Discrete Structures: Sample Questions, Exam 2

1. Let $A = B = \{a, b, c\}$. Consider the relation $g = \{(a, b), (b, c), (c, c)\}$. Is g one-to-one? Is g onto? Why?

2. Consider $f : Z^+ \to Z^+$ defined by $f(a) = a^2$. Is f one-to-one? Is f onto? Why?

3. Put the following functions in order from lowest to highest in terms of their Θ classes. (Some of the functions may be in the same Θ class. Indicate that on your list also.)

   (a) $f_1(n) = n \log n$,
   (b) $f_2(n) = n^{\frac{3}{2}}$,
   (c) $f_3(n) = 10{,}000$,
   (d) $f_4(n) = \sqrt{n}(n + \log n)$,
   (e) $f_5(n) = 3^n$,
   (f) $f_6(n) = 2^{n+2}$,
   (g) $f_7(n) = 0.0001$.

4. The following arrays describe a relation R on the set $A = \{1, 2, 3, 4, 5\}$. Compute both the digraph of R and the matrix $\mathbf{M}_R$.

$$\begin{aligned} \text{VERT} &= [6, 2, 8, 7, 10] \\ \text{TAIL} &= [2, 2, 2, 2, 1, 1, 4, 3, 4, 5] \\ \text{HEAD} &= [4, 3, 5, 1, 2, 3, 5, 4, 2, 4] \\ \text{NEXT} &= [3, 1, 4, 0, 0, 5, 9, 0, 0, 0] \end{aligned}$$

5. Given the partition $\mathcal{P} = \{\{1, 2\}, \{3\}, \{4, 5\}\}$ of the set $A = \{1, 2, 3, 4, 5\}$, consider R the associated equivalence relation on A. Draw the digraph associated to R and write down the matrix $\mathbf{M}_R$.

6. Let $S = \{x, y, z\}$, and consider the set $P(S)$ with relation R given by set inclusion. Is R a partial order? Why or why not? (Carefully check the conditions needed for a relation to be a partial order.) Is R a linear order? Again carefully check the conditions for R to be a linear order.

7. Show that $(P(S), R)$ from the previous problem is isomorphic to the poset D_{42} of divisors of 42 with relation given by divisibility.

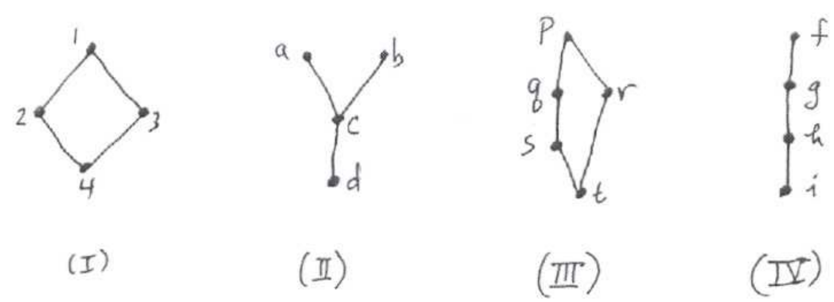
8. Consider the relation on $A = \{a, b, c, d, e\}$.

$$R = \{(a, a), (a, c), (b, c), (c, d), (e, a), (d, b)\}.$$

Draw the corresponding digraph. Use Warshall's algorithm to compute the matrix $\mathbf{M}_{R^\infty}$. Show all the intervening steps.

9. Is there an infinite poset with a least element? Either write down an example or prove that this is impossible.

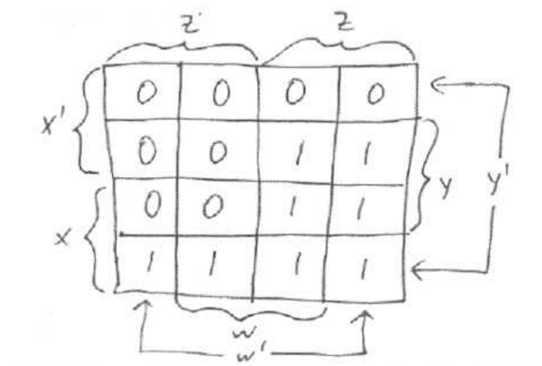
10. Which of the following Hasse diagrams represent lattices?



