Mathematics Colloquium

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On the Prescribing $\sigma_\mathbf{k}$ Curvature Equation

Abstract: In this talk, we will discuss several analytic issue that arise from the prescribing σ_k curvature equation subject to keeping the metric within a fixed conformal class. Here the σ_k curvature of a Riemannian metric refers to the k^{th} elementary symmetric function of the eigenvalue of the Schouten-Weyl tensor of a metric with respect to itself, and the Schouten-Weyl tensor A_g of a metric g on M^n is defined as

$$A_g = \frac{1}{n-2}\left(Ric_g - \frac{R_g}{2(n-1)}\, g\right).$$

More specifically, for a given function $K(x)$ on M^n, we ask whether there exists an admissible conformal metric $g_w = e^{2w(x)}g$ solving

$$\sigma_k(g_w^{-1} \circ A_{g_w}) = K(x) \quad on \quad M^n. \tag{1}$$

Here an admissible *metric refers to a metric g_w which makes (1)* elliptic. *For $k = 1$, we are dealing with a Yamabe type semilinear PDE, while for $1 < k \leq n$, we are dealing with a fully nonlinear PDE in the conformal factor w. First we will give an elementary and unified discussion to the Kazdan-Warner type necessary conditions for the solvability of (1). A proof of such condtions in this context was first given by Jeff Viaclovsky. Then we will discuss the potential loss of compactness to solutions of (1), and show that under appropriate non-degeneracy conditions on $K(x)$, no such loss of compactness can happen. This latter part is joint work with Alice S.-Y. Chang and Paul Yang.*

Wednesday, November 15
4:00-5:00 pm
204 Smith Hall