# Third International Workshop on Differential Algebra and Related Topics

**November 13-16, 2008**

**Rutgers, The State University of New Jersey**
**Newark, New Jersey, USA**

# Program

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The organizers of the Workshop gratefully acknowledge funding from the National Security Agency, the Rutgers University Research Council and the Office of the Chancellor, the Office of the Dean of the Faculty of Arts and Sciences-Newark and the Department of Mathematics and Computer Science of the Newark campus of Rutgers, The State University of New Jersey.

# Contents

# 1 Workshop Schedule

*The Bove Auditorium is located on the first floor of Engelhard Hall, 190 University Ave. When you walk from the Robert Treat Hotel along New Street, Engelhard Hall is on your left at the intersection with University Avenue. Ackerson Hall is the building facing Engelhard Hall across New Street Plaza.* *See* Figure 2 *on page* 24.

**Thursday, November 13 in Bove Auditorium, Engelhard Hall**

| | | |
|---|---|---|
| 8:00–8:45 | Registration and Refreshments | |
| 8:45–9:00 | Welcoming remarks | |
| 9:00–9:50 | Matthias Aschenbrenner | Degree bounds for Gröbner bases in algebras of solvable type |
| 10:00–10:50 | Evelyne Hubert | Constructive algebra for differential invariants |
| 11:00–11:30 | Refreshments | |
| 11:30–12:20 | Robert Bryant | On Cartan's generalizations of Lie's Third Theorem |
| 12:30–2:00 | Lunch Break | |
| 2:00–2:25 | Lisi D'Alfonso | On the index and the order of quasi-regular DAE systems |
| 2:30–2:55 | François Ollivier | A proof of the Dimension Conjecture and of Jacobi's Bound |
| 3:00–4:00 | Refreshments | |
| 4:00–4:25 | A. A. Mikhalev | Free differential calculus for free algebras of Schreier varieties |
| 4:30–4:55 | V. Ravi Srinivasan | Extensions by antiderivatives and iterated logarithms |
| 5:30 | Banquet Dinner<br>Robeson Galleries, first floor of Robeson Campus Center | |

**Friday, November 14 in Ackerson Hall 123**

| | | |
|---|---|---|
| 8:30–9:00 | Registration and Refreshments | |
| 9:00–9:50 | Julia Hartmann | Applications of patching |
| 10:00–10:50 | Anand Pillay | Logarithmic derivatives on nonconstant commutative algebraic groups, and transcendence |
| 11:00–11:30 | Refreshments | |
| 11:30–12:20 | Jerry Kovacic | Differential schemes |
| 12:30–2:00 | Lunch Break | |
| 2:00–2:25 | B. Heinrich Matzat | Galois theory of Frobenius modules |
| 2:30–2:55 | Arne Ledet | $PGL_n$ as a differential Galois group |
| 3:00–4:00 | Poster Session* and Refreshments | |
| 4:00–4:25 | Alexey Ovchinnikov | Differential elimination and bounding orders in effective differential Nullstellensatz |
| 4:30–4:55 | David Blázquez-Sanz | Lie-Vessiot systems, algebraic superposition laws and strongly normal extensions |

*** Poster presentations**

*Poster presentations will be available 3-4pm on November 14-16 in Ackerson Hall 123.*

| | |
|---|---|
| Ruyong Feng | Liouvillian solutions of linear differential-difference equations |
| Xiaoshan Gao | A criterion for testing whether a difference ideal is reflexive and prime |
| Hani Shaker | On factorization of multivariate polynomials and de Rham cohomology |
| Ekaterina Shemyakova | Differential transformations for second-order bivariate parabolic LPDOs (linear partial differential operators) |
| Max Wakefield | Derivations of an effective divisor on the complex projective line |

**Saturday, November 15 in Ackerson Hall 123**

| | | |
|---|---|---|
| 8:30–9:00 | Registration and Refreshments | |
| 9:00–9:50 | Sylvie Paycha | “Locality” as a driving principle for renormalisation |
| 10:00–10:50 | Bin Zhang | Renormalization on toric varieties |
| 11:00–11:30 | Refreshments | |
| 11:30–12:20 | Snigdhayan Mahanta | Holomorphic bundles over noncommutative tori and $\mathrm{Rep}(\hat{\mathbb{Z}})$ |
| 12:30–2:00 | Lunch Break | |
| 2:00–2:25 | Earl Taft | Differentiably finite power series and combinatorial identities |
| 2:30–2:55 | Zongzhu Lin | Representations of differential Rota-Baxter algebras |
| 3:00–4:00 | Poster Session* and Refreshments | |
| 4:00–4:25 | Robert Grossman | Some differential algebraic structures induced by Hopf algebras of labeled trees |
| 4:30–4:55 | A.G. Khovanskiĭ | Topological Galois Theory |

**Sunday, November 16 in Ackerson Hall 123**

| | | |
|---|---|---|
| 8:30–9:00 | Registration and Refreshments | |
| 9:00–9:50 | Anton Leykin | Computation of Bernstein-Sato polynomials |
| 10:00–10:50 | Emma Previato | Isomonodromic Garnier system and geometry of higher Painlevé VI equations |
| 11:00–11:30 | Refreshments | |
| 11:30–12:20 | Guy Casale | Non-linear differential Galois theory |
| 12:30–2:00 | Lunch Break | |
| 2:00–2:25 | Richard Cleyton | Differential Gerstenhaber algebras and weak mirror symmetry of nilpotent Lie algebras |
| 2:30–2:55 | Tom Scanlon | Differential Galois theories from model theory |
| 3:00–4:00 | Poster Session* and Refreshments | |
| 4:00–4:25 | Lucia Di Vizio | q-difference equations with $\|q\| = 1$ |
| 4:30–4:55 | Alexander Levin | Dimension polynomials of intermediate fields of a finitely generated difference-differential field extension |

# 2 Abstracts of Workshop Presentations

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## Degree Bounds for Gröbner Bases in Algebras of Solvable Type

**Matthias Aschenbrenner, UCLA**

**Abstract:** The class of algebras of solvable type was introduced in the 1980s by Kandri-Rody and Weispfenning as a common generalization of commutative polynomal rings, Weyl algebras, and universal enveloping algebras of finite-dimensional Lie algebras. They also developed an algorithmic theory of Gröbner bases in this setting, which is in wide use today. I will explain how a method due to Dubé can be adapted to establish doubly-exponential degree bounds for Gröbner bases in algebras of solvable type. This yields doubly-exponential degree bounds for ideal membership and syzygies, generalizing classical results of Hermann and Seidenberg (in the polynomial case) and Grigoriev (in the case of Weyl algebras). (Joint work with Anton Leykin.)

