

Calculus 1: Sample Questions, Exam 2

1. Short answer. Put your answer in the blank. **NO PARTIAL CREDIT!**

(a) Evaluate $\int \cos x \, dx$. $+ C$

(b) Evaluate $G'(x)$ for $G(x) = \int_0^x t^2 \, dt$.

(c) Compute $\int (x^3 + x) \, dx$. $+ C$

(d) Compute the sum $\sum_{n=1}^4 n^2$. Your answer should be
in the form of *an integer*.

2. Identify the critical points and find the maximum value and the minimum value for $f(x) = x^3 - 3x + 2$ on the interval $[0, 2]$. Show your work.

3. Sketch the graph of a function $y = g(x)$ that has all the following properties:
- (a) The domain of g is the open interval $(-3, 3)$.
 - (b) g is an odd function.
 - (c) $\lim_{x \rightarrow 3^-} g(x) = +\infty$.
 - (d) $g'(x) < 0$ for x in $(-1, 1)$, while $g'(x) > 0$ for x in $(-3, -1)$ and for x in $(1, 3)$.
 - (e) $g'(-1) = g'(1) = 0$.
 - (f) $g''(x) < 0$ for x in $(-3, 0)$, while $g''(x) > 0$ for x in $(0, 3)$.
 - (g) $g''(0) = 0$.
 - (h) $g(1) = -2, g(-1) = 2$.

4. Show that for a rectangle of given perimeter 4, the one with the maximum area is a square. (Recall that for a rectangle with side lengths a and b , the perimeter is given by $2a + 2b$, and the area is given by ab .)

5. Find the particular solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ which satisfies the condition that $y = 4$ when $x = 1$. Show your work.

6. Consider $h(t) = 2t^3 - 3t^2 + 5$.

(a) Compute the critical points of $h(t)$. Show your work.

(b) Compute the intervals where $h(t)$ is increasing, and where $h(t)$ is decreasing. Show your work.

(c) Compute the intervals where $h(t)$ is concave up, and where $h(t)$ is concave down. Show your work.