Analysis of a Revenue-Sharing Contract in Supply Chain Management

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Abstract:
We consider a supply chain involving one supplier and one retailer in which a revenue-sharing contract is adopted. Under this contract, the retailer can obtain the product from the supplier at a discounted price. As a compensation, the retailer must share his revenue with the supplier at a certain revenue-sharing rate, say \( r \) \((0 \leq r \leq 1)\), where \( r \) represents the portion of the revenue to be kept by the retailer. Our ultimate objective is to maximize the overall supply chain’s total profit while upholding the individual components’ incentives. We use a two-stage (Stackelberg) game to model the problem, where one player is the game’s leader and the other the game’s follower. Our analysis reveals that to maximize the supply chain’s total profit, it should be that, the party who keeps more than half of the revenue should also be the leader of the Stackelberg game.

Keywords: Supply Chain; Revenue-sharing Contract; Stackelberg Game.
1 Introduction

We consider a supply chain with one supplier and one retailer (newsvendor), who are involved with a new product with a short life cycle, such as a new film, a new video, or a new style clothing. Normally the demand for the new product peaks at the beginning of its life cycle, and it quickly recedes afterwards. If as tradition has it, a wholesale price contract is adopted, the retailer must have enough cash to make a one-time purchase to brace for the short sales season. A retailer without enough cash will inevitably lose many sales opportunities if nothing were done. And his loss will also translate into lost opportunities for the supplier. However, if a revenue-sharing contract were in place, both parties would potentially be benefited.

A revenue-sharing contract lets the retailer pay the supplier at two time points, with the later time point being after sales have been realized. Under such a contract, for every product unit he gets from the supplier, the retailer first pays the supplier a portion of the unit production cost; and then for every product unit it has sold, the retailer pays the supplier a portion of the unit revenue earned on the sold product. In this paper, we shall denote the first portion by $\alpha$ ($0 < \alpha < 1$) and the second portion by $1 - r$ ($0 < r < 1$) ($r$ is the portion of the revenue per unit kept by the retailer).

Revenue-sharing contracts have been adopted by many retail supply chains and have been proven to be effective in generating market shares and total profits. Warren and Peers [17] reported that Blockbuster’s market share of video rentals increased from 24% in 1997 to 40% in 2002 after a revenue-sharing contract was adopted. Mortimer [10] estimated that the adoption of revenue-sharing contracts increased the video-rental industry’s total profit by 7%.

In this paper, we present and analyze the adoption of a revenue-sharing contract in a supply chain with one supplier and one retailer. We use a two-stage game where one player is the game’s leader and the other one is the game’s follower. In the game, the revenue-sharing rate $r$ is a pre-determined constant. When the supplier is the Stackelberg game’s leader (the supplier’s Stackelberg game), she decides the transfer cost rate $\alpha$ and then the retailer decides the quantity $q$ to be ordered from the supplier; while when the retailer is Stackelberg game’s leader (the retailer’s Stackelberg game), he decides the transfer cost rate $\alpha$ and then the supplier decides the quantity $q$ to be provided to the retailer.

The major finding of our analysis is that, the supplier’s Stackelberg game should be preferred when the revenue-sharing rate $r$ is less than 1/2, while the retailer’s Stackelberg game should be preferred when $r$ is more than 1/2. This finding offers a deeper understanding to revenue-sharing contracts and will assist the design of such contracts in real life.
The paper is organized as follows: We present a review of the literature in Section 2, introduce and analyze the supply chain model in Section 3, and make our conclusions in Section 4.

2 Literature Review

Recent years has seen a growing interest in supply chain contracts. Supply chain contracts such as buy-back (Pasternack [12]), quantity flexibility (Tsay [14]) and penalty scheme (Lariviere [8]) were well analyzed when demand is stochastic and the retail price is given. Research works based on returns policies included those on retailer competitions (Padmanabhan and Png [11]), two echelon inventory systems (Cachon and Zipkin [4]), and risk-free returns to the supplier (Webster and Weng [18]). There are many other forms of contracts with a given retail price such as wholesale price contract (Lariviere and Porteus [9]), linear rebates (Taylor [13]) and revenue-sharing contract (Dana and Spier [6]).

Wang, Li, and Shen [16]) studied the management of a supply chain under a consignment contract with revenue-sharing. There, the supplier decides on the retail price and the quantity of goods to be delivered to the retailer and the retailer decides on the percent of the revenue to be shared by the supplier. Dana and Spier [6] studied a revenue-sharing contract in the context of a competitive market among retailers. Cachon and Lariviere [3] presented a model where a single retailer chooses the optimal price and quantity to maximize the expected profit. They found that a revenue-sharing contract is equivalent to buy-backs in the newsvendor case and price discounts in the price-setting newsvendor case.