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## Generators and Defining Relations for the Ring of Differential Operators on a Smooth Affine Algebraic Variety

**V. Bavula, University of Sheffield**

**Abstract:** For the ring of differential operators on an arbitrary smooth affine algebraic variety over a field of characteristic zero, a finite set of algebra generators and a finite set of defining relations are found explicitly. As a consequence, a finite set of generators and a finite set of defining relations are given for the module of derivations on the algebra of regular functions on the variety.

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## Lie-Vessiot Systems, Algebraic Superposition Laws And Strongly Normal Extensions

**David Blázquez-Sanz, Universidad Sergio Arboleda**

**Abstract:** In this talk we discuss the notion of algebraic Lie-Vessiot system. These systems are algebraic differential equations which admit superposition laws. They are the natural generalization of linear differential equations, and cover the case of Riccati equations, Weierstrass equation, linear flow in abelian varieties and so on. One particular and interesting case is that of Vessiot's automorphic systems. They are the geometrical models for G-primitive extensions of Kolchin. In general Lie-Vessiot systems are solved by terms of strongly normal extensions.

References:

[1] David Blazquez-Sanz "Differential Galois Theroy and Lie-Vessiot Systems" PhD. Thesis, Barcelona 2008.
[2] David Blazquez-Sanz and Juan J. Morales-Ruiz, Global Conditions for Lie's Superposition Theorem, 2008, preprint.

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## On Cartan's Generalizations of Lie's Third Theorem

**Robert Bryant, Duke & MSRI**

**Abstract:** In his fundamental works in the first decade of the 20th century on what are now called Lie pseudo-groups, Elie Cartan proved several generalizations of Lie's Third Theorem on the (local) existence of Lie groups. These results anticipate by many years the theory of Lie groupoids and Lie algebroids that is still being developed.

In this talk, I will review Cartan's results in this direction and illustrate their importance for differential geometry by showing how they apply to solve several recent problems in differential geometry, particularly problems involving the existence of metrics with special holonomy and prescribed curvature problems.

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## Non-linear Differential Galois Theory

**Guy Casale, IRMAR UMR 6625, Université de Rennes 1**

**Abstract:** I will present Malgrange definition of the Galois groupoid (or pseudo group) of a vector field on a algebraic variety (over C). Notions of frames bundles and prolongations will be recalled. Some applications to reducibility or integrability of differential systems will be presented. To finish, I will examine the possibility of a definition over any differential field.

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## Differential Gerstenhaber Algebras and Weak Mirror Symmetry of Nilpotent Lie Algebras

**Richard Cleyton, Humboldt University**

**Abstract:** (Co-authors Yat Sun Poon, UCR , Riverside, CA and Jorge Lauret, CIMAT, Cordoba, Argentina) Weak mirror symmetry is the quasi-isomorphism of (1) the differential Gerstenhaber algebra (DGA) governing the deformation theory of a complex structure on a compact space, with (2) the complexification of differential Gerstenhaber algebra associated to the deformations of a symplectic structure on another. We study this problem for the case of left-invariant structures on (compact quotients of) Lie groups. The isomorphy problem for the DGAs associated to complex structures on 6-dimensional nilpotent algebras is treated. As an application, all pseudo-Kählerian complex structures on six-dimensional

nilpotent algebras such that the DGA of its complex structure is quasi-isomorphic to that of its symplectic structure are classified. For Lie algebras that decompose into a semi-direct product of a subgroup g with an abelian ideal V the existence of a left symmetric algebra structure on g gives rise to a canonically defined complex structure on g+V and a symplectic structure on $g + V^*$. We verify that the associated differential Gerstenhaber algebras controlling the deformation theories of the complex and symplectic form are isomorphic thus giving examples of "weak mirror symmetry".

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## On the Index and the Order of Quasi-regular DAE Systems

**Lisi D'Alfonso, Universidad de Buenos Aires**

**Abstract:** The talk will discuss a joint paper with G. Jeronimo, G. Massaccesi and P. Solerno (http://arxiv.org/abs/0804.4842). We study the differentiation index and the order for quasi-regular ordinary differential algebraic equation (DAE) systems. We give an algebraic definition of the differentiation index and prove a Jacobi-type upper bound for the sum of the order and the differentiation index. Our techniques also enable us to obtain an alternative proof of a combinatorial bound proposed by Jacobi for the order. As a consequence of our approach we deduce an upper bound for the Hilbert-Kolchin regularity and an effective ideal membership test for quasi-regular DAE systems. Finally, we prove a theorem of existence and uniqueness of solutions for implicit differential systems.

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## $q$-Difference Equations with $|q| = 1$

**Lucia Di Vizio, Institut de Mathématiques de Jussieu**

**Abstract:** The paper is on my web page. I establish, under convenient diophantine assumptions, an analytic classification of $q$-difference modules over the field of germs of meromorphic functions at zero, proving some analytic analogs of the results by Soibelman and Vologodsky, cf. math.AG/0205117, and by Baranovsky and Ginzburg, cf. alg-geom/9607008.

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## Liouvillian Solutions of Linear Differential-Difference Equations

**Ruyong Feng, Key Laboratory of Mathematics Mechanization,
Institute of Systems Science, AMSS**

**Abstract:** For a field $k$ with an automorphism $\sigma$ and a derivation $\delta$, we introduce the notion of liouvillian solutions of linear difference-differential systems $\{\sigma(Y) = AY, \delta(Y) = BY\}$ over $k$ and characterize the existence of liouvillian solutions in terms of the Galois group of the systems. We will give an algorithm to decide whether such a system has liouvillian solutions when $k = C(x,t), \sigma(x) = x + 1, \delta(t) = d/dt$ and the size of the system is a prime.

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## A Criterion for Testing Whether a Difference Ideal is Reflexive and Prime

**Xiao-Shan Gao, Institute of Systems Science, Chinese Academy of Sciences**

**Abstract:** It is known that a difference ideal is a reflexive prime ideal if and only if it can be written as the saturation ideal of a strong irreducible difference ascending chain. But, there exist no methods to test whether a difference ascending chain is strong irreducible except in the linear case. In this poster, we give a criterion to test whether a class of nonlinear difference ascending chains is strong irreducible. As a consequence, we can test whether the saturation ideal of such an ascending chain is reflexive and prime.

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## Some Differential Algebraic Structures Induced by Hopf Algebras of Labeled Trees

**Robert Grossman, University of Illinois at Chicago**

**Abstract:** Over the past two decades, a number of different Hopf algebras associated with trees, labeled trees, heap ordered trees, and planar trees have been defined. We review these constructions and describe a construction based upon Hopf algebras of trees that produces natural families of differential algebraic structures.

This is joint work with Richard Larson.

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## Applications of Patching

**Julia Hartmann, RWTH Aachen University**

**Abstract:** Patching methods - building a global object by constructing it locally - have been used in various contexts, in particular, in inverse Galois theory. This talk explains the method of patching over fields,n which was recently developed in joint work with D. Harbater to attack inverse problems in differential Galois theory. We give applications to differential algebra and other related areas.

