HINTS: PROBLEM SET 3

- §1.4, #1.
  - Let $y = f(x) \in Y$. We want to find an open set V so that $y \in V \subset f(U)$.
  - Use the Local Submersion Theorem. In particular, we have coordinate neighborhoods $U \ni x$ and $W \ni y$ on which f is particularly nice. Does this guarantee such an open set V? Why?
  - If g is the canonical submersion from $\mathbb{R}^k$ to $\mathbb{R}^\ell$ ($\ell \leq k$), and B_ϵ is the open ball of radius ϵ centered at the origin in $\mathbb{R}^k$ or $\mathbb{R}^\ell$, then show that $g(B_\epsilon) = B_\epsilon$.
- §1.4, # 2(a).
  - Use the previous problem to show that $f(X)$ is open in Y.
  - Is $f(X)$ also closed in Y? Why will this help?
- §1.4, # 2(b).
  - The Euclidean space in question is $\mathbb{R}^k$ for $k > 0$. Derive a contradiction based on 2(a).
- §1.5, # 2.
  - All these spaces are finite dimensional vector spaces. For a finite-dimensional vector space V, recall that $\forall x \in V$, $T_xV = V$.
  - Then apply linear algebra to compute the $T_xX + T_xZ$ as required in the definition of transversality.
- §1.5, # 4.
  - Show that $T_x(X \cap Z) \subset T_xX \cap T_xZ$. You should find the problem §1.2, # 1 useful.
  - Compute the dimension of both $T_x(X \cap Z)$ in terms of the dimensions of X, Y, Z. Use the Theorem on page 30.
  - Compute the dimension of $T_xX \cap T_xZ$. Use the fact $T_xX + T_xZ = T_xY$. The linear algebra requires some work. Think about what happens in the special case that $X = \{0\} \times \mathbb{R}^2$, $Z = \mathbb{R}^2 \times \{0\}$, and $Y = \mathbb{R}^3$.
  - Note the dimensions are equal and conclude as we did in the proof of the Proposition on page 24-25.
- §1.5, # 8.
  - Notice that both the hyperboloid and the sphere are of the form $f^{-1}(c)$ for a value $c \in \mathbb{R}$. So (after verifying that

the appropriate c is a regular value), calculate the tangent space using the Proposition at the bottom of page 24.

- Show there are 3 cases depending on a. Note that in one case, they don't intersect (are they transversal or not in this case?).
- To check transversality, it may be useful to use the discussion on the bottom of page 23. In particular, show that $f^{-1}(c)$ and $g^{-1}(b)$ are transversal if and only if f and g are independent on the intersection.