Lariviere [8] analyzed a supply chain contract (penalty scheme) under stochastic demand. It was found there that using the contract would not induce the best outcome for the supply chain as a whole. Anupindi and Bassok [1] considered a supply chain where a supplier sets the wholesale price using an approximation of the normal distribution. Van Mieghem [15] analyzed how an exogenously set transfer price can influence the capacity decisions of a manufacturer and an upstream supplier.

Papers that bore more similarity to ours are as follows. Cachon [2] considered a setting close to ours, but his model was based primarily on a wholesale price contract. Caldentey and Wein [5] used Stackelberg solutions to analyze a supply chain, but their model was under a continuous-state approximation and based on an M/M/1 make-to-stock queue. Drezner and Pasternack [7] analyzed the video rental problem and demonstrated that it was analogous to the single-period newsboy problem.

In this paper, we will study a revenue-sharing contract using the Stackelberg game in a single-period newsboy setting, and our overall goal is to maximize the overall supply chain’s
total profit while upholding the individual components’ incentives. Such a study for the revenue-sharing contract has not been done.

3 Model and Analysis

3.1 Preliminaries

We suppose that the demand level faced by the retailer is randomly distributed according to a positive and continuous probability distribution function $f(x)$ on interval $[a, b)$ with a corresponding cumulative distribution function $F(x)$. We also let $g(x) = \frac{xf(x)}{(1 - F(x))}$ be the generalized failure rate of the demand, which roughly gives the percentage decrease in the probability of a stock out by increasing the stocking quantity by 1% over the existing $x$ level. The random demand is considered to have an increasing generalized failure rate (IGFR) if $g(x)$ is nondecreasing for all $x$ with $F(x) < 1$ (Lariviere and Porteus [9]).

In this paper, we assume that

1) the random demand is with an increasing generalized failure rate (IGFR);
2) the unit retail price is fixed at $p$;
3) the revenue sharing rate $r$ has been settled beforehand;
4) the unit production cost is fixed at $c$;
5) the unit product salvage value is fixed at $s$;
6) $rp > s$ and $(1 - r)p > s$; and
7) $p - c < rp - s$ and $p - c < (1 - r)p - s$.

The remaining factors that are still to be determined are:

a) the transfer cost rate $\alpha$, i.e., the percentage of the unit production cost the retailer pays to the supplier for every unit product he receives; and

b) the quantity $q$ of the product that is to be delivered from the supplier to the retailer.

A central decision maker would maximize the expected channel profit $B(q)$, which can be expressed as in

$$B(q) = p \cdot \int_0^q xf(x)dx + pq \cdot \int_q^{+\infty} f(x)dx + s \cdot \int_0^q (q - x)f(x)dx - cq, \quad (1)$$

where the transfer and revenue-sharing elements are not involved. It can be derived from the optimality conditions that the optimal quantity $q_0^*$ to be delivered must satisfy

$$F(q_0^*) = \frac{(p - c)}{(p - s)}. \quad (2)$$
3.2 The Supplier is the Leader of the Stackelberg game

When the supplier is the Stackelberg game’s leader, she announces the transfer rate \( \alpha \) to the retailer with the purpose of optimizing her own expected profit given the retailer’s best response. The retailer decides the optimal quantity \( q \) to order to maximize his own expected profit. At the end of the sales season, the retailer sells the leftover items at the unit salvage value \( s \).

Given \( \alpha \) and \( q \), the retailer’s expected profit \( \Theta_R(\alpha, q) \) can be expressed as in

\[
\Theta_R(\alpha, q) = rp \cdot \int_0^q xf(x)dx + rpq \cdot \int_q^{+\infty} f(x)dx + s \cdot \int_0^q (q - x)f(x)dx - \alpha cq. \tag{3}
\]

Therefore, given \( \alpha \), the optimal quantity \( q^*_R(\alpha) \) that the retailer chooses to maximize \( \Theta_R(\alpha, q) \) satisfies

\[
F(q^*_R(\alpha)) = (rp - \alpha c)/(rp - s). \tag{4}
\]

From (4), we can get the inverse function of \( q^*_R(\alpha) \):

\[
\alpha^*_R(q) = (rp(1 - F(q)) + sF(q))/c. \tag{5}
\]

Combining (5) and (3), we can get the expression \( \Theta^*_R(q) = \Theta_R(\alpha^*_R(q), q) \):

\[
\Theta^*_R(q) = rp \cdot \int_0^q xf(x)dx + rpq \cdot \int_q^{+\infty} f(x)dx + s \cdot \int_0^q (q - x)f(x)dx
- (rp(1 - F(q)) + sF(q))q. \tag{6}
\]

Just because

\[
d\Theta^*_R(q)/dq = (rp - s)f(q)q \geq 0, \tag{7}
\]

we know that the retailer’s expected profit is nondecreasing in \( q \).