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## Constructive Algebra for Differential Invariants

**Evelyne Hubert, INRIA Sophia Antipolis**

**Abstract:** Whether algebraic or differential, one can distinguish two families of applications for invariants of group actions: equivalence problems, together with classification and canonical forms, and symmetry reduction. In this latter case invariants are used to take into account the symmetry of a problem, mainly in order to reduce its size or its analysis. The

computational requirements include then four main components: the explicit computation of a generating set of invariants, and the relations among them (syzygies); procedures for rewriting the problem in terms of the invariants; and finally procedures for computing in the algebra of invariants. Those have been coherently addressed in a series of papers. They form a full algorithmic suite that is implemented in the Maple package AIDA that works on top of the library DifferentialGeometry and the extension of the library diffalg to non commutative derivations.

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## Topological Galois Theory

**A.G. Khovanskiĭ, University of Toronto**

**Abstract:** A Topological Galois Theory which I recently completed provides topological restrictions to solvability of algebraic and differential equations "in finite terms". This question has a rich history.

First proofs of unsolvability of algebraic equations by radicals were found by Abel and Galois. Thinking on the problem of explicit indefinite integration of an algebraic differential form, Abel founded the theory of algebraic curves. Liouville continued Abel's work and proved the non-elementarity of indefinite integrals of many algebraic and elementary differential forms. Liouville was also the first to prove the unsolvability of many linear differential equations by quadratures.

The relationship between the solvability by radicals and the properties of a certain finite group (the so called Galois group of an algebraic equation) goes back to Galois. The notion of finite group introduced by Galois was motivated exactly by this question. Sophus Lie introduced the notion of continuous transformation group while trying to solve differential equations explicitly and to reduce them to a simper form. With each linear differential equation, Picard associated its Galois group, which is a Lie group (and, moreover, a linear algebraic group). Picard and Vessiot showed that this particular group is responsible for the solvability of equations by quadratures. Kolchin developed the theory of algebraic groups and elaborated the Picard–Vessiot theory.

Arnold discovered that many classical mathematical questions are unsolvable for topological reasons. In particular, he showed that the general algebraic equation of degree $\geqslant 5$ is unsolvable by radicals exactly for topological reasons. While developing Arnold's approach, in the beginning of 70s, I constructed a peculiar one-dimensional topological variant of the Galois theory. According to this theory, the way how the Riemann surface of an analytic function covers the complex plane can obstruct the representability of this function by explicit formulas. In this way, the strongest known results on non-expressibility of functions by explicit formulas are obtained. Recently, I succeeded to generalize these topological results to the multivariable case.

No preliminary knowledge is required.

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## Differential Schemes

**Jerry Kovacic, CCNY (CUNY)**

**Abstract:** We use the definition of differential scheme that is analogous to the definition of scheme in Hartshorne. Immediately we find problems: for example the global section functor does not recover the ring of an affine differential scheme. We give some examples to show what goes wrong. However reduced differential scheme seem to behave nicely. We present some of the more recent results.

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## $PGL_n$ as a Differential Galois Group

**Arne Ledet, Texas Tech University**

**Abstract:** Using a recent result about Galois cohomology by Chernousov, Gille and Reichstein, we attempt to give a complete description of $PGL_n$-torsors over a standard differential field, leading to a parameterized description of Picard-Vessiot extensions with Galois group $PGL_n$.

This work is part of my joint research with Lourdes Juan.

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## Dimension Polynomials of Intermediate Fields of a Finitely Generated Difference-Differential Field Extension

**Alexander Levin, The Catholic University of America**

**Abstract:** Let $K$ be a difference-differential field of zero characteristic with basic sets $\Delta = \{\delta_1, \dots, \delta_m\}$ and $\sigma = \{\alpha_1, \dots, \alpha_n\}$ of derivation operators and automorphisms of $K$, respectively. Let $\Lambda$ denote the free commutative semigroup of all elements of the form $\lambda = \delta_1^{k_1} \dots \delta_m^{k_m} \alpha_1^{l_1} \dots \alpha_n^{l_n}$ where $k_i \in \mathbf{N}$, $l_j \in \mathbf{Z}$ $(1 \leqslant i \leqslant m,\, 1 \leqslant j \leqslant n)$, let the order of such an element be defined as $ord\,\lambda = \sum_{i=1}^{m} k_i + \sum_{j=1}^{n} |l_j|$, and for any $r \in \mathbf{N}$, let $\Lambda(r) = \{\lambda \in \Lambda \,|\, ord\,\lambda \leqslant r\}$.

Let $L = K\langle \eta_1, \dots, \eta_s \rangle$ be a difference-differential field extension of $K$ generated by a finite set $\eta = \{\eta_1, \dots, \eta_s\}$ and let $F$ be an intermediate difference-differential field of the extension $L/K$. Furthermore, for any $r \in \mathbf{N}$, let $L_r = K(\bigcup_{i=1}^{s} \Lambda(r)\eta_i)$ and $F_r = L_r \cap F$.

We prove the existence and describe some properties of a polynomial $\phi_{\eta,F}(t) \in \mathbf{Q}[t]$ such that $\phi_{\eta,F}(r) = trdeg_K F_r$ for all sufficiently large $r \in \mathbf{N}$. This result implies the existence of a dimension polynomial that describes the strength of a system of differential equations with a group action in the sense of A. Einstein. We shall also present a more general result, a theorem on a multivariate dimension polynomial associated with an intermediate difference-differential field $F$ and partitions of basic sets $\Delta$ and $\sigma$.

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## Computation of Bernstein-Sato Polynomials

**Anton Leykin, University of Illinois at Chicago**

**Abstract:** Obtaining Bernstein-Sato polynomials is one of the central problems of algorithmic D-modules theory. It can be solved using Groebner bases in rings of linear differential operators. We will discuss the notions of both classical and generalized Bernstein-Sato polynomials, algorithms to compute them, as well as possible applications. The presentation will be accompanied by a demonstration of computations in Macaulay 2 using the D-modules package.

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## Representations of Differential Rota-Baxter Algebras

**Zongzhu Lin, Kansas State University**

**Abstract:** It is known that for a differential algebra $A$ over $k$, the category of differential representations is an abelian category and equivalent to the category of all representations of the smash product algebra $A\#k[x]$. We will discuss the representations of the Rota-Baxter representations of Rota-Baxter algebras and prove that the category of Rota-Baxter representations of a Rota-Baxter algebra is an abelian category which is equivalent to the module category of an associative algebra. We will discuss properties of representations of differential Rota-Baxter algebras as well. Some examples of the representation theory will also be discussed.

