Calculus I, Sections 1-5, Exam 1, Version A
Solution

1. (15 pts) Compute the following derivatives, and simplify your answers. Show your work if you want to be eligible for partial credit.

(a) (3 pts) $D_x(x^4 - 4x + 9)$.
Solution: $D_x(x^4 - 4x + 9) = 4x^3 - 4$.

(b) (3 pts) $D_x[\sin x (\cos x - 2)]$.
Solution:

\[
D_x[\sin x (\cos x - 2)] = \sin x D_x(\cos x - 2) + (\cos x - 2) D_x \sin x
\]

\[
= \sin x(-\sin x) + (\cos x - 2)(\cos x)
\]

\[
= -\sin^2 x + \cos^2 x - 2 \cos x
\]

(c) (3 pts) $D_x\sqrt{x^3 - 2}$.
Solution:

\[
D_x\sqrt{x^3 - 2} = \frac{1}{2\sqrt{x^3 - 2}} D_x(x^3 - 2)
\]

\[
= \frac{3x^2}{2\sqrt{x^3 - 2}}.
\]

(d) (3 pts) $D_x \sec x$.
Solution: $D_x \sec x = \sec x \tan x$.

(e) (3 pts) $D_x \frac{x^3 - 2x^2 + 1}{3x - 9}$.
Solution:

\[
D_x \frac{x^3 - 2x^2 + 1}{3x - 9} = \frac{(3x - 9)D_x(x^3 - 2x^2 + 1) - (x^3 - 2x^2 + 1)D_x(3x - 9)}{(3x - 9)^2}
\]

\[
= \frac{(3x - 9)(3x^2 - 4x) - (x^3 - 2x^2 + 1)(3)}{(3x - 9)^2}
\]

\[
= \frac{2x^3 - 11x^2 + 12x - 1}{3(x - 3)^2}.
\]

2. (a) (3 pts) Give an example of a function $p(x)$ whose derivative at 0 is equal to 2. In other words, $p'(0) = 2$. Verify your answer is correct.
Solution: For example, $p(x) = 2x$, $p'(x) = 2$, $p'(0) = 2$.

(b) (3 pts) Give an example of a function $g(x)$ which is not differentiable at $x = 0$. In other words, $g'(0)$ should not exist. Give a reason for your answer.
Solution: For example, $g(x) = |x|$. $g'(0)$ does not exist since the graph of $g$ has a corner at $x = 0$. 

1
3. (4 pts) Use the definition of the derivative \( f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \) to compute the derivative of \( f(x) = 2x^2 - 3x + 1 \). Show your work.

Solution:

\[
\begin{align*}
  f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
        &= \lim_{h \to 0} \frac{2(x + h)^2 - 3(x + h) + 1 - (2x^2 - 3x + 1)}{h} \\
        &= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h} \\
        &= \lim_{h \to 0} (4x + 2h - 3) \\
        &= 4x - 2(0) - 3 \\
        &= 4x - 3.
\end{align*}
\]

4. (3 pts) Let \( h(x) \) be a function which satisfies \( h'(4) = 3 \). If \( k(x) = h(x^2) \), compute \( k'(2) \). Show your work.

Solution: The Chain Rule says \( k'(x) = h'(x^2)D_x(x^2) = h'(x^2) \cdot 2x \). Then plug in \( k'(2) = h'(2^2) \cdot 2(2) = h'(4) \cdot 4 = 3 \cdot 4 = 12 \).

5. (3 pts) Compute the limit \( \lim_{y \to 3} \frac{y^2 - 3y}{y^2 + 5y - 24} \). Show your work.

Solution:

\[
\begin{align*}
  \lim_{y \to 3} \frac{y^2 - 3y}{y^2 + 5y - 24} &= \lim_{y \to 3} \frac{y(y - 3)}{(y - 3)(y + 8)} \\
        &= \lim_{y \to 3} \frac{y}{y + 8} \\
        &= \frac{3}{3 + 8} \quad = \frac{3}{11}.
\end{align*}
\]

6. (4 pts) Compute the tangent line to the graph of the relation \( y^3 + 3xy = 6 + y \) at the point \( (x, y) = (2, 1) \). Put your answer in the form \( y = mx + b \). Show your work.

Solution: Use Implicit Differentiation to compute \( y' \) when \( (x, y) = (2, 1) \):

\[
\begin{align*}
  (y^3 + 3xy)' &= (6 + y)' \\
  3y^2 y' + 3y + 3xy' &= y' \\
  3(1)^2 y' + 3(1) + 3(2)y' &= y' \\
  3y' + 3 + 6y' &= y' \\
  y' &= -\frac{3}{8}.
\end{align*}
\]

For the tangent line, the slope \( m = y' = -\frac{3}{8} \). So use the point slope formula for \( (x_0, y_0) = (2, 1) \) to show

\[
\begin{align*}
  y - y_0 &= m(x - x_0) \\
  y - 1 &= -\frac{3}{8}(x - 2) = -\frac{3}{8}x + \frac{3}{4} \\
  y &= -\frac{3}{8}x + \frac{7}{4}.
\end{align*}
\]
7. (6 pts) At time $t$ (measured in hours), the position $s(t)$ of a car along a road is given by $s(t) = 3t^3 - 2t$ mi.

(a) (2 pts) Compute the velocity of the car at time $t$.
   \textbf{Solution:} $v(t) = s'(t) = (9t^2 - 2)$ mi/hr.

(b) (2 pts) Compute the acceleration of the car at time $t$.
   \textbf{Solution:} $a(t) = v'(t) = 18t$ mi/hr^2.

(c) (2 pts) At what time is the acceleration equal to $9$ mi/hr^2?
   \textbf{Solution:} $a(t) = 18t = 9$ when $t = 1/2$ hour.