Abstracts of the Second International Workshop on Differential Algebra and Related Topics

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Communicated by
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The Second International Workshop on Differential Algebra and Related Topics will be held on April 12–13, 2007 at the Newark Campus of Rutgers, the State University of New Jersey, USA. It is jointly organized by the Department of Mathematics and Computer Science at Rutgers University at Newark, and the Kolchin Seminar in Differential Algebra of the City University of New York. The abstracts of the tutorial talks to be presented at the Workshop are given below, in alphabetical order by speaker. Participation at the Workshop is open to all interested researchers (Preregistration is appreciated). For further information, please visit `http://newark.rutgers.edu/~liguo/diffalg.html`.

Overview of Baxter Algebras

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We discuss old and recent results on Baxter algebras, from work of Cartier and Rota in the 60's to current work of Guo and others. We will touch on topics such as Spitzer's identity, Loday's dendriform algebras, and the Yang-Baxter equation, among others.

The Painlevé Equations—Nonlinear Special Functions

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The six Painlevé equations ($\mathrm{P_I}$–$\mathrm{P_{VI}}$) were first discovered around the beginning of the twentieth century by Painlevé, Gambier and their colleagues in an investigation of nonlinear second-order ordinary differential equations. Recently there has been considerable interest in the Painlevé equations primarily due to the fact that they arise as reductions of the soliton equations which are solvable by inverse scattering. Although first discovered from strictly mathematical considerations, the Painlevé equations have arisen in a variety of important physical applications including statistical mechanics, random matrices, plasma physics, nonlinear waves, quantum gravity, quantum field theory, general relativity, nonlinear optics and fibre optics. Further the Painlevé equations may be thought of a nonlinear analogues of the classical special functions.

In this lecture I will give an introduction to the Painlevé equations. In particular I shall discuss many of the remarkable properties which the Painlevé equations possess including connection formulae, Bäcklund transformations associated discrete equations, and hierarchies of exact solutions.

Hopf Algebras of Labeled Trees and Some Associated Differential Algebra Structures

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It is well known that the vector space spanned by rooted trees forms a Hopf algebra. We survey several such Hopf algebras and describe some of the their duals. In particular, we consider Hopf algebras H of trees that are labeled by derivations in $\mathrm{Der}(R)$. Here k is a field, R is a commutative k-algebra, and $\mathrm{Der}(R)$ is the Lie algebra of derivations of R.

We describe a construction that gives R an H-module algebra structure and show this induces a differential algebra structure of H acting on R. The construction extends the notion of a R/k-bialgebra introduced by Nichols and Weisfeiler.

This is joint work with Richard Larson.

The Complete Picard-Vessiot Closure of the Constants

Andy R. Magid

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The compositum of all the Picard-Vessiot extensions of a given base differential field, unlike the algebraic closure of the field, may itself have proper Picard-Vessiot extensions. Iterating this, in general countably many times, produces a differential field that has no proper Picard-Vessiot extensions, and is minimal over the base with this property. This field is called the complete Picard-Vessiot closure. Its group of differential automorphisms over the base controls the differential subfield structure, even though the group is not (pro)algebraic and the correspondence is not a full Galois connection. We will focus on the natural special case when the base field is the (algebraically closed, characteristic zero) field of constants.

Model Theory and Differential Algebra

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Many model theoretic phenomena arise naturally in differntial fields. We will describe some model theoretic questions that lead to interesting questions in differential algebraic geometry.

Differential Galois Theory in Positive Characteristic

An Introduction

B. Heinrich Matzat

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We will give an introduction to differential Galois theory in positive characteristic and explain interrelations between Picard-Vessiot extensions in positive characteristic and in characteristic zero. The lecture summarizes work of M. van der Put and the speaker.

Computable Model Theory and Differential Algebra

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Model theory is the study of mathematical structures and the extent to which they can be described by statements and formulas. Computable model theory considers the effectiveness of results in model theory: whether they can actually be given or realized by algorithms. For example, a computable field is a field F in which the basic operations of addition and multiplication can be computed algorithmically, and one can then ask whether there exists a *splitting algorithm* for deciding whether a given polynomial in $F[X_1, \ldots, X_n]$ is reducible there.

We will give a tutorial in computable model theory, oriented towards results on fields and towards an audience with no serious background in either computability or model theory. Differential algebra is a natural subject for study by computable model theorists, yet there are precious few results for computable differential fields. (It should be understood that this is not the same thing as *computational* differential algebra, although there certainly should be some relation between the two.) As an example, we will describe Rabin's well-known result that every computable field F has a computable algebraic closure, but that F itself can be a computable subfield of the algebraic closure if and only if there is a splitting algorithm for $F[X]$. One would expect some sort of analogous result for computable differential fields and their differential closures, yet to the speaker's knowledge, no such work has been done.

Computable model theory has always restricted itself to countable structures, since the natural domain for computability is the natural numbers. However, we will present work by the speaker which also considers certain uncountable structures $\mathcal{S}$, called *locally computable* structures, by effectively describing the finitely generated substructures of $\mathcal{S}$, rather than giving a global description of $\mathcal{S}$. This concept was only recently developed and has not as yet been widely applied, but fields and differential fields are natural choices for its use.

Introduction to Symbolic-Numeric Completion Methods for PDE

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Differential elimination methods apply a finite sequence of differentiations and eliminations to general systems of PDE to extract potent information about their solutions. Much recent progress has been made in the design and implementation of exact algorithms, applying to exact input sytems, by researchers such as Boulier, Hubert, Mansfied, Seiler, Wittkopf and others. Though powerful, such methods cannot be applied to approximate systems, since the strong underlying use of rankings of partial derivatives, often induces instability, by forcing such methods to pivot on small quantities.

The talk will be an introduction to the new area of symbolic-numeric methods for completion of PDE. Main features include the focus on geometric methods and the use of Homotopy continuation methods for the detection of new constraints by slicing varieties in jet space with random hyperplanes. Our most recent work on this topic will be presented by Wenyuan Wu at the related AMS Special Session on Differential Algebra.

Differential Dependence and Differential Groups

Michael F. Singer

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I will develop a Galois theory of linear difference equations where the Galois groups are linear differential groups. These groups measure the differential dependence among solutions of linear difference equations. We will show how this theory can be used to prove anew Hölder's Theorem that the Gamma function satisfies no differential polynomial equation, Hardouin's recent results concerning differential dependence of solutions of first order difference equations and new results concerning differential dependence of solutions of higher order difference equations.

Solving Linear Differential Equations

Marius van der Put

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We concentrate on linear differential equations (or differential modules) over the differential field $\mathbb{C}(z)$. The theme, probably introduced by L. Fuchs, is to solve a differential equation in terms of equations of lower order. This problem has led to the highly interesting paper of G. Fano (1900). The work of M. F. Singer opened a new perspective on this theme. We continue this direction and apply the powerful theory of representations of semi-simple Lie algebras in order to obtain a systematic way for solving the problem. This involves differential Galois theory, Tannaka theory, simple algebraic groups and it leads to algorithms.

Solving Second and Third Order Linear ODE's in Terms of Special Functions

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In this talk an algorithm will be presented for solving any second or third order linear ordinary differential equation with rational function coefficients that is solvable in terms of Bessel, Kummer, or Whittaker functions.
