Elementary Differential Equations: Quiz #11, April 28, 2010

SOLUTION

1. Use the Laplace transform to solve the initial value problem below. Show your work.

$$y'' - y' - 6y = 0, \qquad y(0) = 1, \quad y'(0) = -1.$$

Solution: Let $F(s) = \mathcal{L}\{y\}$. Then take the Laplace transform of the differential equation and compute

$$\begin{aligned}
0 &= \mathcal{L}\{0\} = \mathcal{L}\{y'' - y' - 6y\} \\
&= \mathcal{L}\{y''\} - \mathcal{L}\{y'\} - 6\mathcal{L}\{y\} \\
&= [s^2F(s) - sy(0) - y'(0)] - [sF(s) - y(0)] - 6F(s) \\
&= (s^2 - s - 6)F(s) - 1 \cdot s - (-1) + 1, \\
F(s) &= \frac{s-2}{s^2 - s - 6} = \frac{s-2}{(s-3)(s+2)}, \\
\frac{s-2}{(s-3)(s+2)} &= \frac{a}{s-3} + \frac{b}{s+2}, \\
s - 2 &= a(s+2) + b(s-3) \\
-2 - 2 &= a(-2+2) + b(-2-3), \qquad b = \frac{4}{5}, \\
3 - 2 &= a(3+2) + b(3-3), \qquad a = \frac{1}{5}, \\
F(s) &= \frac{s}{(s-3)(s+2)} = \frac{\frac{1}{5}}{s-3} + \frac{\frac{4}{5}}{s+2}, \\
y &= \mathcal{L}^{-1}\{F(s)\} = \frac{1}{5}\,\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + \frac{4}{5}\,\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} \\
&= \frac{1}{5}\,e^{3t} + \frac{4}{5}\,e^{-2t}.
\end{aligned}$$