Calculus 1: Sample Questions, Exam 1

1. (a) Use the definition of the derivative to compute the derivative \( \frac{d}{dx} \left( \frac{1}{x} \right) \). Show your work.

(b) Compute the tangent line to the graph \( y = \frac{1}{x} \) at the point \((x, y) = (2, \frac{1}{2})\). Put your answer in the form \( y = mx + b \). Show your work.

2. Compute the derivative \( f'(3) \) for \( f(t) = t^2 - 3t + 4 \). Show your work.
3. Let $a$ be a constant, and let $y = \cos ax$. Compute the derivative $y'$.
   Show your work.

4. If $z = \frac{1}{x^2 + 1}$, compute $\frac{dz}{dx}$ and $\frac{d^2z}{dx^2}$. Show your work.

5. If $g(x)$ is a function satisfying $g(3) = 1$ and $g'(3) = 4$, and $f(x) = x^3g(x)$, compute $f'(3)$. Show your work.
6. Compute the limit \( \lim_{y \to 2} \frac{y^2 - 4y + 4}{y^2 - 5y + 6} \). Show your work.

7. Put the letter of the derivative of the function in the blank.

<table>
<thead>
<tr>
<th>( f(x) )</th>
<th>choices for ( f'(x) )</th>
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</thead>
<tbody>
<tr>
<td>( \cot x )</td>
<td>(a) ( \sin )</td>
</tr>
<tr>
<td>( \sqrt{x} )</td>
<td>(b) ( \csc^2 x )</td>
</tr>
<tr>
<td>( \sec 2x )</td>
<td>(c) ( \cos x )</td>
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<tr>
<td>( x^{-1} )</td>
<td>(d) ( -\frac{1}{x^2} )</td>
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<tr>
<td>( \sin x )</td>
<td>(e) ( -\frac{1}{2}x^{-\frac{1}{2}} )</td>
</tr>
<tr>
<td>( \tan x )</td>
<td>(f) ( -\csc^2 x )</td>
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<tr>
<td>( 2 \sec x \tan x )</td>
<td>(g) ( 2 \sec x \tan 2x )</td>
</tr>
<tr>
<td>( 2 \sec^2 x \tan x )</td>
<td>(h) ( 2 \sec^2 x \tan x )</td>
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<tr>
<td>( 1^{-1} )</td>
<td>(i) ( 1^{-1} )</td>
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<tr>
<td>( 2 \tan^2 2x )</td>
<td>(j) ( 2 \tan^2 2x )</td>
</tr>
<tr>
<td>( \frac{1}{2\sqrt{x}} )</td>
<td>(k) ( \frac{1}{2\sqrt{x}} )</td>
</tr>
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</table>
8. What is the slope of the tangent line to the graph of the relation

\[ x^2y^2 + y^3 - y = \sqrt{x} \]

at the point \((x, y) = (1, 1)\)? Show your work.

9. Assume the position of a bicycle on a road in miles is given by \(s(t) = -t^2 + 20t\), where \(t\) is measured in hours. Compute the velocity and the acceleration. What is the position of the bicycle when the velocity is 10 mi/hr? Show your work.