

## Discrete Structures: Sample Questions, Final Exam

1. Show the results of a PREORDER search for the labeled positional binary tree in Figure 1 of the picture page.
2. Consider the following example. Let  $G = \{V, S, \langle \text{integer} \rangle, \mapsto\}$  for

$$\begin{aligned} S &= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -\}, \\ V &= S \cup \{\langle \text{integer} \rangle, \langle \text{unsigned-integer} \rangle, \langle \text{digit} \rangle\}, \end{aligned}$$

and let part of the production relation given in BNF notation be given by

$$\begin{aligned} \langle \text{unsigned-integer} \rangle &::= \langle \text{digit} \rangle | \langle \text{digit} \rangle \langle \text{unsigned-integer} \rangle \\ \langle \text{digit} \rangle &::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 \end{aligned}$$

Design the production relation for  $\langle \text{integer} \rangle$  so that an integer can be either an unsigned integer, or an unsigned integer preceded by a + or - (not both). So +324, 009, -8922 are all valid integers, but + -87, 00-2+, and + are not valid integers. Write the remaining part of the production relation in BNF notation.

3. Consider  $G = (V, I, v_0, \mapsto)$ , where  $V = \{v_0, w, a, b, c\}$ ,  $I = \{a, b, c\}$ , and  $\mapsto$  defined by

$$v_0 \mapsto aw, \quad w \mapsto bbw, \quad w \mapsto bc.$$

- (a) Write the production relation in BNF notation.
- (b) Draw the syntax diagrams of  $v_0$  and  $w$  separately, and then draw a master syntax diagram for  $v_0$ . (Recall a master diagram is one that involves no nonterminal symbols.)
- (c) Show that the sentence  $ab^5c$  is in the language  $L(G)$ . Draw a derivation tree for this sentence.
- (d) Describe the language  $L(G)$  in words, and find the regular expression over  $I = \{a, b, c\}$  it corresponds to.
- (e) Find a Moore machine  $M = (S, I, \mathcal{F}, v_0, T)$  which produces this language  $L(G)$ . Draw the state diagram and the labeled digraph for  $M$ .

4. Given the BNF representation for the master syntax diagram given in Figure 2 of the picture page. You may provide nonterminal symbols as needed (in addition to  $v_0$ ) to use in the BNF productions, and you may use several BNF statements if needed.
5. Construct a Moore machine  $M$  that will accept any string ending in  $ab$  from input strings of  $a, b, c$ . In other words,  $I = \{a, b, c\}$ , and  $L(M) = (a \vee b \vee c)^*ab$ . Draw the labeled digraph of  $M$  and draw its state transition table.
6. Consider the regular expression  $0(0 \vee 1)^*1$  over the input set  $I = \{0, 1\}$ .
  - (a) Construct a Moore machine with input set  $I$  whose language corresponds to this regular expression.
  - (b) Construct a Type 3 grammar  $G = (V, I, s_0, \mapsto)$  corresponding to the Moore machine constructed the previous part.
7. (a) Define a monoid.
  - (b) Let  $I$  be a set, and let  $I^*$  be the set of strings on  $I$ . Show that  $I^*$  is a monoid with operation given by catenation (i.e., if  $w, z \in I^*$ , define  $w * z = w \cdot z$ ).
8. (a) Let  $f: Z \rightarrow \{0, 1, 2\}$  be the mod 3 function. Complete the following table:

| $f(n)$ | $f(2n + 0)$ | $f(2n + 1)$ |
|--------|-------------|-------------|
| 0      | 0           |             |
| 1      |             |             |
| 2      |             | 2           |

For example, the lower left corner means that if  $n$  is an integer with  $f(n) = 2$ , then we must have  $f(2n + 1) = 2$ . This can be proved as follows:  $f(n) = 2$  if and only if there is an integer  $k$  so that  $n = 3k + 2$ . Then

$$2n + 1 = 2(3k + 2) + 1 = 6k + 5 = (6k + 3) + 2 = 3(2k + 1) + 2,$$

and so the mod 3 function  $f$  applied to  $2n + 1$  is 2.

