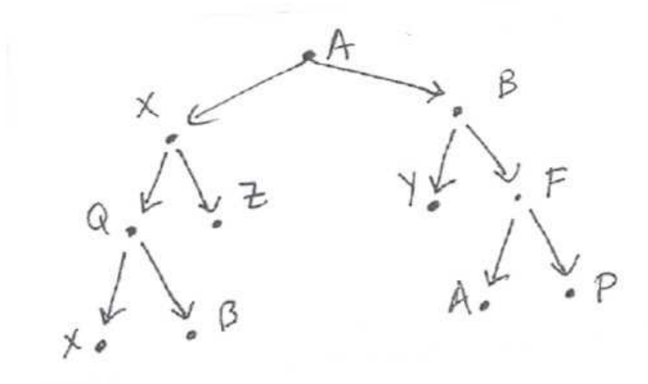
Discrete Structures: Sample Questions, Final Exam

1. Construct the labeled tree of the algebraic expression

$$(((x - y) * z) - 3)/(19 + (x * x)).$$

2. Show the results of a PREORDER search for the following labeled positional binary tree.



3. Consider the following example. Let $G = \{V, S, \langle\text{integer}\rangle, \mapsto\}$ for

$$\begin{aligned} S &= \{0,1,2,3,4,5,6,7,8,9,+,-\}, \\ V &= S \cup \{\langle\text{integer}\rangle, \langle\text{unsigned-integer}\rangle, \langle\text{digit}\rangle\}, \end{aligned}$$

and let part of the production relation given in BNF notation be given by

$$\begin{aligned} \langle\text{unsigned-integer}\rangle &::= \langle\text{digit}\rangle | \langle\text{digit}\rangle\langle\text{unsigned-integer}\rangle \\ \langle\text{digit}\rangle &::= 0|1|2|3|4|5|6|7|8|9 \end{aligned}$$

Design the production relation for $\langle\text{integer}\rangle$ so that an integer can be either an unsigned integer, or an unsigned integer preceded by a + or − (not both). So +324, 009, −8922 are all valid integers, but +−87, 00−2+, and + are not valid integers. Write the remaining part of the production relation in BNF notation.

4. Consider $G = (V, I, v_0, \mapsto)$, where $V = \{v_0, w, a, b, c\}$, $I = \{a, b, c\}$, and $\mapsto$ defined by

$$v_0 \mapsto aw, \qquad w \mapsto bbw, \qquad w \mapsto bc.$$

   (a) Write the production relation in BNF notation.

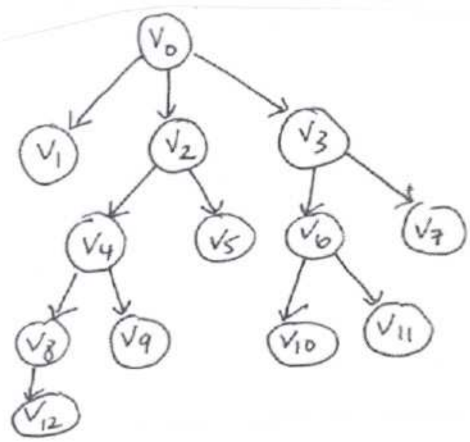
   (b) Draw the syntax diagrams of v_0 and w separately, and then draw a master syntax diagram for v_0. (Recall a master diagram is one that involves no nonterminal symbols.)

   (c) Show that the sentence ab^5c is in the language $L(G)$. Draw a derivation tree for this sentence.

   (d) Describe the language $L(G)$ in words, and find the regular expression over $I = \{a, b, c\}$ it corresponds to.

   (e) Find a Moore machine $M = (S, I, \mathcal{F}, v_0, T)$ which produces this language $L(G)$. Draw the state diagram and the labeled digraph for M.

