{\varepsilon} = 20\%$. So by (37), we have

$$\bar{v}^L = 0.2 \cdot \left( 1 + \frac{0.2}{0.6} \right) \cdot \left( 1 + \frac{0.2}{2 \cdot 0.6} \right) \simeq 0.311. \quad (38)$$

We can find $\theta$ and $\omega$ for the manufacturer to be willing to pay the premium because the current $\bar{v} = 0.8$ is well above the threshold level 0.311. From (35), we obtain

$$\bar{x}^U(\theta) = \frac{2 \cdot 500}{1 + 0.6/(0.2 + \theta)} = \frac{1000 \cdot (\theta + 0.2)}{\theta + 8}. \quad (39)$$

When $\omega$ is fixed at 5%, we may see from Proposition 3 that

$$\begin{align*}
\Delta \pi_M(\theta) &= (0.8 - \theta) \cdot J_{250}^{1000(\theta+0.2)/(\theta+0.8)}(1 - 0.001u) \, du - \theta \cdot J_0^{250}(1 - 0.001u) \, du \\
&
- 0.05 \cdot J_0^{1000(\theta+0.2)/(\theta+0.8)} 0.001u \, du \\
&= (0 \cdot (220 - 5750 - 5000^2) - 25 \cdot (\theta + 0.2)^2)/ (\theta + 0.8)^2, \\
\Delta \pi_S(\theta) &= \theta \cdot J_0^{250}(1 - 0.001u) \, du + (1 + \theta - 0.2) \cdot J_{250}^{1000(\theta+0.2)/(\theta+0.8)}(1 - 0.001u) \, du \\
&
- 0.6 \cdot (1000 \cdot (\theta + 0.2)/(\theta + 0.8) - 250) + 0.05 \cdot J_0^{250} 0.001u \, du \\
&= (5000 \cdot (\theta + 0.35) \cdot (\theta + 0.8) + 25 \cdot (\theta + 0.2)^2)/(\theta + 0.8)^2, \\
\Delta \pi_{SM}(\theta) &= (1 + \theta - 0.2) \cdot J_{250}^{1000(\theta+0.2)/(\theta+0.8)}(1 - 0.001u) \, du \\
&
- 0.6 \cdot (1000 \cdot (\theta + 0.2)/(\theta + 0.8) - 250) = 3600/(\theta + 0.8)^2. \quad (40)
\end{align*}$$

Through simple search, we find that the manufacturer would most prefer the premium rate $\theta_M^* \simeq 12.77\%$, for which he would gain $\Delta \pi_M^* \simeq \$17.42$ while the supplier $\Delta \pi_S^*(\theta_M^*) \simeq \$35.99$. On the other hand, we have $\Delta \pi_M(0.8) \simeq -$184.77, and hence both the supplier and the channel optimizer would most prefer $\theta_S^* = \theta_{SM}^* \simeq 27.91\%$, for which both the supplier and the channel would gain $\Delta \pi_S^* = \Delta \pi_{SM}^* \simeq \$86.29$ while the manufacturer nothing. Our earlier choice of $\theta = 20\%$ took the middle ground between the two extremes.

Functional Example 2. When the demand level $U$ is exponentially distributed with a positive constant $\overline{D}$ as its mean, we have $f(u) = 0$ and $F(u) = 0$ for $u < 0$, and $f(u) = e^{(-u/\overline{D})}/\overline{D}$ and $F(u) = 1 - e^{(-u/\overline{D})}$ for $u \geq 0$. So

$$\begin{align*}
\bar{x}^L(\theta) &= \overline{D} \cdot \ln((1 - \bar{\alpha})/\bar{\gamma}) = \overline{D} \cdot \ln(1 + \bar{\varepsilon}/\bar{\gamma}), \\
\bar{x}^U(\theta) &= \overline{D} \cdot \ln((1 + \theta - \bar{\alpha})/\bar{\gamma}) = \overline{D} \cdot \ln(1 + (\bar{\varepsilon} + \theta)/\bar{\gamma}). \quad (41)
\end{align*}$$