Given \( \alpha \) and \( q \), the supplier’s expected profit \( \Theta_S(\alpha, q) \) can be expressed as in

\[
\Theta_S(\alpha, q) = (1 - r)p \cdot \int_0^q xf(x)dx + (1 - r)pq \cdot \int_q^{+\infty} f(x)dx + (\alpha c - c)q. \tag{8}
\]

Therefore, suppose that the supplier has chosen the \( \alpha = \alpha^*_R(q) \) that induces the retailer to respond with a particular \( q \), the expected profit \( \Theta^*_S(q) = \Theta_S(\alpha^*_R(q), q) \) she can get can be determined by

\[
\Theta^*_S(q) = (1 - r)p \cdot \int_0^q xf(x)dx + (1 - r)pq \cdot \int_q^{+\infty} f(x)dx + (rp(1 - F(q)) + sF(q) - c)q. \tag{9}
\]

We can get the first- and second-order derivatives of the function \( \Theta^*_S(q) \) in \( q \) as follows:

\[
d\Theta^*_S(q)/dq = p - c + (s - p)F(q) + (s - rp)g(q)(1 - F(q)); \tag{10}
\]

\[
d^2\Theta^*_S(q)/dq^2 = (s - p)f(q) - (s - rp)g(q)f(q) + (s - rp)g'(q)(1 - F(q)). \tag{11}
\]

The following Lemma 1 shows that the \( q^*_S \) that maximizes the supplier’s profit \( \Theta^*_S(q) \) can be found by solving the first-order optimality condition.
Lemma 1 The supplier’s profit $\Theta^*_S(q)$ is quasi-concave in $q$.

Proof: From Assumption 1), $g(q)$ is nondecreasing for all $q$. Hence, we can define $y$ as follows:

$$y = \max\{q \mid g(q) \leq 1 \text{ and } a \leq q < b\}.$$ 

When $a \leq q \leq y$, by (11) and Assumption 6), we have

$$d^2\Theta_S(q)/dq^2 \leq (s - rp)f(q)(1 - g(q)) + (s - rp)dg(q)/dq(1 - F(q)).$$

We know that $dg(q)/dq \geq 0$ because $g(q)$ is nondecreasing in $q$. By the definition of $y$, we also know that $g(q) \leq 1$. Therefore, we have

$$d^2\Theta_S(q)/dq^2 \leq 0.$$ 

That is, $\Theta_S(q)$ is concave in $[a, y]$.

When $y \leq q < b$, by (10) and Assumption 7), we get

$$d\Theta_S(q)/dq \leq (rp - s)(1 - F(q))(1 - g(q)).$$

By Assumption 6) and the fact that $q(q) \geq 1$ due to the definition of $y$, we have

$$d\Theta_S(q)/dq \leq (rp - s)(1 - F(q))(1 - g(q)) \leq 0.$$ 

That is, $\Theta_S(q)$ is nonincreasing in $[y, b)$. Combining the above two facts, we know that $\Theta_S(q)$ is quasi-concave over $[a, b)$.

According to Lemma 1 and (10), $q^*_S$ satisfies the following equation:

$$p - c - (p - s)F(q^*_S) + (s - rp)q^*_Sf(q^*_S) = 0.$$  

(12)

The following proposition summarizes what we have just derived.

Proposition 1 When the supplier is the leader of the Stackelberg game, she would choose the transfer cost rate $\alpha^*_R(q^*_S)$, and the retailer would respond with $q^*_S = q^*_R(\alpha^*_R(q^*_S))$.

3.3 The Retailer is the leader of the Stackelberg game

When the retailer is the Stackelberg game’s leader, he announces the transfer rate per unit $\alpha$ to the supplier with the purpose of optimizing his own expected profit given the supplier’s best response. The supplier decides the optimal quantity $q$ to provide to maximize her own expected profit. At the end of the sales season, the supplier sells the leftover items at the unit salvage value $s$.  

