1. (15 pts) Compute the following derivatives, and simplify your answers. Show your work if you want to be eligible for partial credit.

(a) (3 pts) \( \frac{d}{dx} \sin(5x + 4) \).
   **Solution:** Use the Chain Rule: \( \cos(5x + 4) \cdot 5 = 5 \cos(5x + 4) \).

(b) (3 pts) \( \frac{d}{dx} \left( \frac{x^2}{x^4 + 3x + 2} \right) \).
   **Solution:** Use the Quotient Rule:
   \[
   \frac{(x^4 + 3x + 2)(2x) - x^2(4x^3 + 3)}{(x^4 + 3x + 2)^2} = \frac{-2x^5 + 3x^2 + 4x}{(x^4 + 3x + 2)^2}.
   \]

(c) (3 pts) \( \frac{d}{dx} (x^9 - 3x^4 + 20) \).
   **Solution:** \( 9x^8 - 3(4x^3) + 0 = 9x^8 - 12x^3 \).

(d) (3 pts) \( \frac{d}{dx} (x^3 \cot x) \).
   **Solution:** Use the Product Rule: \( 3x^2 \cdot \cot x + x^3 \cdot (-\csc^2 x) = 3x^2 \cot x - x^3 \csc^2 x \).

(e) (3 pts) \( \frac{d}{dx} e^{5+\sqrt{x}} \).
   **Solution:** Use the Chain Rule:
   \[
   e^{5+\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{e^{5+\sqrt{x}}}{2\sqrt{x}}.
   \]

2. (3 pts) Compute \( \frac{dy}{dx} \) for the relation \( \cos(x^2y) - y^3 = x + 2 \).
   **Solution:**
   \[
   \frac{d}{dx} [\cos(x^2y) - y^3] = \frac{d}{dx} (x + 2),
   \]
   \[
   -\sin(x^2y) \cdot \frac{d}{dx} (x^2y) - 3y^2 \frac{dy}{dx} = 1,
   \]
   \[
   -\sin(x^2y) \cdot \left( 2xy + x^2 \frac{dy}{dx} \right) - 3y^2 \frac{dy}{dx} = 1,
   \]
   \[
   -x^2 \sin(x^2y) \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 1 + 2xy \sin(x^2y),
   \]
   \[
   [-x^2 \sin(x^2y) - 3y^2] \frac{dy}{dx} = 1 + 2xy \sin(x^2y),
   \]
   \[
   \frac{dy}{dx} = \frac{1 + 2xy \sin(x^2y)}{-x^2 \sin(x^2y) - 3y^2}.
   \]
3. (4 pts) Use the definition of the derivative to compute the derivative of \( g(x) = 3x^2 - x + 1 \). Show your work.

Solution:
\[
g'(x) = \lim_{h \to 0} \frac{g(x + h) - g(x)}{h} \\
= \lim_{h \to 0} \frac{3(x + h)^2 - (x + h) + 1 - [3x^2 - x + 1]}{h} \\
= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - x - h + 1 - 3x^2 + x - 1}{h} \\
= \lim_{h \to 0} \frac{6xh + 3h^2 - h}{h} \\
= \lim_{h \to 0} (6x + 3h - 1) \\
= 6x + 3(0) - 1 = 6x - 1.
\]

4. (3 pts) Let \( f \) be a function which satisfies \( f(2) = 4 \), \( f(4) = 9 \), \( f'(2) = 2 \), and \( f'(4) = -6 \). Consider the function \( h(x) = f(\sqrt{x}) \). Compute \( h'(4) \). Show your work.

Solution: Use the Chain Rule to compute \( h'(x) = \frac{d}{dx}f(\sqrt{x}) = f'(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} \) and \( h'(4) = f'(\sqrt{4}) \cdot \frac{1}{2\sqrt{4}} = f'(2) \cdot \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \).

5. (3 pts) Compute the limit \( \lim_{y \to 1} \frac{y^2 - 2y + 1}{y^2 - 1} \). Show your work.

Solution:
\[
\lim_{y \to 1} \frac{y^2 - 2y + 1}{y^2 - 1} = \lim_{y \to 1} \frac{(y - 1)(y - 1)}{(y - 1)(y + 1)} \\
= \lim_{y \to 1} \frac{y - 1}{y + 1} \\
= \frac{1 - 1}{1 + 1} = 0.
\]

6. (4 pts) Compute the tangent line to the graph of the function \( y = \sqrt{x^2 - 5} \) at the point \( (x, y) = (3, 2) \). Put your answer in the form \( y = mx + b \). Show your work.
Solution:

\[
y' = \frac{1}{2\sqrt{x^2 - 5}} \cdot 2x \]
\[
= \frac{1}{2\sqrt{x^2 - 5}} \cdot x \]
\[
y'(3) = \frac{3}{\sqrt{3^2 - 5}} = \frac{3}{\sqrt{4}} = \frac{3}{2}
\]
\[
y - y_0 = m(x - x_0),
\]
\[
y - 2 = \frac{3}{2}(x - 3),
\]
\[
y = \frac{3}{2} \cdot x - \frac{9}{2} + 2 = \frac{3}{2} \cdot x - \frac{5}{2}.
\]

7. (4 pts) Assume the position of a bicycle traveling along a road is \(10t + \cos 30t\) miles, with \(t\) in hours.

(a) (2 pts) Compute the velocity of the bicycle at time \(t\).

**Solution:** The position is \(s = 10t + \cos 30t\), and so the velocity \(v = s' = 10 + (-\sin 30t) \cdot 30 = 10 - 30 \sin 30t\).

(b) (2 pts) Compute the acceleration of the bicycle at time \(t\).

**Solution:** The acceleration \(a = v' = 0 - 30 \cos(30t) \cdot 30 = -900 \cos(30t)\).

8. (3 pts) On the axes provided below, sketch the graph of a function \(y = f(x)\) which satisfies

\[
f(1) = 2, \quad \lim_{x \to 1^-} f(x) = 2, \quad \lim_{x \to 1^+} f(x) = -1.
\]