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## Holomorphic Bundles over Noncommutative Tori and $\mathrm{Rep}(\hat{\mathbb{Z}})$

**Snigdhayan Mahanta, University of Toronto & IHES**

**Abstract:** The category of holomorphic bundles over noncommutative tori will be introduced. This will be done in the setting of differential algebras. The construction of the subcategory of semistable bundles of degree zero will be outlined. Finally it will be shown that the category of finite dimensional representations of $\hat{\mathbb{Z}}$ lives in this subcategory. The motivation of this construction is to have a notion of an étale fundamental group of noncommutative tori. This is a joint work with W. D. van Suijlekom.

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## Galois Theory of Frobenius Modules

**B. Heinrich Matzat, University of Heidelberg**

**Abstract:** Frobenius difference modules are very related to differential modules in positive characteristic. In particular, Galois groups of Frobenius modules are linear algebraic groups (or group schemes, respectively,) over the base field. In this lecture we explain the Galois theory of Frobenius modules and show methods to construct Frobenius modules with nice groups. This applies, among other things, to ordinary Galois theory in positive characteristic, to iterative differential Galois theory, and to the Galois theory of $t$-motives.

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## Free Differential Calculus for Free Algebras of Schreier Varieties

**A. A. Mikhalev, Moscow State University**

**Abstract:** A variety of linear algebras is Schreier if any subalgebra of a free algebra of this variety is free. The main types of Schreier varieties of algebras are the variety of all algebras, the variety of all commutative algebras, the variety of all anti-commutative algebras, the variety of all Lie algebras, the variety of all Lie superalgebras and varieties of (all) restricted Lie algebras and superalgebras. In the talk we present results for free algebras of Schreier varieties of algebras based on the free differential calculus (the technique of universal derivations). In particular, we study automorphic orbits of elements of free algebras and construct a series of computer algebra algorithms for symbolic computation in free algebras. We discuss the structure of the automorphism groups of free algebras. We consider also similar problems for free associative algebras and for free Leibniz algebras.

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## A Proof of the Dimension Conjecture and of Jacobi's Bound

**F. Ollivier, École Polytechnique**

**Abstract:** Let $P_1, \ldots, P_n$ be differential polynomials in $\mathcal{F}\{x_1, \ldots, x_n\}$, where $\mathcal{F}$ is an ordinary differential field of characteristic 0. It is conjectured that if $\mathcal{P}$ is a zero dimensional component of $\{P\}$, then the order of $\mathcal{P}$ is bounded by $\max_{\sigma \in S_n} \sum_{i=1}^{n} \mathrm{ord}_{x_i} P_{\sigma(i)}$. This result, first introduced by Jacobi, was proved by Ritt [4] in the linear case, and by Kondratieva *et al.* [3] for ideals that satisfy Johnson's regularity condition [2].

Richard Cohn [1] has proved that Jacobi's bound implies the dimension conjecture: let $P_1, \ldots, P_r$ be differential polynomials in $\mathcal{F}\{x_1, \ldots, x_n\}$, then if $\mathcal{P}$ is a component of $\{P\}$, the dimension of $\mathcal{P}$ is at least $n - r$. This result has been proved by Ritt [5] for $r = 1$.

We extend Ritt's proof, based on the construction of a Puiseux series solutions, to the general case. Then, we adapt to the irregular case some arguments suggested by Jacobi's constructions in order to deduce Jacobi's bound from the dimension conjecture.

**References.**

[1] COHN (Richard M.), "Order and dimension", *Proc. Amer. Math. Soc.* **87** (1983), n° 1, 1–6.

[2] JOHNSON (Joseph), "Systems of $n$ partial differential equations in $n$ unknown functions: the conjecture of M. Janet", *Trans. of the AMS*, vol. 242, Aug. 1978.

[3] KONDRATIEVA (Marina Vladimirovna), MIKHALEV (Aleksandr Vasil'evich), PANKRATIEV (Evgeniĭ Vasil'evich), "Jacobi's bound for systems of differential polynomials" (in Russian), *Algebra.* M.: MGU, 1982, s. 79-85.

[4] RITT, (Joseph Fels), "Jacobi's problem on the order of a system of differential equations", *Annals of Mathematics*, vol. 36, 1935, 303–312.

[5] RITT, (Joseph Fels), 1950. *Differential Algebra*, Amer. Math. Soc. Colloq. Publ., vol. 33, A.M.S., New-York.

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## Differential Elimination and Bounding Orders in Effective Differential Nullstellensatz

**Alexey Ovchinnikov, University of Illinois at Chicago**

**Abstract:** We discuss how one can obtain an explicit upper bound for orders of derivatives in differential Nullstellensatz using differential elimination and dicksonian sequences. After differentiating a polynomial system of PDEs up to this bound one can test consistency of the system just by algebraic methods.

This is a joint work with Oleg Golubitsky, Marina Kondratieva, and Agnes Szanto.

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## "Locality" as a Driving Principle for Renormalization

**Sylvie Paycha, Université Blaise Pascal**

**Abstract:** The locality principle in physics according to which distant objects cannot have direct influence on one another, can be transposed to a multiplicative property of renormalized multiple sums or integrals arising in mathematical physics; renormalized multiple sums and integrals with constraints should factorize over disjoint sets of constraints. Renormalized (or generalized) evaluators that provide a way to extract a finite part from meromorphic functions in several variables fulfill a similar multiplicative property. We shall show how they can be combined to meromorphic extensions of multiple integrals and sums in order to define renormalized multiple integrals with linear constraints reminiscent of Feynman integrals in physics, and renormalized multiple sums with conical constraints, which generalize multiple zeta functions in number theory.

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## Logarithmic Derivatives on Nonconstant Commutative Algebraic Groups, and Transcendence

**Anand Pillay, University of Leeds**

**Abstract:** This is joint work with D. Bertrand. I will discuss statements and proofs of a functional/differential algebraic analogue of the classical theorem that the exponentials of a set of $\mathbb{Q}$-linearly independent algebraic numbers are algebraically independent.

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## Isomonodromic Garnier System and Geometry of Higher Painlevé VI Equations

**Emma Previato, Boston University**

**Abstract:** R. Fuchs in 1905 revealed that the isomonodromy condition for a second-order differential equation with four regular singular points is equivalent to Painlevé VI. R. Garnier in one part of his thesis extended the analysis to the case of $g + 3$ regular singular points, expressing the isomonodromy property by a system of ODEs, and interpreting its integrability by means of hyperelliptic integrals on a genus-$g$ curve. In joint work with V.Z. Enolskii and F.W. Nijhoff, we study Garnier's system, establish its Lagrangian structure, and propose its connection to Picard-(L. )Fuchs equations via Klein's hyperelliptic $\sigma$ function, giving a reformulation to be thought of as a generalized Painlevé equation, in analogy to the $g = 1$ case of R. Fuchs. To further connect Garnier's equations and hyperelliptic curves, the last part of the talk will outline recent work by A.M. Levin, M.A. Olshanetsky and A.V. Zotov, which connects integrable Lax-pair models with their quantized isomonodromy counterpart; in their work, Painlevé VI results as the nonautonomous (*ad hoc*) version of a linear integrable hierarchy of elliptic type; the time parameter is introduced by performing a Hecke transformation. The proposal is to deform hyperelliptic Lax pairs, serving as quasi-classical limit of the Garnier system.