11. For the examples in the previous problem, which represent finite Boolean algebras?

12. Draw the Hasse diagram for the lattice D_{18} consisting of the divisors of 18 with the partial order of divisibility.

13. Given the following Karnaugh map of a Boolean function, write the function as an equivalent Boolean polynomial.



14. Let x, y, z be Boolean variables. Use the rules of Boolean algebra (or a truth table with Karnaugh map) to simplify the following expression. Your answer should be as simple as possible.

$$(x \wedge z) \vee (y' \vee (y' \wedge z)) \vee ((x \wedge y') \wedge z')$$

15. If $b = 1011 \in B_4$, write down the minterm E_b in terms of the Boolean variables x_1, x_2, x_3, x_4.

16. True/False. Circle T or F. No explanation needed.

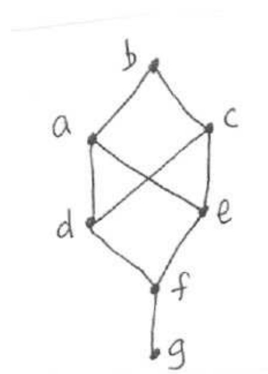
(a)	T	F	If f is a one-to-one function from an infinite set A to itself, then f must be onto.
(b)	T	F	If g is a one-to-one function from a finite set A to itself, then g must be onto.
(c)	T	F	Let $A = Z^+$ the set of positive integers. Define the relation R on A by aRb if and only if $a\mid b$. R is transitive.
(d)	T	F	R the relation from (c) is symmetric.
(e)	T	F	R the relation from (c) is asymmetric.
(f)	T	F	R the relation from (c) is antisymmetric.
(g)	T	F	R the relation from (c) is irreflexive.
(h)	T	F	R the relation from (c) is an equivalence relation.
(i)	T	F	If x is a Boolean variable, then $x \vee x = x$.
(j)	T	F	If y is a Boolean variable, then $y \vee I = y$.
(k)	T	F	Let $B = \{0, 1\}$ with the standard partial order and let $A = B \times B \times B$ with the product partial order. Then A is isomorphic as a lattice to D_{60}.
(l)	T	F	D_{85} is a Boolean algebra.
(m)	T	F	Every finite lattice has a least element.
(n)	T	F	Every poset has a greatest element.
(o)	T	F	$f(n) = \log_5(n)$ is $O(\lg(n))$.
(p)	T	F	If g is the mod-10 function, then $g(405) = 4$.
(q)	T	F	If x and y are Boolean variables, then $(x \wedge y)' = x' \wedge y'$.
(r)	T	F	Given the digraph of a relation R on A, the digraph of the inverse relation R^{-1} is given by reversing the direction of each arrow in the digraph.
(s)	T	F	Given the digraph of a relation R on A, the digraph of complementary relation $\bar{R}$ is given by reversing the direction of each arrow in the digraph.
(t)	T	F	If R is a reflexive relation, then the connectivity relation R^∞ is equal to the reachability relation R^*.

17. Consider the relation R on $A = \{1, 2, 3, 4\}$ given by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Is R a partial order? Why or why not?

18. Consider the poset $(A, \leq)$ given by the following Hasse diagram.



(a) List all the minimal elements of $(A, \leq)$.

(b) List all the upper bounds of $B = \{c, d\} \subseteq A$.

(c) List a pair of incomparable elements of A (if such a pair exists).

(d) Is $(A, \leq)$ a lattice? Why or why not?

19. Let $A = \{*, q, 1\}$, with partial order determined by $q < 1 < *$. Put the following elements of $A \times A \times A$ in lexigraphic order:

$(q, q, 1)$, $(q, 1, *)$, $(*, *, q)$, (q, q, q), $(*, 1, 1)$, $(1, *, q)$, $(q, 1, 1)$, $(*, *, *)$