

HINTS: PROBLEM SET 4

- §1.6, #1. In order to find ρ , you should use the notes on “bump functions.”
- §1.7, #5. Again use the “bump function” techniques.
- §1.7, #6. Simply connected is defined in §1.6, Problem 6. As part of this problem, you should use §1.6, problems 5 and 6. (Or at least the conclusion that \mathbb{R}^k is simply connected). For §1.6, Problem 5, check that $f_t: \mathbb{R}^k \rightarrow \mathbb{R}^k$, $f_t(x) = (1 - t)x$, is a homotopy from the identity map to the constant map with image 0.

Note that the definition of simply connected for us is *not* quite the same as the usual one. In this course, we say a smooth manifold X is simply connected if every *smooth* map $f: S^1 \rightarrow X$ is homotopic to a constant map by a *smooth* homotopy. In other words, there is a *smooth* map $F: S^1 \times I \rightarrow X$ and $F(x, 0) = f(x)$, and $F(x, 1)$ is constant. This makes some things a little easier: for example Sard’s theorem says that any smooth map from S^1 to S^k is not onto for $k > 1$.