- (b) Consider the Moore machine with input set  $I = \{0, 1\}$  given in Figure 3 of the picture page.

We think of elements of  $I^*$  as binary integers. Provide a proof by mathematical induction that the Moore machine accepts exactly those binary integers which are divisible by 3. (Hint: we may identify  $s_0$  as the set of integers that are  $0 \pmod 3$ ,  $s_1$  as the set of integers which are  $1 \pmod 3$ , and  $s_2$  as the set of integers which are  $2 \pmod 3$ . What does this have to do with the table you completed above?)

9. Consider the finite state machine whose state transition table is

|                       |                       |                       |                       |
|-----------------------|-----------------------|-----------------------|-----------------------|
|                       | <i>a</i>              | <i>b</i>              | <i>c</i>              |
| <i>s</i> <sub>0</sub> | <i>s</i> <sub>0</sub> | <i>s</i> <sub>1</sub> | <i>s</i> <sub>2</sub> |
| <i>s</i> <sub>1</sub> | <i>s</i> <sub>1</sub> | <i>s</i> <sub>0</sub> | <i>s</i> <sub>0</sub> |
| <i>s</i> <sub>2</sub> | <i>s</i> <sub>2</sub> | <i>s</i> <sub>0</sub> | <i>s</i> <sub>1</sub> |

List all the values of the transition function  $f_{abcc}$ .

10. Evaluate the expression  $3\ 4\ -\ 6\ +\ 6\ 12\ \div\ +$ , which is in reverse Polish (postfix) notation.
11. For the Moore machine  $M$  given in Figure 4 of the picture page, construct a type 3 grammar  $G = (V, I, s_0, \mapsto)$  so that  $L(G) = L(M)$ . Express the production relation in terms of both  $\mapsto$  notation and BNF notation.

12. True/False. Circle T or F. No explanation needed. For (a)-(g), refer to the digraph of the Moore machine  $M = (S, I, \mathcal{F}, s_0, T)$  represented in Figure 4 on the picture page.

- (a) T F  $f_{02}(s_1) = s_2$ .
- (b) T F  $f_{11112}(s_0) = (f_{112} \circ f_{11})(s_0)$ .
- (c) T F  $0100 \in L(M)$ . (Recall  $L(M)$  is the language of  $M$ .)
- (d) T F  $121 \in L(M)$ .
- (e) T F There is a Type 3 grammar  $G$  with terminal symbols  $I = \{0, 1, 2\}$  so that  $L(M) = L(G)$ .
- (f) T F If  $w \in I^*$  contains an odd number of 2's, then we must have  $w \in L(M)$ .
- (g) T F  $f_{201}(s_1) = s_2$ .
- (h) T F In BNF notation,  $\langle v_0 \rangle ::= \langle v_1 \rangle a$  is an acceptable production relation for a Type 1 phase structure grammar.
- (i) T F In BNF notation,  $\langle v_0 \rangle ::= \langle v_1 \rangle a$  is an acceptable production relation for a Type 2 phase structure grammar.
- (j) T F In BNF notation,  $\langle v_0 \rangle ::= \langle v_1 \rangle a$  is an acceptable production relation for a Type 3 phase structure grammar.
- (k) T F Let  $G = (V, S, v_0, \mapsto)$  be a phase structure grammar with  $S = \{a, b, c\}$ ,  $V = S \cup \{v_0, v_1\}$ , and the production relation determined by

$$v_0 \mapsto av_0, \quad v_0 \mapsto av_1, \quad v_1 \mapsto bcv_0, \quad v_1 \mapsto c.$$

Then  $v_0 \Rightarrow^\infty abc$ .

- (l) T F For the grammar  $G$  in part (k), the regular expression for the language  $L(G)$  is  $(a \vee abc)^*c$ .
- (m) T F The string  $xyx$  is in the regular set determined by the regular expression  $(xx \vee (xy)^* \vee xy^*)^*$ .
- (n) T F  $0^*(1 \vee \Lambda)$  is a regular expression over the set  $I = \{0, 1\}$ .
- (o) T F  $37 \times 4 - 9 \times 65 \times 2 +$  is a valid mathematical expression in reverse Polish (postfix) notation.