We then have

$$\theta \cdot (1 + h(\theta)) = \theta \cdot \left( 1 + \left( \int_{0}^{\overline{D} \cdot \ln((1 - \bar{\alpha})/\bar{\gamma})} e^{(-u/\overline{D})} \, du \right)/\left( \int_{0}^{\overline{D} \cdot \ln((1 + \theta - \bar{\alpha})/\bar{\gamma})} e^{(-u/\overline{D})} \, du \right) \right)$$

$$= \theta + ((1 - \bar{\alpha} - \bar{\gamma}) \cdot (1 + \theta - \bar{\alpha}))/\bar{\gamma}, \quad (42)$$

which is increasing in $\theta$ for $\theta > 0$. Therefore, we have

$$\bar{v}^L = \bar{v}^0 = ((1 - \bar{\alpha} - \bar{\gamma}) \cdot (1 - \bar{\alpha}))/\bar{\gamma} = \bar{\varepsilon} \cdot \left( 1 + \frac{\bar{\varepsilon}}{\bar{\gamma}} \right). \quad (43)$$

That is, whenever $\bar{v} > \bar{\varepsilon} \cdot (1 + \bar{\varepsilon}/\bar{\gamma})$, there will exist a premium-payment level $\theta$ as well as an under-order penalty level $\omega$ such that the manufacturer will have the incentive to make the premium-payment for up to the quantity of $\overline{D} \cdot \ln((1 + (\bar{\varepsilon} + \theta)/\bar{\gamma})$ and the supplier will be willing to build her capacity to exactly this level.
5. Channel coordination

We first derive the first-best capacity acquisition level for the entire supply chain. If the realized demand level is \( u \) and the supplier decides a delivery level \( y \) to the manufacturer, the total channel profit \( \pi_{SM}(u,y) \) should satisfy

\[
\pi_{SM}(u,y) = (1 + \psi)\bar{c} \cdot u - \bar{z}\bar{c} \cdot y - (1 + \bar{v})\bar{c} \cdot (u - y)^+.
\]

(44)

It is apparently an increasing function of \( y \) for \( y \in [0,u] \). Given that the supplier’s capacity is \( w \) and the realized demand level is \( u \), we can obtain the optimal delivery level \( y_{SM}^*(w,u) \) by solving

\[
\pi_{SM}^*(w,u) = \max \{ \pi_{SM}(u,y) \mid 0 \leq y \leq \min\{w,u\} \}.
\]

(45)

Hence, \( y_{SM}^*(w,u) = \min\{w,u\} \) and

\[
\pi_{SM}^*(w,u) = (1 + \psi - \bar{z})\bar{c} \cdot u - (1 + \bar{v} - \bar{z})\bar{c} \cdot (u - w)^+.
\]

(46)

The optimal overall profit \( \pi_{SM}^{**} \) and the optimal supplier capacity \( w_{SM}^* \) can be obtained by solving

\[
\pi_{SM}^{**} = \max \{ \pi_{SM}^*(w) \mid w \geq 0 \}.
\]

(47)

where

\[
\pi_{SM}^*(w) = E[\pi_{SM}(w,U)] - \bar{c}\cdot w
\]

\[
= (1 + \psi - \bar{z})\bar{c} \cdot \int_0^{+\infty}(1 - F(u)) \, du - (1 + \bar{v} - \bar{z})\bar{c} \cdot \int_w^{+\infty}(1 - F(u)) \, du - \bar{c}\cdot w.
\]

(48)

The above is merely a newsboy problem with an underage cost \((1 + \bar{v} - \bar{z} - \bar{g})\bar{c}\) and an overage cost \(\bar{c}\). So we have

\[
w_{SM}^* = \bar{w} = F^{-1}((1 + \bar{v} - \bar{z} - \bar{g})/(1 + \bar{v} - \bar{z})).
\]

(49)

By studying (49), (16), and Proposition 4, we see that for \( \bar{v} > \bar{v}^l \), we have \( w_{SM}^* > \bar{x}^U(\theta) \) for any \( \theta \) that satisfies \( \theta \cdot (1 + h(\theta)) < \bar{v} \). That is,

**Proposition 5.** The channel-optimal capacity level \( w_{SM}^* \) is higher than the best capacity level \( w_{SM}^*(\bar{x}^U(\theta)) = \bar{x}^U(\theta) \) that the manufacturer can lure the supplier to build using the premium-payment scheme.