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Given $\alpha$ and $q$, the supplier’s expected profit $\theta_S(\alpha, q)$ can be expressed as in

$$\theta_S(\alpha, q) = (1 - r)p \cdot f_0^q x f(x)dx + (1 - r)pq \cdot f_q^{+\infty} f(x)dx + s \cdot f_0^q(q - x)f(x)dx - (1 - \alpha)cq. \quad (13)$$

Hence, given $\alpha$, the optimal quantity $q_S^*(\alpha)$ that the supplier would provide to maximize her own profit $\theta_S(\alpha, q)$ satisfies

$$F(q_S^*(\alpha)) = ((1 - r)p - (1 - \alpha)c)/(1 - r)p - s). \quad (14)$$

From (14), we can get the inverse function of $q_S^*(\alpha)$:

$$\alpha_S^*(q) = (c - (1 - r)p(1 - F(q)) - sF(q))/c. \quad (15)$$

Combining (15) with (13), we can obtain the expression $\theta_S^*(q) = \theta_S(\alpha_S^*(q), q)$:

$$\theta_S^*(q) = (1 - r)p \cdot f_0^q x f(x)dx + (1 - r)pq \cdot f_q^{+\infty} f(x)dx + s \cdot f_0^q(q - x)f(x)dx - ((1 - r)p(1 - F(q)) + sF(q))q. \quad (16)$$

Just because

$$d\theta_S^*(q)/dq = ((1 - r)p - s)f(q)q \geq 0. \quad (17)$$

we know that the supplier’s expected profit is nondecreasing in $q$.

Given $\alpha$ and $q$, the retailer’s expected profit $\theta_R(\alpha, q)$ can be expressed as in

$$\theta_R(\alpha, q) = rp \cdot f_0^q x f(x)dx + rpq \cdot f_q^{+\infty} f(x)dx - \alpha cq. \quad (18)$$

Therefore, suppose that the retailer chooses the $\alpha = \alpha_S^*(q)$ that induces the supplier to respond with a particular $q$, the expected profit $\theta_R^*(q) = \theta_R(\alpha_S^*(q), q)$ he can get should be determined by

$$\theta_R^*(q) = rp \cdot f_0^q x f(x)dx + rpq \cdot f_q^{+\infty} f(x)dx - q(c - p(1 - r)(1 - F(q)) - sF(q)). \quad (19)$$

We can get the first- and second-order derivatives of the function $\theta_R^*(q)$ in $q$ as follows:

$$d\theta_R^*(q)/dq = p - c - (p - s)F(q) - g(q)(1 - F(q))(1 - r)p - s]. \quad (20)$$

$$d^2\theta_R^*(q)/dq^2 = -(p - s)f(q) + g(q)f(q)[(1 - r)p - s] - g'(q)(1 - F(q))(1 - r)p - s]. \quad (21)$$

The following Lemma 2 shows that the $q^*_R$ that maximize the retailer’s profit $\theta_R^*(q)$ can be found by solving the first-order optimality condition $d\theta_R^*(q^*_R)/dq = 0$.

**Lemma 2** The retailer’s profit $\theta_R^*(q)$ is quasi-concave in $q$. 

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**Proof:** As in the proof of Lemma 1, we define $y$ as in

$$y = \max\{q \mid g(q) \leq 1 \text{ and } a \leq q < b\}.$$ 

When $a \leq q \leq y$, from (21) and Assumption 6), we know that

$$d^2\theta_R(q)/dq^2 \leq -f(q)[(1 - r)p - s][1 - g(q)] - dg(q)/dq(1 - F(q))(1 - r)p - s).$$

We know that $dg(q)/dq \geq 0$ because $g(q)$ is nondecreasing in $q$. We also know that $g(q) \leq 1$ by the definition of $y$. Hence, it is true that

$$d^2\theta_R(q)/dq^2 \leq 0.$$ 

That is, $\theta_R(q)$ is concave in $[a, y]$. When $y \leq q < b$, by (20) and Assumption 7), we get

$$d\theta_R(q)/dq \leq (p - s)(1 - F(q))[1 - g(q)].$$

By the definition of $y$, we have $q(q) \geq 1$. This and Assumption 6) lead to that

$$d\theta_R(q)/dq \leq (p - s)(1 - F(q))(1 - g(q)) \leq 0.$$ 

That is, $\theta_R(q)$ is nonincreasing in $[y, b]$. Combining the above two facts, we see that $\theta_R(q)$ is quasi-concave in $[a, b]$.

By (20), we see that $q^*_R$ satisfies the following equation:

$$p - c - (p - s)F(q^*_R) - q^*_Rf(q^*_R)[(1 - r)p - s] = 0.$$ 

The following proposition summarizes what we have just derived.

**Proposition 2** When the retailer is the leader of the Stackelberg game, the retailer would choose the transfer cost rate $\alpha^*_S(q^*_R)$, and the supplier would respond with $q^*_R = q^*_S(\alpha^*_S(q^*_R))$.

