Colloquium

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Triangle Groups, Complex Hyperbolic Geometry, and Dehn Surgery

Abstract: My talk will have 4 parts.

1. One of the most familiar objects in geometry is the tiling of the Euclidean plane by equilateral triangles. There are related tilings of the non-Euclidean, or hyperbolic, plane whose associated symmetry groups are known as the Schwarz reflection triangle groups. (It's a different Schwarz!) For the first part of my talk I will introduce the hyperbolic plane, construct the Schwarz triangle groups, and explain some of their mathematical significance.

2. The complex hyperbolic plane is a kind of exotic cousin of the ordinary hyperbolic plane. A model for the complex hyperbolic plane is the open unit ball in $\mathbb{C}^2$*, equipped with a highly symmetric and beautiful metric. One can consider the complex hyperbolic plane as the complexification of the ordinary hyperbolic plane. It turns out that the Schwarz reflection groups also act on the complex hyperbolic plane, sometimes in very exotic ways. In the second part of the talk I will construct the complex hyperbolic plane and explain how the Schwarz reflection triangle groups act on it.*

3. As I mentioned above, a model for the complex hyperbolic plane is the open unit ball in $\mathbb{C}^2$*. The "ideal boundary" of the complex hyperbolic plane is the unit 3-sphere. In many cases, when a group acts on the complex hyperbolic plane, it gives rise to a 3 dimensional "geometric manifold" modelled on the 3-sphere. This manifold is called "the manifold at infinity." In particular, such 3-manifolds can be associated to the actions of the Schwarz reflection triangle groups on the complex hyperbolic plane. In the third part of my talk I will explain the general notion of a geometric manifold, due to Thurston, and I will explain how the relevant special cases arise in connection with complex hyperbolic geometry.*

4. In the fourth part of my talk I will explain how one can understand the geometric 3-manifolds associated to the complex hyperbolic actions of the triangle groups, in terms of a topological operation called Dehn surgery. These examples are part of, and illustrate, my recent theorem, "The Horotube Surgery Theorem." The HST explains how Dehn surgery arises in the context of complex hyperbolic geometry. One consequence of the HST is the existence of infinitely many commensurability classes of closed hyperbolic 3-manifolds which bound complex hyperbolic 4-manifolds. I hope to give at least some hint of what the Horotube Surgery Theorem says.

Wednesday, February 11

4:00 pm
204 Smith Hall