The supplier’s share of the profit, \( \pi_{SSM}^{**} \), can be calculated by

\[
\pi_{SSM}^{**} = (1 - \bar{z})\bar{c} \cdot E[\min\{w_{SM}^*, U\}] - \bar{g}\bar{c} \cdot w_{SM}^* = (1 - \bar{z})\bar{c} \cdot \int_0^{w_{SM}^*}(1 - F(u)) \, du - \bar{g}\bar{c} \cdot w_{SM}^*.
\]

(50)

The manufacturer’s share \( \pi_{MSM}^{**} \) can therefore be obtained by

\[
\pi_{MSM}^{**} = \pi_{SM}^{**} - \pi_{SSM}^{**} = \psi\bar{c} \cdot \int_0^{w_{SM}^*}(1 - F(u)) \, du - \bar{c}\cdot w_{SM}^* - \bar{c}\cdot \int_w^{+\infty}(1 - F(u)) \, du.
\]

(51)

**Numerical Example 3.** We may continue with the earlier numerical example. We first still let \( \theta \) be 20%. According to (49), (47) and (5), we have

\[
\begin{align*}
\bar{w} &= F^{-1}((1 + 0.8 - 0.2 - 0.6)/(1 + 0.8 - 0.2)) = F^{-1}(0.625) = 625, \\
\pi_{SM}^{**} &= \pi_{SM}^*(625) = (1 + 0.5 - 0.2) \cdot \int_{0}^{1000}(1 - 0.001u) \, du \\
&
\quad - (1 + 0.8 - 0.2) \cdot \int_{625}^{1000}(1 - 0.001u) \, du - 0.6 \cdot 625 = 162.5.
\end{align*}
\]

(52)
So the channel-optimal capacity level is 625 and the channel profit is $162.5. Using (50), we get
\[
\pi_{SSM}^* = (1 - 0.2) \cdot \int_0^{625} (1 - 0.001u) \, du - 0.6 \cdot 625 = -$31.25.
\] (53)

Hence, \( \pi_{SSM}^* \) is $193.75. At the same time, we have, by (48),
\[
\begin{align*}
\pi_{SM}^{**}(250) & = (1 + 0.5 - 0.2) \cdot \int_0^{100} (1 - 0.001u) \, du - (1 + 0.8 - 0.2) \cdot \int_{250}^{100} (1 - 0.001u) \, du \\
& - 0.6 \cdot 250 = $50, \\
\pi_{SM}^{**}(400) & = (1 + 0.5 - 0.2) \cdot \int_0^{100} (1 - 0.001u) \, du - (1 + 0.8 - 0.2) \cdot \int_{400}^{100} (1 - 0.001u) \, du \\
& - 0.6 \cdot 400 = $122;
\end{align*}
\] (54)

while by (19),
\[
\begin{align*}
\pi_{S}^{**}(0) & = (1 - 0.2) \cdot \int_0^{250} (1 - 0.001u) \, du - 0.6 \cdot 250 = $25, \\
\pi_{S}^{**}(400) & = (1 + 0.2 - 0.2) \cdot \int_0^{400} (1 - 0.001u) \, du + 0.05 \cdot \int_0^{400} 0.001u \, du - 0.6 \cdot 400 = $84. 
\end{align*}
\] (55)

From the above, we see that the manufacturer’s profits are $25 and $38, respectively, depending on whether he withholds or goes with the premium-payment. This result is consistent with our earlier ones on relative gains.

When \( \theta = \theta_M^* \simeq 12.77\% \), however, we have
\[
w_S^* \simeq F^{-1}((1 + 0.1277 - 0.2 - 0.6)/(1 + 0.1277 - 0.2)) \simeq F^{-1}(0.353) \simeq 353,
\] (56)

while when \( \theta = \theta_S^* \simeq 27.91\% \), we have
\[
w_S^* \simeq F^{-1}((1 + 0.2791 - 0.2 - 0.6)/(1 + 0.2791 - 0.2)) \simeq F^{-1}(0.444) \simeq 444.
\] (57)

From the \( \Delta \pi_M \) and \( \Delta \pi_S \) values, we can obtain the profits earned by the parties under the above \( \theta \) values.

