

Calculus I, Sections 1-5, Exam 1, Version A
Solution

1. (15 pts) Compute the following derivatives, and simplify your answers. Show your work if you want to be eligible for partial credit.

(a) (3 pts) $D_x(x^4 - 4x + 9)$.

Solution: $D_x(x^4 - 4x + 9) = 4x^3 - 4$.

(b) (3 pts) $D_x[\sin x(\cos x - 2)]$.

Solution:

$$\begin{aligned} D_x[\sin x(\cos x - 2)] &= \sin x D_x(\cos x - 2) + (\cos x - 2) D_x \sin x \\ &= \sin x(-\sin x) + (\cos x - 2)(\cos x) \\ &= -\sin^2 x + \cos^2 x - 2 \cos x \end{aligned}$$

(c) (3 pts) $D_x\sqrt{x^3 - 2}$.

Solution:

$$\begin{aligned} D_x\sqrt{x^3 - 2} &= \frac{1}{2\sqrt{x^3 - 2}} D_x(x^3 - 2) \\ &= \frac{3x^2}{2\sqrt{x^3 - 2}}. \end{aligned}$$

(d) (3 pts) $D_x \sec x$.

Solution: $D_x \sec x = \sec x \tan x$.

(e) (3 pts) $D_x \frac{x^3 - 2x^2 + 1}{3x - 9}$.

Solution:

$$\begin{aligned} D_x \frac{x^3 - 2x^2 + 1}{3x - 9} &= \frac{(3x - 9)D_x(x^3 - 2x^2 + 1) - (x^3 - 2x^2 + 1)D_x(3x - 9)}{(3x - 9)^2} \\ &= \frac{(3x - 9)(3x^2 - 4x) - (x^3 - 2x^2 + 1)(3)}{(3x - 9)^2} \\ &= \frac{2x^3 - 11x^2 + 12x - 1}{3(x - 3)^2}. \end{aligned}$$

2. (a) (3 pts) Give an example of a function $p(x)$ whose derivative at 0 is equal to 2. In other words, $p'(0) = 2$. Verify your answer is correct.

Solution: For example, $p(x) = 2x$, $p'(x) = 2$, $p'(0) = 2$.

- (b) (3 pts) Give an example of a function $g(x)$ which is not differentiable at $x = 0$. In other words, $g'(0)$ should not exist. Give a reason for your answer.

Solution: For example, $g(x) = |x|$. $g'(0)$ does not exist since the graph of g has a corner at $x = 0$.

3. (4 pts) Use the definition of the derivative $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to compute the derivative of $f(x) = 2x^2 - 3x + 1$. Show your work.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 3h}{h} \\ &= \lim_{h \rightarrow 0} (4x + 2h - 3) \\ &= 4x - 2(0) - 3 \\ &= 4x - 3. \end{aligned}$$

4. (3 pts) Let $h(x)$ be a function which satisfies $h'(4) = 3$. If $k(x) = h(x^2)$, compute $k'(2)$. Show your work.

Solution: The Chain Rule says $k'(x) = h'(x^2)D_x(x^2) = h'(x^2) \cdot 2x$. Then plug in $k'(2) = h'(2^2) \cdot 2(2) = h'(4) \cdot 4 = 3 \cdot 4 = 12$.

5. (3 pts) Compute the limit $\lim_{y \rightarrow 3} \frac{y^2 - 3y}{y^2 + 5y - 24}$. Show your work.

Solution:

$$\begin{aligned} \lim_{y \rightarrow 3} \frac{y^2 - 3y}{y^2 + 5y - 24} &= \lim_{y \rightarrow 3} \frac{y(y-3)}{(y-3)(y+8)} \\ &= \lim_{y \rightarrow 3} \frac{y}{y+8} = \frac{3}{3+8} = \frac{3}{11}. \end{aligned}$$

6. (4 pts) Compute the tangent line to the graph of the relation $y^3 + 3xy = 6 + y$ at the point $(x, y) = (2, 1)$. Put your answer in the form $y = mx + b$. Show your work.

Solution: Use Implicit Differentiation to compute y' when $(x, y) = (2, 1)$:

$$\begin{aligned} (y^3 + 3xy)' &= (6 + y)' \\ 3y^2y' + 3y + 3xy' &= y' \\ 3(1)^2y' + 3(1) + 3(2)y' &= y' \\ 3y' + 3 + 6y' &= y' \\ y' &= -\frac{3}{8}. \end{aligned}$$

For the tangent line, the slope $m = y' = -\frac{3}{8}$. So use the point slope formula for $(x_0, y_0) = (2, 1)$ to show

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 1 &= -\frac{3}{8}(x - 2) = -\frac{3}{8}x + \frac{3}{4} \\ y &= -\frac{3}{8}x + \frac{7}{4}. \end{aligned}$$

7. (6 pts) At time t (measured in hours), the position $s(t)$ of a car along a road is given by $s(t) = 3t^3 - 2t$ mi.

(a) (2 pts) Compute the velocity of the car at time t .

Solution: $v(t) = s'(t) = (9t^2 - 2)$ mi/hr.

(b) (2 pts) Compute the acceleration of the car at time t .

Solution: $a(t) = v'(t) = 18t$ mi/hr².

(c) (2 pts) At what time is the acceleration equal to 9 mi/hr²?

Solution: $a(t) = 18t = 9$ when $t = 1/2$ hour.