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## Differential Galois Theories from Model Theory

**Tom Scanlon, UC Berkeley**

**Abstract:** I will present three different approaches to differential Galois theory based on ideas from model theory:

(1) Differential Galois theory from the theory of liaison groups with special emphasis on infinite dimensional Galois theory for PDEs,

(2) Linearization and the associated differential Galois groupoids based on the constructions of differential arc, jet and prolongation spaces, and

(3) Comparison of differential and difference Galois groups based on the model theory of valued D-fields.

The first of these topics has been developed by Poizat, Pillay, Marker, Hrushovski and Kamensky though the specific consequences for infinite dimensional extensions of partial differential fields has not been clearly exposed in the literature even though all of the ingredients are there. I will compare this theory with that of Landesman which has been explained in important examples by Cassidy and Singer. The second theory is based on constructions of Moosa and myself and bears close formal analogies to the theories of Malgrange and Umemura. The last theory is based on my theory of D-henselian fields and a general Galois theory of valued fields with operators that is being developed by Anderson for her doctoral dissertation and is related to earlier constructions of Kreimer and Matzat.

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## On Factorization of Multivariate Polynomials and de Rham Cohomology

**Hani Shaker, COMSATS Institute of Information Technology (CIIT)**

**Abstract:** For any polynomial $P \in \mathbb{C}[X_1, X_2, ..., X_n]$, we describe a $\mathbb{C}$-vector space $F(P)$ of solutions of a linear system of equations coming from some algebraic partial differential equations such that the dimension of $F(P)$ is the number of irreducible factors of $P$. Moreover, the knowledge of $F(P)$ gives a complete factorization of the polynomial $P$ by taking gcds. This generalizes previous results by Ruppert and Gao in the case $n = 2$.

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## Differential Transformations for Second-Order Bivariate Parabolic LPDOs (Linear Partial Differential Operators)

**Ekaterina Shemyakova, University of Linz**

**Abstract:** The Laplace transformations (not to be confused with the Laplace transforms) method is probably the oldest symbolic method for integration of PDEs. This Laplace method was suggested for the second-order bivariate hyperbolic LPDOs. Many generalizations and modifications of the Laplace methods are known (and mainly developed at the end of the last century). However, the case of parabolic second-order LPDOs remained uncovered.

For such LPDOs I present differential transformations, which are analogous to the Laplace ones. Recent results on their invertibility, nice determinant formulae for their computation, whenever some partial solutions are known, and others are shown.

The presentation is based on the joint work with S.Tsarev.

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## Extensions by Antiderivatives and Iterated Logarithms

**V. Ravi Srinivasan, University of Oklahoma**

**Abstract:** We will discuss a new proof for the Kolchin-Ostrowski theorem and use that theorem to classify finitely differentially generated subfields of a special tower of extensions by antiderivatives, namely, the extensions by all iterated logarithms. In the process, we will show that iterated logarithms are algebraically independent over the ground differential field and discuss the normality of such extensions. We will also produce an algorithm to compute the finitely differentially generated subfields of these extensions. For further information please visit http://math.ou.edu/ vsrinivasan/Thesis-I.pdf

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## Differentiably Finite Power Series and Combinatorial Identities

**Earl Taft, Rutgers University at New Brunswick**

**Abstract:** The sequence of coefficients of a $D$-finite power series is a polynomially recursive sequence, i.e., satisfies a finite-order recursion with polynomial coefficients. The difference algebra of all such sequences forms a topological bialgebra which is dual to the polynomial algebra $k[x]$, $k$ the base field. The coproduct of a polynomially recursive sequence, when evaluated at a suitable basis of $P \otimes P$, yields a quadratic identity on the coordinates of the sequence, often of a combinatorial nature. For details,see "Bialgebras of Recursive Sequences and Combinatorial Identities" by C. A. Futia, E. F. Muller and E. J. Taft, *Advances in Appl. Math.* **28** (2002), 203-230, in which we derived two examples of such combinatorial identities by comultiplying the sequences $(n!)$ and $(n(n!))$, the first yielding the Chu-Vandermonde identity. Now we have extended these coproduct formulas to one for each sequence $((n/k)(n!))$ for each non-negative $k$, $(n/k)$ the binomial coefficient. This yields corresponding hypergeometric combinatorial identities. (Joint with E. F. Muller)

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## Derivations of an Effective Divisor on the Complex Projective Line

**Max Wakefield, US Naval Academy**

**Abstract:** In this talk we consider an effective divisor on the complex projective line and associate with it the module $D$ consisting of all the derivations $\theta$ such that $\theta(I_i) \subset I_i^{m_i}$ for every $i$, where $I_i$ is the ideal of $p_i$. The module $D$ is graded and free of rank 2; the degrees of its homogeneous basis, called the exponents, form an important invariant of the divisor. It turns out that under certain conditions on $(m_i)$ the exponents do not depend on $\{p_i\}$. Our main result asserts that if these conditions do not hold for $(m_i)$ then there exists a general position of $n$ points for which the exponents do not change. We give an explicit

formula for them. If time permits we will also exhibit some examples of degeneration of the exponents, in particular those where the degeneration is defined by vanishing of certain Shur functions. As application and motivation, we show that our results imply Terao's conjecture (about the combinatorial nature of the freeness of hyperplane arrangements) for certain new classes of arrangements of lines in the complex projective plane.

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## Renormalization on Toric Varieties

**Bin Zhang, Sichuan University**

**Abstract:** Toric varieties are a family of very important algebraic varieties, with applications in number theory, algebraic geometry, mathematical physics and geometry analysis. The Todd classes of toric varieties play key roles in applications. Because of combinatorial nature of a toric variety, it is natural to search for combinatorial ways to handle toric varieties. In this talk, we will present a natural way by adapting renormalization method. We will construct a Hopf algebra of cones and an algebraic homomorphism from this Hopf algebra to the algebra of meromorphic functions. And then the application of renormalization method to toric varieties will be discussed.