Indeed, channel coordination can be achieved when the supplier is forced to build her capacity to the level \( x \), announced by the manufacturer. When such is the case, the identity
\[
w_S^*(x) = x
\] (58)
replaces (17). When this happens, from (11) and (1), we obtain
\[
\iota_{MS}(u, y_S^*(x), u, x) = (1 + \theta)\bar{c} \cdot \min\{x, u\} + \omega\bar{c} \cdot (x - u)^+.
\] (59)

Plugging this into (8), we have
\[
z_M(x) = (1 + \theta)\bar{c} \cdot \int_0^x (1 - F(u)) \, du + \omega\bar{c} \cdot \int_0^x F(u) \, du + (1 + \bar{v})\bar{c} \cdot \int_x^{\infty} (1 - F(u)) \, du.
\] (60)

Through differentiation, we can find the \( x_M^* \) level to satisfy
\[
x_M^* = F^{-1}\left(\frac{\bar{v} - \theta}{\bar{v} - \theta + \omega}\right).
\] (61)

When it so happens that
\[
\bar{v} - \theta + (1 + \bar{v} - \bar{x} - \bar{\gamma})\omega = \bar{\gamma}\bar{v},
\] (62)
we will have \( x_M^* = w_{SM}^* \) as described in (49), i.e., the manufacturer will be induced to pay a premium for the channel-optimal quantity. From (61), we also note that requirements on \( \theta \) and \( \omega \) as stated in Proposition 4 should be replaced by the simple \( 0 \leq \theta \leq \bar{v} \) and \( \omega \geq 0 \).

In view of (15), the supplier’s compliance may be enforced by imposing a penalty \( \delta \bar{c} \cdot (x - w)^+ \) for her capacity under-building, where the parameter \( \delta \) is so high so that
\[
(1 + \theta - \bar{x})\bar{c} \cdot (1 - F(w_{SM}^*)) + \delta \bar{c} \geq \bar{\gamma}\bar{c},
\] (63)

or equivalently,
\[
\delta \geq \delta^* \geq \frac{\bar{\gamma}(\bar{v} - \theta)}{1 + \bar{v} - \bar{x}} = \frac{(1 + \bar{v} - \bar{x} - \bar{\gamma})\omega}{1 + \bar{v} - \bar{x}}.
\] (64)
At channel optimality, we note that the supplier’s and manufacturer’s shares are, respectively,
\[
\begin{align*}
\pi_{\text{SSM}}^* &= E[t_{\text{MS}}(U, \min\{w_{\text{SM}}^*, U\}, w_{\text{SM}}^*)] - \bar{\alpha}c \cdot E[\min\{w_{\text{SM}}^*, U\}] - \bar{\gamma}c \cdot w_{\text{SM}}^*, \\
\pi_{\text{MSM}}^* &= (1 + \theta - \bar{\gamma})c \cdot \int_0^{w_{\text{SM}}^*} (1 - F(u)) \, du + \omega c \cdot \int_0^{w_{\text{SM}}^*} F(u) \, du - \bar{\gamma}c \cdot w_{\text{SM}}^*, \\
\pi_{\text{SM}}^* &= (1 + \bar{\psi})c \cdot E[U] - E[t_{\text{MS}}(U, \min\{w_{\text{SM}}^*, U\}, w_{\text{SM}}^*)] - (1 + \bar{\gamma})c \cdot (U - w_{\text{SM}}^*)^+. \\
\end{align*}
\]
(65)

We summarize the above in the following.

**Proposition 6.** When the supplier is forced or induced to build her capacity level \( w \) to the premium-payment level \( x \) announced by the manufacturer, such as when \( \theta \) and \( \omega \) are so agreed upon that \( 0 \leq \theta \leq \bar{\gamma}, \omega \geq 0 \), and \( \bar{\gamma} / (1 + \bar{\gamma} - \bar{\gamma}) \omega = \bar{\gamma}v \), and the supplier is charged with an under-capacity penalty \( \delta c \cdot (x - w)^+ \) for some \( \delta \geq \delta^* = \bar{\gamma}(v - \theta)/(1 + \bar{\gamma} - \bar{\gamma}) \), channel coordination will be reached.

