

Elementary Differential Equations, Section 2
Prof. Loftin: Practice Test Problems for the Material
after Test 2

1. Consider the following function

$$f(t) = \begin{cases} 1 & \text{for } t < 2 \\ t + 1 & \text{for } 2 \leq t < 5 \\ 3 & \text{for } t \geq 5 \end{cases}$$

- (a) Rewrite $f(t)$ using a single formula in terms of $u_2(t)$ and $u_5(t)$.
(b) Find the Laplace transform of $f(t)$.
(c) Use the Laplace transform to find the solution to the initial value problem

$$y' - 2y = f(t), \quad y(0) = 1.$$

Show your work.

2. Determine all the singular points of the following differential equation:

$$x^2(x-1)^2(x+2)y'' + (x+x^3)y' + y = 0.$$

Determine which of the singular points are regular. For each regular singular point, calculate the indicial equation and its roots (i.e., the exponents). Show your work.

3. Consider the differential equation

$$xy'' + (1+2x)y' + y = 0.$$

- (a) Show that $x = 0$ is a regular singular point, and determine the indicial equation. Show that 0 is a double root of the indicial equation.
(b) Find the first three terms of a solution to the equation of the form

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots.$$

- (c) Find the first three terms of the general solution to the equation, by finding all solutions of the form

$$ay_1(x) \ln x + b_0 + b_1x + b_2x^2 + b_3x^3 + \cdots,$$

where $y_1(x)$ is the solution to part (b) with $a_0 = 1$.

4. Derive the formula for the Laplace transform $\mathcal{L}\{t\}$. (In other words, compute it from the definition, and don't just look it up in the table.)
5. Consider the differential equation

$$2x^2y'' + xy' + (x - x^2)y = 0.$$

- (a) Write down the indicial equation for $x = 0$ and find its roots r_1, r_2 .
- (b) Find the recurrence relation for a solution of the differential equation of the form

$$|x|^r(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots),$$

where r is a root of the indicial equation.

- (c) For each exponent r_1, r_2 , write down the first three nonzero terms of the solution to the differential equation.