

**Elementary Differential Equations, Section 2**  
**Prof. Loftin: Practice Test Problems for Test 2**

1. Find the radius of convergence of the power series

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots + \frac{x^n}{n} + \cdots$$

Show your work.

2. (a) Find the general solution to  $y'' + 4y = 0$ .  
(b) Use the method of undetermined coefficients to find a solution to  $y'' + 4y = \sin 2t$ .  
(c) Find the general solution to  $y'' + 4y = \sin 2t$ .  
(d) Find the solution to the initial value problem

$$y'' + 4y = \sin 2t, \quad y(0) = 2, \quad y'(0) = 1.$$

3. Find the general solution to  $y''' - y'' + y' - y = 0$ . Show your work.  
4. Find the recurrence relation for a power series around  $x = 0$  for the equation  $y'' - xy' + y = 0$ . Find the general term of the power series. Show your work.  
5. Find the general solution to  $x^2y'' + 7xy' + 5y = 0$ . Show your work.  
6. Find all the singular points of  $(x^4 - x^2)y'' + (x + 2)y' + y = 0$ . Classify each as a regular or irregular singular point, and show your work.  
7. Use Abel's Theorem to determine the Wronskian of two solutions to the differential equation  $xy'' + y' + 3y = 0$  (for  $x > 0$ ). Show your work.  
8. Find all the terms up to the  $a_4$  term of power series representation of the solution to

$$y'' + 6xy' + 4y = x, \quad y(0) = 1, \quad y'(0) = 3.$$

Show your work.