Calculus 1: Sample Questions, Final Exam

1. Short answer. Put your answer in the blank. **NO PARTIAL CREDIT!**

   (a) Evaluate $\int_{e^2}^{e^3} \frac{1}{x}\,dx$. Your answer should be in the form of *an integer.*

   (b) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\,d\theta$. Your answer should be in the form of *an integer.*

   (c) Compute $e^{\ln 3+\ln 2}$. Your answer should be in the form of *an integer.*

   (d) Compute $\int_{5}^{5} \frac{x^3}{e^x+9}\,dx$. Your answer should be in the form of *an integer.*

   (e) Evaluate $\int 3x^2 \sin(x^3+1)\,dx$. $+\,C$

   (f) Evaluate the derivative $D_x\, e^x$.

   (g) Compute the derivative $D_x \ln(x^2+1)$.

   (h) Evaluate $\int_{-1}^{1} \frac{y^5}{y^2+1}\,dy$. Your answer should be in the form of *an integer.*

   (i) Evaluate $\int_{2}^{10} 3\sqrt{z-1}\,dz$. Your answer should be in the form of *an integer.*

2. Let f be a continuous function on the interval $[0, 2]$ which satisfies $\int_0^2 f(x)\,dx = 5$. Given this information, compute the integral $\int_0^1 f(2y)\,dy$. Show your work and justify your answer.

3. A population of bacteria undergoes exponential growth. If at noon, there are 1000 bacteria, and there are 2000 by 2pm, when does the number of bacteria reach 8000? Show your work and simplify your answer.

4. Find the particular solution to the differential equation $\frac{dy}{dx} = xy + x$ which satisfies $y = 3$ when $x = 0$. Show your work.

5. Consider the following functions. Circle the one(s) which are concave up on an open interval containing $x = 0$. No explanation necessary.

- $\ln x$
- x^2
- $\cos x$
- $\frac{1}{x^2 - 1}$
- $\tan x$

6. Consider the function $g(x) = \frac{1}{2}x + \cos x$ for $0 < x < 2\pi$.

   (a) Find all the critical points of $g(x)$ for $0 < x < 2\pi$. Show your work. Hint: there are two of them.

   (b) Classify each of the critical points you found in part (a) as a local maximum or a local minimum (or neither). Justify your answers.

7. Compute the derivative $\dfrac{d}{dx}\left[\dfrac{x^2+1}{x^3(x-1)^2}\right]$. Show your work.

8. Consider the function $h(x) = \dfrac{e^x + e^{-x}}{2}$ for $-\infty < x < \infty$.

(a) Find the interval(s) on which $h(x)$ is increasing. Show your work.

(b) Show that $h(x)$ is always concave up.

9. Recall that $[\![x]\!]$ represents the greatest integer function ($[\![x]\!]$ is the greatest integer $\leq x$). Compute $\int_0^4 [\![x]\!]\, dx$. Show your work. (Hint: Draw the graph.)

10. (a) If $f(x) = \sin x$, what is $f'(x)$?

(b) For $f(x) = \sin x$, write down the formula for $f'(0)$ using the definition of the derivative.

(c) Use parts (a) and (b) to compute the limit $\lim_{h \to 0} \frac{\sin h}{h}$. Your answer should be in the form of *an integer*. Justify your answer.

11. Consider the function $p(r) = r^3 + 6r^2 + 9r - 4$.

(a) Find all the critical points of $p(r)$ for r in the interval $[-2, 2]$. Show your work.

(b) For which r does $p(r)$ attain its minimum and maximum on the interval $[-2, 2]$? Show your work.

12. Compute the limit $\lim_{u \to 3} \frac{u^2 - 9}{u^2 - 4u + 3}$. Show your work.

13. (a) Sketch the graph of a function $y = F(x)$ which satisfies all the following properties:

- $F(x)$ has domain $(0, \infty)$.
- $F(x)$ has a vertical asymptote at $x = 0$.
- $F(x)$ is increasing on the interval $(0, \infty)$.
- $F(x)$ is concave down on the interval $(0, \infty)$.

(b) Give a formula for a function $F(x)$ which satisfies all the properties listed in part (a). Justify your answer.