5. Consider the rooted tree (T, v_0) given below.



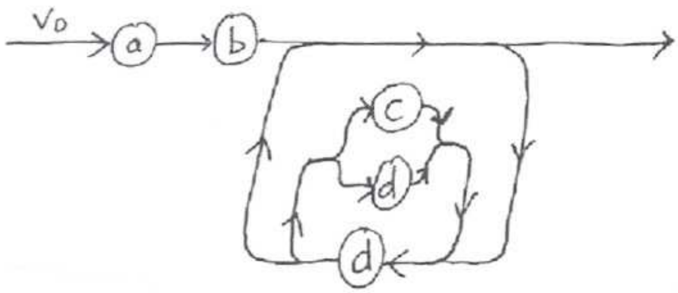
   (a) List all the level-2 vertices of the tree.

   (b) List all the leaves of the tree.

   (c) List all the siblings of v_3.

(d) Draw the digraph of the subtree $T(v_4)$ with root v_4.

6. Given the BNF representation for the master syntax diagram given below. You may provide nonterminal symbols as needed (in addition to v_0) to use in the BNF productions, and you may use several BNF statements if needed.



7. Construct the digraph of the positional binary tree for the following doubly linked list. Label each vertex with the corresponding data.

INDEX	LEFT	DATA	RIGHT
1	11		0
2	10	M	7
3	0	Q	0
4	8	T	0
5	3	V	4
6	0	X	2
7	0	K	0
8	0	D	0
9	6	G	5
10	0	C	0
11	9	Y	0

8. Construct a Moore machine M that will accept any string ending in ab from input strings of a, b, c. In other words, $I = \{a, b, c\}$, and $L(M) = (a \vee b \vee c)^*ab$. Draw the labeled digraph of M and draw its state transition table.

9. Consider the regular expression $0(0 \vee 1)^*1$ over the input set $I = \{0, 1\}$.

(a) Construct a Moore machine with input set I whose language corresponds to this regular expression.

(b) Construct a Type 3 grammar $G = (V, I, s_0, \mapsto)$ corresponding to the Moore machine constructed the previous part.

10. (a) Let $f \colon Z \to \{0, 1, 2\}$ be the mod 3 function. Complete the following table:

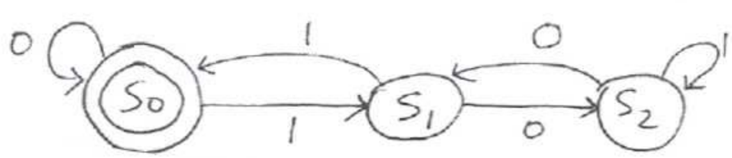
$f(n)$	$f(2n+0)$	$f(2n+1)$
0	0	
1		
2		2

For example, the lower left corner means that if n is an integer with $f(n) = 2$, then we must have $f(2n+1) = 2$. This can be proved as follows: $f(n) = 2$ if and only if there is an integer k so that $n = 3k + 2$. Then

$$2n + 1 = 2(3k + 2) + 1 = 6k + 5 = (6k + 3) + 2 = 3(2k + 1) + 2,$$

and so the mod 3 function f applied to $2n + 1$ is 2.

(b) Consider the Moore machine with input set $I = \{0, 1\}$ given below.



We think of elements of I^* as binary integers. Provide a proof by mathematical induction that the Moore machine accepts exactly those binary integers which are divisible by 3. (Hint: we may identify s_0 as the set of integers that are 0 mod 3, s_1 as the set of integers which are 1 mod 3, and s_2 as the set of integers which are 2 mod 3. What does this have to do with the table you completed above?)