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# 3 Workshop Organization

## 3.1 Program Committee

- Li Guo, Rutgers University at Newark (liguo@rutgers.edu)
  Phone: (O) 1-973-353-5156 ext. 3917, (H) 1-609-275-5191
- Matilde Marcolli, Max-Planck Institute in Mathematics (marcolli@mpim-bonn.mpg.de)
- Michael Singer, North Carolina State University (singer@math.ncsu.edu )
- William Sit, City College of CUNY (wyscc@sci.ccny.cuny.edu)
  Phone: (O) 1-212-650-5179, (H) 1-718-549-2765, (C) 1-347-251-2390
- Jim Stasheff, University of Pennsylvania (jds@math.upenn.edu)

## 3.2 Local Committee

- Li Guo, Rutgers University at Newark (liguo@rutgers.edu)
  Phone: (O) 1-973-353-5156 ext. 3917, (H) 1-609-275-5191
- William Keigher, Rutgers University at Newark (keigher@rutgers.edu)
  Phone: (O) 1-973-353-5156 ext. 3916, (H) 1-732-574-1835

(H)=Home, (O)=Office, (C)=Cell
*Feel free to contact any of the organizers if you need help.* Please note that 1 and the area codes may have to be dialed even for local calls.

# 4 Workshop Participants

Matthias Aschenbrenner, UCLA
Vlad Bavula, University of Sheffield
Yuri Berest, Cornell University
David Blázquez-Sanz, Universidad Sergio Arboleda
Robert Bryant, Duke University & MSRI
Guy Casale, IRMAR UMR 6625, Universite de Rennes 1
Sheng Chen, Kansas State University and Harbin Institute of Technology
Zao Chen, City Tech (CUNY)
Richard Churchill, Hunter College and the Graduate Center (CUNY)
Richard Cleyton, Humboldt University of Berlin
Lisi D'Alfonso, Universidad de Buenos Aires
Lucia Di Vizio, Institut de Mathmatiques de Jussieu
Ruyong Feng, North Carolina State University and Acad. Math and System Sciences
Scott Forrest, Oakland University
Xiaoshan Gao, Chinese Academy of Sciences
Bob Grossman, University of Illinois at Chicago
Li Guo, Rutgers University of Newark
Charlotte Hardouin, University of Paul Sabatier
Julia Hartmann, RWTH Aachen University
Evelyne Hubert, INRIA Sophia Antipolis
Roy Joshua, Ohio State University
Lourdes Juan, Texas Tech University
William Keigher, Rutgers University Newark
Askold Khovanskiĭ, University of Toronto
Jerry Kovacic, CCNY (CUNY)
Arne Ledet, Texas Tech University
Alexander Boris Levin, Catholic University of America
Anton Leykin, University of Illinois at Chicago
Fang Li, Zhejiang University
Zongzhu Lin, Kansas State University
Ling Long, Iowa State University
Andy Magid, University of Oklahoma
Snigdhayan Mahanta, University of Toronto
Matilde Marcolli, Max-Planck Institute in Mathematics in Bonn
B. Heinrich Matzat, University of Heidelberg
A. A. Mikhalev, Moscow State University
A. V. Mikhalev, Moscow State University
Brian Miller, Texas Tech University
John Michael Nahay, Burlington County College
Richard Ng, Iowa State University
François Ollivier, Ecole Polytechnique

Alexey Ovchinnikov, University of Illinois at Chicago
Sylvie Paycha, Université Blaise Pascal
Bogdan Petrenko, SUNY at Brockport
Anand Pillay, University of Leeds
Emma Previato, Boston University
Safdar Quddus, Washington University in St. Louis
Camilo Sanabria, Graduate Center (CUNY)
Tom Scanlon, U.C. Berkeley
Hani Shaker, COMSATS Institute of Information Technology (CIIT)
Ekaterina Shemyakova, University of Linz
Michael Singer, North Carolina State University
William Sit, CCNY (CUNY)
V. Ravi Srinivasan, University of Oklahoma
Jim Stasheff, University of Pennsylvania
Earl Taft, Rutgers University at New Brunswick
Max Wakefield, US Naval Academy
Ping Xu, Pennsylvania State University
Yifan Yang, National Chiao Tung University
Bin Zhang, Sichuan University
Ronghua Zhang, Yunnan University
Yang Zhang, University of Manitoba

# 5 Transportation and Parking in Newark

## 5.1 Between Newark and the Airports

There are three major airports in the NYC-Newark area: Newark Liberty International Airport (Newark Airport), JFK Airport and LaGuardia Airport. The phone number of the Port Authority of New York and New Jersey, which manages these airports, is 1-212-435-7000. Their web site is http://www.panynj.gov/

**Newark Airport:** The Robert Treat Hotel provides free shuttle service to and from the airport. At the airport, take the monorail train (AirTrain) to P4 to take the shuttles. The hotel shuttle picks up guest at that location on the hour and half past the hour. It is a white van labeled as Best Western Robert Treat Hotel. (There is a second Best Western at Newark Airport; do not take their shuttle!) A taxi between the airport and the hotels costs about $11.00 one way. There are also bus services between the airport and Newark Penn Station (note there is also a Penn Station in New York City). To get from Robert Treat Hotel to Newark Penn Station, you can take a cab or take the Newark Light Rail (see §.5.3) to Newark Penn Station from either Military Park Station or NJPAC/Center Street Station, a better choice since it is at ground level.

**JFK Airport or LaGuardia Airport:** You can take a cab or the Newark Light Rail to Newark Penn Station, as described in the last paragraph. From there you take one of the NJ Transit trains to New York Penn Station which is the last stop. Then you can take the subway or shuttle to the airports.

## 5.2 Between Robert Treat Hotel and Newark Campus

Please refer to maps in Figure 1 on page 22 and Figure 2 on page 24.

To walk from Robert Treat Hotel to Engelhard Hall, start from the front exit of the hotel, cross Park Place, and go directly through Military Park. Cross Broad Street at the New Street traffic light. Walk on New Street for two blocks, cross Washington Street, and continue on the pedestrian path (formerly New Street) for one more block. Engelhard Hall will be the last building on the left before crossing University Avenue. Bove Auditorium is on the first floor. Ackerson Hall will be the last building on the right before crossing University Avenue. Ackerson Hall 123 is the large lecture hall on the first floor.

## 5.3 Getting Around Newark: Newark Light Rail

There are two Light Rail services in Newark. One is between Newark Penn Station and Grove Street/Branch Brook Park. The second one is between Newark Penn Station and Broad Street. The (downtown) fare on the Grove Street service (valid between Rutgers-Newark and Newark Penn Station) is $0.65. The fare on the remainder of the system, including the Broad Street Station service, is $1.35. There are ticket vending machines at the platform.