Note that once \( \theta \) and \( \omega \) are given, and \( \delta \) is selected from \([\delta^*, +\infty)\), the supplier will never have to pay the under-capacity penalty. So \( \delta \) does not appear in (65) and its change will not affect either party’s profit. On the other hand, we may introduce constant \( \bar{\zeta} = \int_0^w (1 - F(u)) \, du/(1 - F(\bar{w})) - \int_0^w F(u) \, du/F(\bar{w}) \) and, we may use \( \omega(\theta) \) to denote \( \omega \)’s dependence on \( \theta \) as prescribed by (62) as well as use \( \theta(\omega) \) for the opposite dependence, and use \( \pi_{\text{SSM}}^*(\theta, \omega) \) and \( \pi_{\text{MSM}}^*(\theta, \omega) \) to denote, respectively, the dependencies of \( \pi_{\text{SSM}}^* \) and \( \pi_{\text{MSM}}^* \) on \( \theta \) and \( \omega \) as described in (65). Utilizing (49), (61), (62), and (65), we may obtain
\[
\begin{align*}
\frac{d\omega(\theta)}{d\theta} &= -\bar{\gamma}/(1 + \bar{\gamma} - \bar{\gamma}) = -(1 - F(\bar{w}))/F(\bar{w}), \\
\frac{d\theta(\omega)}{d\omega} &= -(1 + \bar{\gamma} - \bar{\gamma})/\bar{\gamma} = -F(\bar{w})/(1 - F(\bar{w})), \\
\frac{d\pi_{\text{SSM}}^*(\theta, \omega(\theta))/d\theta} &= \bar{\gamma}c/(1 + \bar{\gamma} - \bar{\gamma}), \\
\frac{d\pi_{\text{SSM}}^*(\theta(\omega), \omega)/d\omega} &= -(1 + \bar{\gamma} - \bar{\gamma})\bar{\zeta}c/(1 + \bar{\gamma} - \bar{\gamma}), \\
\frac{d\pi_{\text{MSM}}^*(\theta, \omega(\theta))/d\theta} &= \bar{\gamma}c/(1 + \bar{\gamma} - \bar{\gamma}), \\
\frac{d\pi_{\text{MSM}}^*(\theta(\omega), \omega)/d\omega} &= (1 + \bar{\gamma} - \bar{\gamma})\bar{\zeta}c/(1 + \bar{\gamma} - \bar{\gamma}).
\end{align*}
\]
(66)

For the constant \( \bar{\zeta} \), we can check that
\[
\bar{\zeta} \geq \frac{1}{1 - F(\bar{w})} \bar{w}(1 - F(\bar{w})) - \frac{1}{F(\bar{w})} \bar{w}F(\bar{w}) = 0.
\]
(67)

So, when we let the premium-payment rate \( \theta \) increase and let the under-capacity penalty rate \( \omega \) decrease linearly in response to the condition dictated in (62), the supplier’s share of profit \( \pi_{\text{SSM}}^* \) will increase linearly, while the manufacturer’s share of profit \( \pi_{\text{MSM}}^* \) will decrease linearly; on the other hand, when we let the under-capacity penalty rate \( \omega \) increase and let the premium-payment rate \( \theta \) decrease linearly in response to the condition dictated in (62), the supplier’s share of profit \( \pi_{\text{SSM}}^* \) will decrease linearly, while the manufacturer’s share of profit \( \pi_{\text{MSM}}^* \) will increase linearly.

Erkoc and Wu [9] studied a partially refundable reservation contract in almost the same setting as ours, except that their capacity cost is general convex instead of our linear form \( \bar{\gamma}c \cdot w \). In their contract, the supplier first announces a reservation price \( \omega c \), and then the manufacturer decides his reservation level \( x \) and pays the supplier the reservation fee \( (\theta + \omega)c \cdot x \). Based on \( x \), the supplier builds her capacity up to a level \( w \). When demand level \( u \) is realized, the supplier delivers a quantity \( y = \min\{w, u\} \) and the manufacturer pays the supplier an additional \( c \cdot y - \omega c \cdot \min\{x, u\} \). Note that \( \partial c \) here is \( r - r_2 \) in the original paper and \( \omega c \) here is \( r_2 \) there. The manufacturer’s total payment to the supplier is
\[
\tilde{c}_{\text{MS}}(u, y, x) = \theta c \cdot x + c \cdot y + \omega c \cdot (x - u)^+.
\]
(68)

