

Discrete Structures: Sample Questions, Exam 1

1. Prove by mathematical induction that $3 \mid (n^3 - n)$ for every positive integer n .
2. Write down a formula for the sequence

$$3, 4, 6, 9, 13, 18, 24, 31, 38, \dots$$

Is your formula recursive or explicit?

3. Consider the matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

Compute $\mathbf{A} \odot \mathbf{B}$, \mathbf{AB} , $\mathbf{B} \odot \mathbf{A}$ and $\mathbf{A} \wedge \mathbf{B}$.

Assume \mathbf{A} is the matrix of a relation. Draw the corresponding digraph.

4. Use the Euclidean algorithm to compute the greatest common divisor $\text{GCD}(75, 12)$. Show your work. Compute the least common multiple $\text{LCM}(75, 12)$.
5. How many ways can a committee of 3 faculty members and 2 students be selected from 7 faculty members and 8 students? Show your work.
6. Two fair six-sided dice are rolled and the sum s of the numbers coming up is recorded. What is the probability that $s \geq 10$? Show your work.

7. True/False. Circle T or F. No explanation needed.

- (a) T F If \mathbf{A} and \mathbf{B} are any 2×2 matrices, then $\mathbf{AB} = \mathbf{BA}$.
- (b) T F If \mathbf{A} , \mathbf{B} and \mathbf{C} are 2×2 matrices, then $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$.
- (i) T F Let $R = (A, \$)$ be the mathematical structure where A is the set of even integers and $\$$ is a unary operation on A given by $\$a = \frac{a}{2}$ for every $a \in A$. A is closed with respect to the operation $\$$.
- (m) T F 49 and 77 are relatively prime.
- (n) T F Let A and B be subsets of a universal set U . Then it is always true that $|A \cup B| = |A| + |B|$.
- (o) T F Two cards are dealt in succession from a standard shuffled 52-card deck. The number of possible 2-card hands is 1326.

8. Let a_n be the sequence recursively defined by $a_1 = 2$, $a_2 = 3$, $a_n = a_{n-1}a_{n-2}$ for $n \geq 3$. Compute the first five terms a_1, \dots, a_5 .

9. A 6-sided die is rolled twice. What is the probability that the sum of the two rolls is exactly 8?

10. Compute the truth table for the statement $[(p \wedge q) \vee r] \Rightarrow (\sim q)$. Show your work.

11. Consider the relation R on $A = \{1, 2, 3, 4\}$ given by

$$1R2, \quad 2R3, \quad 3R3, \quad 3R4, \quad 4R3.$$

Draw the digraph of R and compute its connectivity relation R^∞ . Draw the digraph of R^∞ . (Hint: Try to determine R^∞ by inspection, not by computing with formulas.)