Mathematics Colloquium

Samit Dasgupta

Harvard

Conjectures on Hilbert's 12^{th} Problem

Abstract: It is well known that the square root of any integer can be written as a linear combination of roots of unity. A generalization of this fact is the "Kronecker-Weber Theorem," which states that in fact any element which generates a Galois extension of the field of rational numbers $\mathbb{Q}$ with abelian Galois group can also be written as such a linear combination. The roots of unity may by viewed as the special values of the analytic function $\mathrm{e}(x) = \exp(2\pi i x)$, where x is taken to be a rational number. Broadly speaking, Hilbert's 12^{th} problem is to find an analogous result when $\mathbb{Q}$ is replaced by a general algebraic number field $\mathbb{F}$, and in particular to find the analytic functions which play the role of $\mathrm{e}(x)$ in this general setting.

Hilbert's 12^{th} problem has been solved in the case where $\mathbb{F}$ is an imaginary quadratic field, with the role of $\mathrm{e}(x)$ being played by certain modular forms. All other cases are, generally speaking, unresolved. In this talk I will discuss the case where $\mathbb{F}$ is a real quadratic field, and more generally, a totally real field. I will describe conjectures of Stark and Gross, as well as current work using methods of Shintani, which if proven would arguably provide a positive resolution of Hilbert's 12^{th} problem in these cases.

Wednesday, October 24

4:00-5:00 pm

204 Smith Hall