Under forced compliance, the authors found a similar result that, when
\[
(1 + \bar{\gamma} - \bar{\gamma})\theta + (1 + \bar{\gamma} - \bar{\gamma})\omega = \bar{\gamma}v,
\]
(69)
the contract can induce the manufacturer to reserve and the supplier to build, the channel-optimal capacity \( w_{\text{SM}}^* \).
Numerical Example 4. For the previous numerical example, (62) is merely
\[ 0.6 \cdot \theta + (1 + 0.8 - 0.2 - 0.6) \cdot \omega = 0.6 \cdot 0.8. \]  
(70)

We may pick \( \theta = 20\% \) and \( \omega = 36\% \). Following (64), we obtain
\[ \hat{\delta}^t = \frac{0.6 \cdot (0.8 - 0.2)}{1 + 0.8 - 0.2} = 22.5\%. \]  
(71)

We may pick \( \delta = 25\% \), for instance. Plugging numbers into (65), we get
\[
\begin{align*}
\pi_{SSM}^* &= (1 + 0.2 - 0.2) \cdot \int_0^{625} (1 - F(u)) \, du + 0.36 \cdot \int_0^{625} F(u) \, du - 0.6 \cdot 625 = \$125, \\
\pi_{MSM}^* &= (0.5 - 0.2) \cdot \int_0^{625} (1 - F(u)) \, du - 0.3 \cdot \int_0^{1000} (1 - F(u)) \, du \\
&\quad - 0.36 \cdot \int_0^{625} F(u) \, du = \$37.5.
\end{align*}
\]  
(72)

We may as well pick \( \theta = 30\% \) and \( \omega = 30\% \). This will result in \( \hat{\delta}^t = 18.75\% \). We may still let \( \delta = 25\% \), for instance. The corresponding profit shares will be \( \pi_{SSM}^* = \$156.25 \) and \( \pi_{MSM}^* = \$6.25 \). The trends in the changes agree with our earlier observations. Table 1 re-captures results concerning all the numerical examples.

6. Formulation for a general payment schedule

Now we lay out the formulation for finding the optimal manufacturer payment schedule, no longer limited to the special form of a fixed premium for the first few items. The manufacturer may offer the supplier a payment schedule \( p \) in some set \( P \) of allowable schedules. Here, a schedule \( p \) is a function whose \( p(y) \) value dictates the price the manufacturer is willing to pay when the delivery level is \( y \). For \( \theta \) fixed and \( \omega = 0 \), what we have just studied prior to Section 5 are payment schedules \( p \) with
\[ p(y) = (1 + \theta) \hat{c} \cdot y - \theta \bar{c} \cdot (y - x)^+. \]  
(73)

That is, we limited our \( p \)'s to the special set \( P_0 = \{ p | p(y) = (1 + \theta) \hat{c} \cdot y - \theta \bar{c} \cdot (y - x)^+ \} \) for some positive \( x \).

We may suppose that even the general \( P \) contains only \( p \)'s satisfying the following assumptions:

1. \( p(y) - (\hat{z} + \gamma) \hat{c} \cdot y \) is increasing in \( y \)
2. \( p(y) \) is concave in \( y \).

Since the payment should allow the supplier to more than recover her cost, we have assumption (1) naturally. From (9), we see that \( E[\tilde{z}_{MD}(U,y)] \) decreases with \( y \) and its absolute decreasing rate is
\[
\left| \frac{dE[\tilde{z}_{MD}(U,y)]}{dy} \right| = \frac{dE[\tilde{z}_{MD}(U,y)]}{dy} = (1 + \bar{v}) \hat{c} \cdot (1 - F(y)).
\]  
(74)

Hence, the “worth” of each additional product unit delivered to the manufacturer, being \( (1 + \bar{v}) \hat{c} \cdot (1 - F(y)) \), is also decreasing with the quantity \( y \) already delivered. This fact motivates us to impose assumption (2).