### 3.4 Comparison between the Two Stackelberg Games

The supply chain’s total profit is the sum of the supplier’s and retailer’s profits. When the supplier is the leader of the Stackelberg game, the supply chain’s total profit is the sum of (3) and (8), while when the retailer is the leader of the Stackelberg game, the supply chain’s total profit is the sum of (13) and (18). But since the profits earned from internal transactions offset each other, the total profit should be completely earned from the end customers, and hence is a function of only the quantity $q$ of delivery from the supplier to
the retailer: the total profit $B(q)$ as expressed in (1) for the centralized case. The first- and second-order derivatives of $B(q)$ over $q$ are as follows:

\[
\begin{aligned}
\frac{dB(q)}{dq} &= p - c - (p - s)F(q), \\
\frac{d^2B(q)}{dq^2} &= (s - p)f(q).
\end{aligned}
\]

(23)

By our Assumption 6) (hence $p \geq s$), we know that $B(q)$ is concave in $q$. As described in (2), the optimal quantity $q^*_0$ satisfying $F(q^*_0) = (p - c)/(p - s)$ maximizes the supply chain’s total profit. Due to the concavity of $B(q)$, when $q < q^*_0$, the supply chain’s total profit increases in the quantity $q$ of delivery.

Recall that, when the supplier is the leader of the Stackelberg game, the optimal delivery quantity $q^*_S$ is determined by (12), i.e.,

\[
F(q^*_S) = (p - c - (rp - s)q^*_S f(q^*_S))/(p - s).
\]

So by Assumption 6), we have $q^*_S \leq q^*_0$. When the retailer is the leader of the Stackelberg game, the optimal delivery quantity $q^*_R$ is determined by (22), i.e.,

\[
F(q^*_R) = (p - c - ((1 - r)p - s)q^*_R f(q^*_R))/(p - s).
\]

Again by Assumption 6), it is true that $q^*_R \leq q^*_0$. So the optimal delivery quantity for either decentralized case is no larger than the optimal delivery quantity for the centralized case, and hence the total profit for either decentralized case is no more than the total profit for the latter case.

Based on the above analysis, the larger the optimal delivery quantity, the more profit the overall decentralized supply chain will gain. The following proposition gives the criterion of judging whether one of the two alternatives for the decentralized chain is better than the other from the prospective of the total supply chain profit.

**Proposition 3** The delivery quantity $q$ in the supply chain is greater when the supplier is the leader of the Stackelberg game than when the retailer is the leader of the Stackelberg game if $r < 1/2$. Otherwise, it is greater when the retailer is the leader of the Stackelberg game. Therefore, for the supply chain to gain more profit, it should be the same party that both keeps more than half of the revenue and serves as the leader of the Stackelberg game.

**Proof:** We have

\[
\frac{d\Theta^*_S(q)}{dq} - \frac{d\theta^*_R(q)}{dq} = pq f(q)(1 - 2r).
\]

(24)

So the left-hand side of the above equation is greater than 0 if only if $r < 1/2$. By the quasi-concavity of $\Theta^*_S(q)$ and $\theta^*_R(q)$, we know that $q^*_S > q^*_R$ if $r < 1/2$ and $q^*_S < q^*_R$ if $r > 1/2$. 


In the above, we have tackled the case where the retailer keeps all the salvage revenue. Another practical case is the one in which the retailer keeps only the same $r$ portion of the salvage revenue as it does the regular sales revenue. This case can be similarly analyzed as above. Our analysis arrives to the same conclusion as presented in Proposition 3 that, the whole supply chain will be better off if the party that retains more than half of the revenue is also the leader of the Stackelberg game. It can also be proved that, under the same revenue sharing rate $r$, the delivery quantity in this case will be smaller, and hence the supply chain profit will be less, than respectively the delivery quantity and supply chain revenue in the previous case, regardless of the choice of the party to be designated as the Stackelberg game’s leader. So from the perspective of the total supply chain profit, this new case is not as interesting.

4 Conclusion

We have analyzed a supply chain under the revenue-sharing contract. A mild restriction assures that the leader’s expected profit is quasi-concave in the delivery quantity. This means that optimal results can be derived using the first-order optimality condition. These results allow us to compare the values of the optimal quantities, and consequently the optimal total supply chain profits, in the two major cases: one in which the supplier is the leader and another in which the retailer is. From the comparison, we learn that, if the revenue sharing rate is pre-determined, then the alternative where the party with the larger share of the revenue is the leader of the Stackelberg game will result in a larger total supply chain profit. Our model neglects a number of factors such as supplier competition, retailer competition and the retailing price’s change. Future research can take into consideration these factors.

References


