

Elementary Differential Equations, Section 2
Prof. Loftin: Practice Test Problems for Test 1

1. Give the general solution for the following ordinary differential equations. Solve for y if possible. Show your work.

(a) $y' = \frac{x}{y(x^2+1)}$.

(b) $\frac{dy}{dx} + 3y = e^{3x}$.

(c) $y'' + 6y' + 5y = 0$.

(d) $y^3 - \sin x + 3xy^2 \frac{dy}{dx} = 0$.

2. Find the largest interval on which the solution to the initial value problem

$$(t-1)y''' + 6y'' + (\sin t)y' + (\ln t)y = 0, \quad y(2) = 4, \quad y'(2) = 0, \quad y''(2) = -1$$

must exist. (Do not try to solve the equation.)

3. Solve the initial value problems.

(a) $y' + \frac{y}{t} = \sin t$, $y(\pi) = 1$. (Recall $\sin \pi = 0$, $\cos \pi = -1$.)

(b) $4y'' - 9y = 0$, $y(0) = 1$, $y'(0) = 2$.

(c) $x dx = y dy$, $y(0) = -1$.

4. Consider the autonomous equation $y' = y^3 - 4y$.

(a) Find all the equilibrium solutions to the equation.

(b) Draw a graph in the ty plane containing representative solutions to the equation. Be sure to include the equilibrium solutions, and indicate whether each is stable or unstable.

(c) Compute $\lim_{t \rightarrow \infty} y(t)$ if $y(t)$ is the solution to the initial value problem $y' = y^3 - 4y$, $y(0) = 1$. (You should read this from the graph—do not attempt to solve the equation explicitly.)

5. Consider the equation $6y + (3x + \frac{2y}{x}) \frac{dy}{dx} = 0$

(a) Show the equation is not exact.

- (b) Find an integrating factor of the form $\mu(x)$ to make the equation exact.
 - (c) Use the integrating factor to find the general solution to the equation.
6. A lake holds 10^{10} liters of water, and the lake water is currently polluted with 1 mg of a pollutant per liter. A stream of clean water carrying 10^7 liters per day enters the lake. 2×10^6 liters of water evaporate from the lake each day (the pollution does not evaporate), while an outgoing stream takes 8×10^6 liters of (mixed) lake water away from the lake each day.

How long does it take until the pollution level in the lake drops to an acceptable concentration of .1 mg/liter? Show your work, and use a calculator.