In the following, we outline the procedure for finding the schedule \( p_M \in P \) that is most preferred by the manufacturer, when a general \( P \) satisfying assumptions (1) and (2) is given. As expected, the logic flow is

<table>
<thead>
<tr>
<th>( (\theta, \omega, \delta) )</th>
<th>Capacity</th>
<th>Supplier’s $</th>
<th>Manufacturer’s $</th>
<th>Channel $</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>250</td>
<td>$25</td>
<td>$25</td>
<td>$50</td>
</tr>
<tr>
<td>(12.77%, 5%, 0)</td>
<td>353</td>
<td>$60.99</td>
<td>$42.42</td>
<td>$103.41</td>
</tr>
<tr>
<td>(20%, 5%, 0)</td>
<td>400</td>
<td>$84</td>
<td>$38</td>
<td>$122</td>
</tr>
<tr>
<td>(27.91%, 5%, 0)</td>
<td>444</td>
<td>$111.29</td>
<td>$25</td>
<td>$136.29</td>
</tr>
<tr>
<td>(20%, 36%, 25%)</td>
<td>625</td>
<td>$125</td>
<td>$37.5</td>
<td>$162.5</td>
</tr>
<tr>
<td>(30%, 30%, 25%)</td>
<td>625</td>
<td>$156.25</td>
<td>$6.25</td>
<td>$162.5</td>
</tr>
<tr>
<td>Forced coordination</td>
<td>625</td>
<td>$31.25</td>
<td>$193.75</td>
<td>$162.5</td>
</tr>
</tbody>
</table>
as follows: first, the supplier will figure out her best response \( w^*_S(p) \) in terms of her capacity acquisition level to any \( p \in P \) that is offered to her by the manufacturer and then, while knowing this response, the manufacturer will select the \( p^*_M \in P \) that makes \( w^*_S(p^*_M) \) work the most towards his own benefit.

Under any payment schedule \( p \in P \), given capacity level \( w \) and demand level \( u \), the supplier’s optimal delivery level \( y^*_S(w, u, p) \) is \( \min\{w, u\} \), and her optimal second-stage profit \( \pi^*_S(w, u, p) \) satisfies
\[
\pi^*_S(w, u, p) = p(\min\{w, u\}) - \bar{c} \cdot \min\{w, u\}.
\]

The supplier’s optimal overall profit \( \pi^*_S(p) \) and optimal capacity level \( w^*_S(p) \) can be determined by solving
\[
\pi^*_S(p) = \max\{\pi^*_S(w, p)|w \geq 0\},
\]
where
\[
\pi^*_S(w, p) = E[\pi^*_S(w, U, p)] - \bar{c} \cdot w = E[p(\min\{w, U\}) - \bar{c} \cdot \min\{w, U\}] - \bar{c} \cdot w.
\]

Taking derivative on \( w \), we get
\[
\frac{d\pi^*_S(w, p)}{dw} = \left( \frac{dp(w)}{dw} - \bar{c} \right) \cdot (1 - F(w)) - \bar{c},
\]
which is a decreasing function since, by the assumptions on \( P \), we have
\[
\frac{d^2\pi^*_S(w, p)}{dw^2} = \frac{d^2p(w)}{dw^2} \cdot (1 - F(w)) - f(w) \cdot \left( \frac{dp(w)}{dw} - \bar{c} \right) \leq 0.
\]

We can let \( w^*_S(p) \) be such that
\[
\begin{cases}
\frac{dp(w^*_S(p))}{dw} \cdot (1 - F(w^*_S(p))) + \bar{c} \cdot F(w^*_S(p)) \geq (\bar{c} + \bar{c})\bar{c}, \\
\frac{dp((w^*_S(p))^+)}{dw} \cdot (1 - F((w^*_S(p))^+)) + \bar{c} \cdot F((w^*_S(p))^+) \leq (\bar{c} + \bar{c})\bar{c}.
\end{cases}
\]

To determine his optimal profit \( \pi^*_M \) and choose the optimal payment schedule \( p^*_M \in P \), the manufacturer has only to solve
\[
\pi^*_M = (1 + \bar{c})\bar{c} \cdot E[U] - \min\{z_M(p)|p \text{ is concave and } dp(+\infty)/dy \geq (\bar{c} + \bar{c})\bar{c}\},
\]
where we have assumed that
\[
z_M(p) = E[p(\min\{w_M(p), U\})] + (1 + \bar{c})\bar{c} \cdot E[(U - w_M(p))^+]
\]
\[
= \int_0^{w_M(p)} p(u)f(u)du + p(w_M(p)) \cdot (1 - F(w_M(p))) + (1 + \bar{v})\bar{c} \cdot \int_{w_M(p)}^{+\infty} (1 - F(u))du.
\]