**Please Note: You need to purchase a ticket** *and* **validate it with a time-stamp** *before* **boarding the subway cars, or there will be a heavy fine ($76 at last report).**

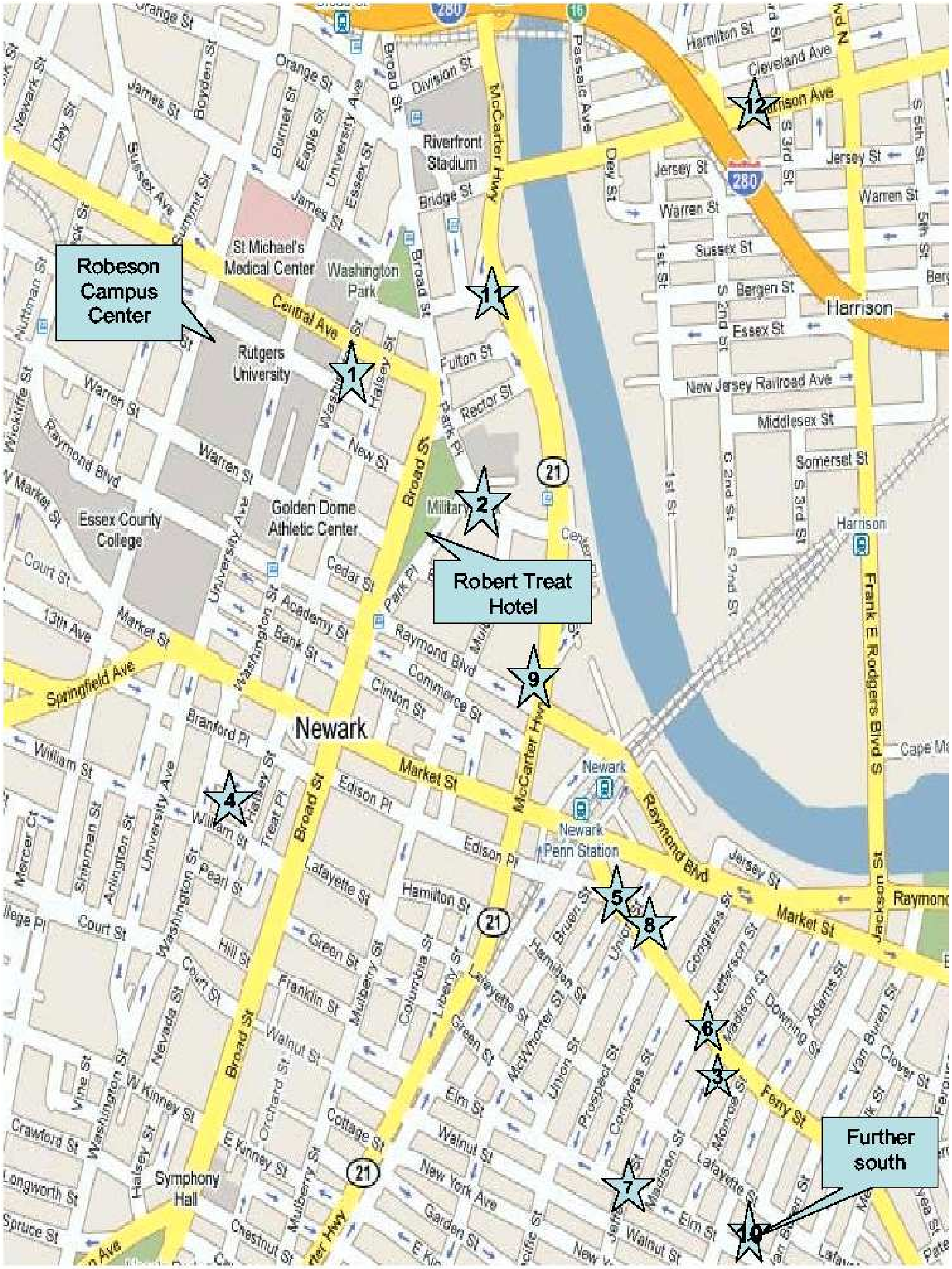


Figure 1: Map showing Robeson Campus Center, Robert Treat Hotel, Newark Penn Station (lower left), Broad Street Station (top) and local restaurants (key: p. 26).

**To get from Newark Penn Station to the Rutgers-Newark campus**: Take the service to either Branch Brook Park or Grove Street, and exit at the second station (Washington Street). When on the platform of the Washington Street Station, walk to the left (in the direction that the subway is traveling) to the stairs or elevator. At the top of the stairs, turn right onto Raymond Boulevard, turn right again at the first traffic light onto University Avenue, and walk two blocks (north) to Engelhard Hall, the first building past the parking garage on the right. Ackerson Hall is next building past Engelhard Hall.

**To get from Robert Treat Hotel to Newark Penn Station**: Both services below are a short (less than 5 minute) walk from the Robert Treat Hotel. The Grove Street service is found at the Military Park station, while the Broad Street service is found at the NJPAC/Center Street station.

**To get from Newark Penn Station to the Broad Street Station**: This is for connections to other rail services offered by NJ Transit. Take the service to Broad Street Station. This service also connects to other Newark cultural institutions such as the Performing Arts Center, the Newark Museum and the Newark Public Library.

More information about the Newark Light Rail and other NJ Transit services is available at `http://www.njtransit.com`.

## 5.4 Parking on the Newark Campus for the Workshop

Robert Treat Hotel provides complimentary parking. If you drive but do not stay at the hotel, you can also park in either of two locations on campus and ask for a coupon from Li Guo or William Keigher at the workshop. The two locations where a visitor can park are Parking Deck I, located at 200 University Avenue (between Bleeker Street and Warren Street), and Parking Deck II, located at 166 Washington Street (between Raymond Boulevard and Warren Street). The fee for parking is about $10. Parking Deck I is available from 7:00 a.m. to midnight on both Thursday and Friday, while Parking Deck II is available on Thursday (only) from 7:00 a.m. until 7:00 p.m. On Saturday and Sunday, ample parking is available in many University surface parking lots.

