Review 1 – Basic Calculus

The exam will cover §0.1-6, §1.1-4,6-8, §2.1,2. No calculators will be allowed.

0. Functions

§0.1. Functions and their graphs: domain of a function (no zero in the denominator, no negative number in a square root).

§0.2. Some important functions: Linear function and quadratic functions, polynomials and rational functions; absolute value |x| and piecewisely defined functions.

§0.3. Algebra of functions: Arithmetic and composition of functions.

§0.4. Zeros of functions: Use quadratic formula to solve equations and factor polynomials.

§0.5. Exponents and power functions: Radicals and exponents: \( \sqrt[n]{b^m} = b^{m/n} \).

Exponent laws: \( b^r b^s = b^{r+s}, (b^r)^s = b^{rs}, (ab)^r = a^r b^r, \frac{b^r}{b^s} = b^{r-s}, 1/b^r = b^{-r}, \left( \frac{a}{b} \right)^r = a^r / b^r \).

Compound interest: \( A = P(1 + i)^n \) with interest rate \( i \) per period for \( n \) periods.

Or \( A = P \left( 1 + \frac{r}{m} \right)^{mt} \) with interest rate \( r \) per year, \( m \) interest period per year for \( t \) years.

§0.6. Functions and graphs in applications: Function models: material needed to make boxes; cost, revenue and profit: \( P(x) = R(x) - C(x) \). Break even points: when \( P(x) = 0 \).

1. The Derivative

§1.1. Straight lines: i) Slope-intercept equation: \( y = mx + b \).

ii) Point-slope equation: \( y - y_1 = m(x - x_1) \). iii) Two point equation: \( y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \).

iv) Two lines are parallel if they have the same slope: \( m_2 = m_1 \).

v) Two lines are perpendicular if their slopes are negative reciprocals of each other: \( m_2 = -1/m_1 \).

§1.2,3,6,7. Derivative rules:

i) (constant rule) \( \frac{d}{dx}(k) = 0 \). ii) (constant-multiple rule) \( \frac{d}{dx}(k f(x)) = k \frac{d}{dx}(f(x)) \).

iii) (power rule) \( \frac{d}{dx}(x^n) = nx^{n-1} \). iv) (sum rule) \( \frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x)) \).

v) (generalized power rule). \( \frac{d}{dx}(g(x)^r) = r(g(x))^{r-1} \frac{d}{dx}(g(x)) \). vi) (2nd derivative) \( \frac{d^2}{dx^2}(f(x)) = \frac{d}{dx} \left( \frac{d}{dx}(f(x)) \right) \).

Notations: \( f'(x) = \frac{dy}{dx} = \frac{d}{dx}(f(x)) \). \( f''(x) = \frac{d^2y}{dx^2} = \frac{d^2}{dx^2}(f(x)) \).

§1.4. Limits and the derivative: i) \( \lim_{x \to a} f(x) = L \) means \( f(x) \) is arbitrarily close to \( L \) as long as \( x \) is close enough (but not equal) to \( a \). Often \( \lim_{x \to a} f(x) = f(a) \).

ii) In other cases, have to use some ”tricks”, such as factoring: \( \lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4 \), or rationalizing, or taking common denominator.

iii) The definition of derivative is \( f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \). Use this for \( f(x) = x^2, \sqrt{x}, 1/x \).

iv) Limits at infinity. How to find them?

§1.2,8. Some common meanings of the derivative \( f'(x) \): rate of change, slope of tangent line, velocity, marginal cost or revenue.

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§2.1,2. Derivative tests rules for graphs.