**Functional Example 3.** For instance, we may suppose that demand is exponentially distributed so that for some \( \bar{u} > 0 \),
\[
f(u) = \bar{u}e^{-\bar{u}u} \quad \text{and} \quad F(u) = 1 - e^{-\bar{u}u};
\]
while
\[
P = \{p|p(y) = \bar{c} \cdot (y + s - te^{-t'})\} \text{ for some positive } s \text{ and } t.
\]
It is easy to check that our \( P \) here satisfies the two assumptions. Since any \( p \) in the current \( P \) is parameterized by \( s \) and \( t \), we may write the supplier’s best response \( w^*_S(p) \) as \( w^*_S(s, t) \) and at the same time, our search for the best schedule \( p^*_M \in P \) boils down to the search for the best parameters \( s^*_M \) and \( t^*_M \).

To use (80) to find the supplier’s optimal capacity acquisition level \( w^*_S(s, t) \), we note that
\[
\frac{dp(y)}{dy} \cdot (1 - F(y)) + \bar{c} \cdot F(y) = \bar{c} \cdot (\bar{z} + (1 - \bar{z})e^{-\bar{v}} + st(e^{-(\bar{u} + t')v})),
\]
Hence, \( w_s^*(s, t) \) is the root of the equation

\[
(1 - \bar{x})e^{-\bar{y}} + st e^{-(\bar{\mu} + \bar{\phi})t} - \bar{y} = 0.
\]  

(86)

On the manufacturer’s side, according to (82), we have

\[
z_M(s, t) = \int_0^{w_s^*(s, t)} p(u)f(u) \, du + p(w_s^*(s, t)) \cdot (1 - F(w_s^*(s, t))) + (1 + \bar{v})\bar{c} \cdot \int_{w_s^*(s, t)}^{+\infty} (1 - F(u)) \, du \]

\[
= \bar{c} \cdot \left[ \int_0^{w_s^*(s, t)} \bar{\mu} e^{-\bar{\mu}u} \cdot (u + s - se^{-u}) \cdot du + e^{-\bar{\mu}w_s^*(s, t)} \cdot (w_s^*(s, t) + s - se^{-w_s^*(s, t)}) \right]
\]

\[
+ (1 + \bar{v}) \cdot \int_{w_s^*(s, t)}^{+\infty} e^{-\bar{\mu}u} \cdot du
\]

\[
= \bar{c} \cdot \left( \frac{1}{\bar{\mu}} - st/(t + \bar{\mu}) + (2s + \bar{v}/\bar{\mu})e^{-\bar{\mu}w_s^*(s, t)} - (s(t + 2\bar{\mu}))/((t + \bar{\mu})e^{-(t+\bar{\phi})w_s^*(s, t)}). \right)
\]  

(87)

The manufacturer may find the best parameters \( s_M^* \) and \( t_M^* \) through solving

\[
\pi_M^* = \max\{ (1 + \bar{\phi})\bar{c} \cdot E[U] - z_M(s, t) \mid s, t \geq 0 \}
\]

\[
= \bar{c} \cdot \max\{ \bar{\phi}/\bar{\mu} + st/(t + \bar{\mu}) - (2s + \bar{v}/\bar{\mu})e^{-\bar{\mu}w_s^*(s, t)} + (s(t + 2\bar{\mu})/(s + \bar{\mu}))/((t + \bar{\mu})e^{-(t+\bar{\phi})w_s^*(s, t)} \mid s, t \geq 0 \}. \]  

(88)

7. Concluding remarks

We have shown that by offering a premium-payment for the first few items, the manufacturer can induce the supplier to acquire bigger capacity, leading to a mutually beneficial outcome, so long as missing demand is costly for the manufacturer. When an under-capacity penalty can be imposed on the supplier, channel coordination can be reached by this scheme. We have also presented the formulation for more general payment schemes, hoping that a better scheme will be found in future research.

In this study, we have kept the wholesale price \( \bar{c} \) fixed. To investigate cases where demands are price-sensitive, it will be imperative that this restriction be lifted. Also, we have thus far only analyzed the payment scheme in a single-manufacturer and symmetric-information framework. More work is needed in settings involving multiple manufacturers and information asymmetry between the supplier and the various manufacturers and themselves. We will pursue these directions in our future research.

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References