For driving instructions and parking deck locations, please visit

`http://www.newark.rutgers.edu/maps/`

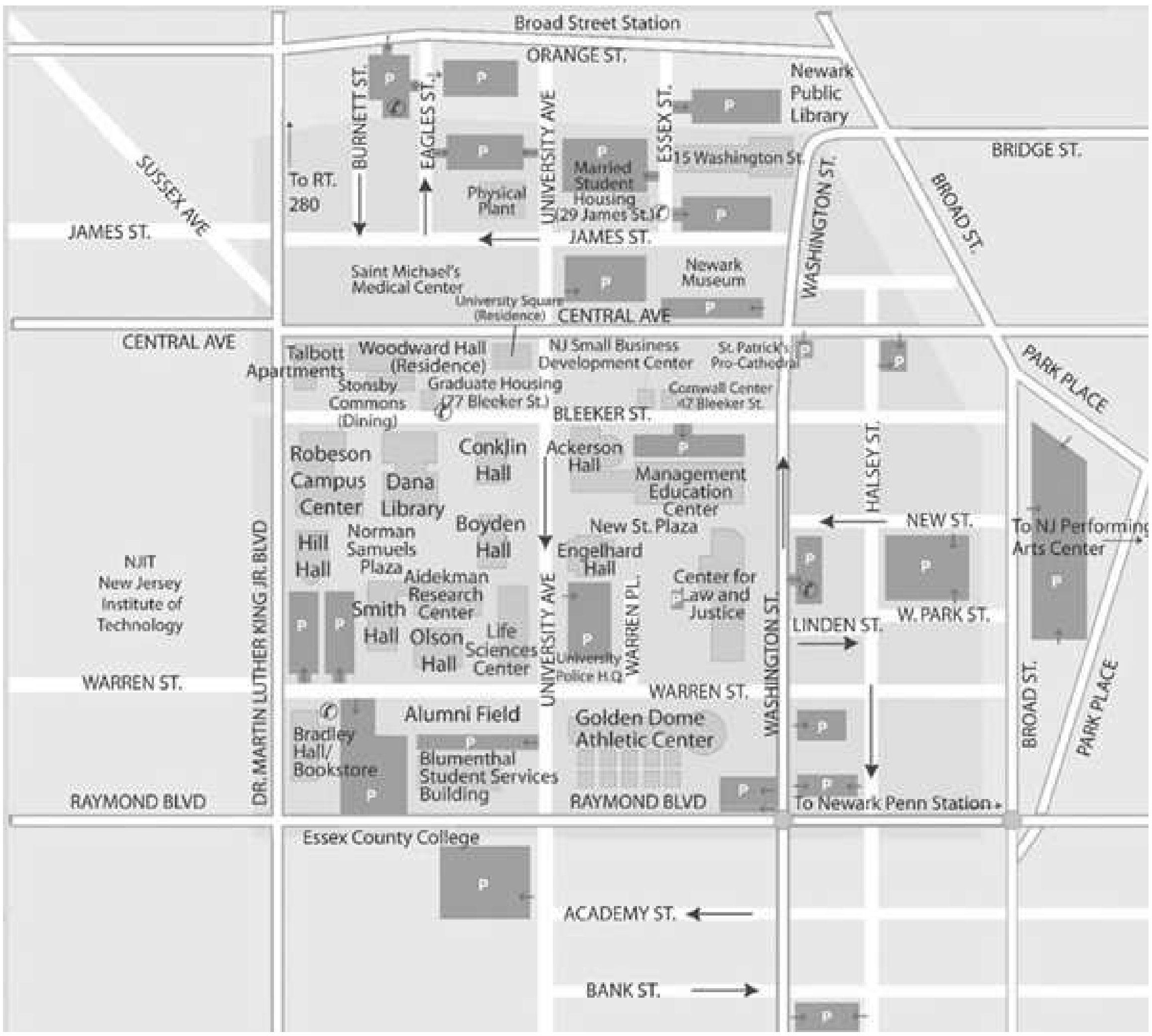


Figure 2: Campus Map for Rutgers University, Newark. The workshop will be in Engelhard Hall for the first day and in Ackerson Hall for the other days. The banquet will be in the Robeson Galleries on the first floor of Robeson Campus Center

# 6 Rutgers and Newark, Other Information

## 6.1 Some useful numbers

- **Emergency:** 911
- **Medical emergency on campus:**
  80 from any campus telephone, or 1-973-353-5111
- **Telephone operator:** 1766 from any campus telephone
- **Phone number inquiry:** 411
- **Department of Mathematics and Computer Science:**
  Phone: 1-973-353-5156, Fax: 1-973-353-5270
  Address: 216 Smith Hall, 101 Warren Street, Rutgers, The State University of New Jersey, Newark NJ 07102-1811
- **Campus Parking**: Phone 1-973-353-1839, Fax: 1-973-353-5873
  Address: 209 Blumenthal Hall, 249 University Avenue, Rutgers, The State University of New Jersey, Newark, NJ 07102

### 6.1.1 Police

- **Rutgers University Police Department (Newark):**
  Main phone: 1-973-353-5581
  Emergency (From University Extension): 5-111 or 80
  Address: 200 University Ave., Newark, NJ 07102
  http://nwkpolice.rutgers.edu/
- **Newark Police Department:**
  Main numbers: 1-973-733-6000
  Address: 31 Green Street, Newark, NJ 07102

### 6.1.2 Hospitals

- **Rutgers University Health Services Office**
  Blumenthal Hall, Room 104, 249 University Avenue, Newark, NJ 07102
  Phone: 1-973-353-5231
- **St Michael's Medical Center**
  111 Central Avenue Newark, NJ 07102, Phone: (973) 877-5000.
  St. Michael's Medical Center E.R. (973) 877 - 5500.

## 6.2 Newark Visitor Information

- Newark Museum (evening music), 49 Washington St., 1-973-596-6550
- New Jersey Performing Arts Center (NJPAC) (performing arts/music), One Center Street, Newark, NJ 07102 (next to Robert Treat), 888-GO-NJPAC
- Ferry St. (main Ironbound street) — Ferry St. is known as an area destination for nightlife — many venues are located along or near the street.
- Newark Symphony Hall (performing arts/music/comedy), 1030 Broad Street 973-643-8014
- Dining options will be listed in the welcoming package for Workshop participants.

For additional visitor information, visit `http://www.gonewark.com`.

## 6.3 Internet

Robert Treat Hotel has internet connection in all rooms. Some computers are open to the public at the Dana Library located at 185 University Ave. Their hours are M–Th 8am–12am, Fri. 8am–7pm, Sat. 10am–6pm and Sun. Noon–6pm.

# 7 Restaurants

See the separate flyer for restaurants near campus. See Figure 1 on page 24 for locations.

1. **27 Mix** 27 Halsey St, Newark, NJ 07102, 973-648-0643.
2. **Theater Square Grill** 1 Center St, New Jersey Performing Arts Center, Newark, NJ 07102, 973-642-1226.
3. **Brasilia Grill** 99 Monroe St, Newark, NJ 07105, 973-589-8682.
4. **Je's** 34 William St, Newark, NJ 07102, 973-623-8848.
5. **Fornos of Spain** 47 Ferry St, Newark, NJ 07105, 973-589-4767.
6. **Adega Grill** 130-132 Ferry St, Newark, NJ 07105, 973-589-8830.
7. **Casa Vasca** 141 Elm St, Newark, NJ 07105, 973-465-1350.
8. **Iberia Peninsula Restaurant** 63-69 Ferry St, Newark, NJ 07105, 973-344-5611.
9. **Don Pepe** 844 McCarter Hwy, Newark, NJ 07102, 973-623-4662.
10. **Assaggini Di Roma** 134 Clifford St, Newark, NJ, 973-466-3344.
11. **Seabra's Rodizio** 1034 Mccarter Hwy Newark, NJ 07102 (973) 622-6221.
12. **5 Grains Rice Restaurant** 231 Harrison Ave Harrison, NJ 07029 (973) 268-2811.