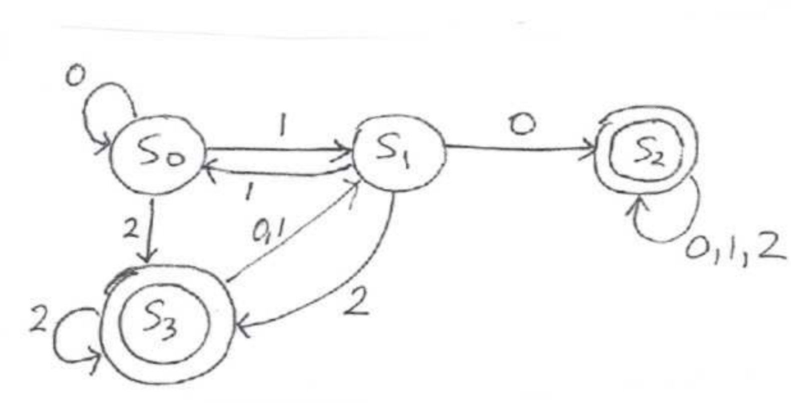
11. Consider the finite state machine whose state transition table is

	a	b	c
s_0	s_0	s_1	s_2
s_1	s_1	s_0	s_0
s_2	s_2	s_0	s_1

List all the values of the transition function f_{abcc}.

12. Evaluate the expression 3 4 − 6 + 6 12 ÷ +, which is in reverse Polish (postfix) notation.

13. For the Moore machine M given below, construct a type 3 grammar $G = (V, I, s_0, \mapsto)$ so that $L(G) = L(M)$. Express the production relation in terms of both $\mapsto$ notation and BNF notation.



14. True/False. Circle T or F. No explanation needed. For (a)-(g)), refer to the digraph of the Moore machine $M = (S, I, \mathcal{F}, s_0, T)$ represented in the previous problem.

(a)	T	F	$f_{02}(s_1) = s_2$.
(b)	T	F	$f_{11112}(s_0) = (f_{112} \circ f_{11})(s_0)$.
(c)	T	F	$0100 \in L(M)$. (Recall $L(M)$ is the language of M.)
(d)	T	F	$121 \in L(M)$.
(e)	T	F	There is a Type 3 grammar G with terminal symbols $I = \{0, 1, 2\}$ so that $L(M) = L(G)$.
(f)	T	F	If $w \in I^*$ contains an odd number of 2's, then we must have $w \in L(M)$.
(g)	T	F	$f_{201}(s_1) = s_2$.
(h)	T	F	If (T, v_0) is a rooted tree on a set A, then the relation T is irreflexive.
(i)	T	F	If $A = \{1, 2, 3, 4, 5, 6\}$ and R is the relation $\{(1, 2), (1, 4), (3, 5), (3, 6)\}$, then R is a tree on A.
(j)	T	F	In BNF notation, $\langle v_0 \rangle ::= \langle v_1 \rangle a$ is an acceptable production relation for a Type 1 phase structure grammar.
(k)	T	F	In BNF notation, $\langle v_0 \rangle ::= \langle v_1 \rangle a$ is an acceptable production relation for a Type 2 phase structure grammar.
(l)	T	F	All the vertices of a complete binary tree have out degree either 0 or 2.
(m)	T	F	Every vertex of a tree has in-degree 1.
(n)	T	F	In BNF notation, $\langle v_0 \rangle ::= \langle v_1 \rangle a$ is an acceptable production relation for a Type 3 phase structure grammar.
(o)	T	F	Let $G = (V, S, v_0, \mapsto)$ be a phase structure grammar with $S = \{a, b, c\}$, $V = S \cup \{v_0, v_1\}$, and the production relation determined by $v_0 \mapsto av_0, \quad v_0 \mapsto av_1, \quad v_1 \mapsto bcv_0, \quad v_1 \mapsto c.$ Then $v_0 \Rightarrow^\infty abc$.
(p)	T	F	For the grammar G in part (k), the regular expression for the language $L(G)$ is $(a \vee abc)^* c$.
(q)	T	F	The string xxy is in the regular set determined by the regular expression $(xx \vee (xy)^* \vee xy^*)^*$.
(r)	T	F	$0^*(1 \vee \Lambda)$ is a regular expression over the set $I = \{0, 1\}$.
(s)	T	F	$3\,7 \times 4 - 9 \times 6\,5 \times 2 +$ is a valid mathematical expression in reverse Polish (postfix) notation.