Discrete Structures: Sample Questions, Exam 2

1. Let $A = B = \{a, b, c\}$. Consider the relation $g = \{(a, b), (b, c), (c, c)\}$. Is $g$ one-to-one? Is $g$ onto? Why?

2. Consider $f : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ defined by $f(a) = a^2$. Is $f$ one-to-one? Is $f$ onto? Why?

3. Put the following functions in order from lowest to highest in terms of their $\Theta$ classes. (Some of the functions may be in the same $\Theta$ class. Indicate that on your list also.)
   
   (a) $f_1(n) = n \log n$,
   (b) $f_2(n) = n^\frac{3}{2}$,
   (c) $f_3(n) = 10,000$,
   (d) $f_4(n) = \sqrt{n(n + \log n)}$,
   (e) $f_5(n) = 3^n$,
   (f) $f_6(n) = 2^{n+2}$,
   (g) $f_7(n) = 0.0001$.

4. Let $S = \{x, y, z\}$, and consider the set $P(S)$ with relation $R$ given by set inclusion. Is $R$ a partial order? Why or why not? (Carefully check the conditions needed for a relation to be a partial order.) Is $R$ a linear order? Again carefully check the conditions for $R$ to be a linear order.

5. Show that $(P(S), R)$ from the previous problem is isomorphic to the poset $D_{42}$ of divisors of 42 with relation given by divisibility.

6. Consider the relation on $A = \{a, b, c, d, e\}$.

   $R = \{(a, a), (a, c), (b, c), (c, d), (e, a), (d, b)\}$.

   Draw the corresponding digraph. Use Warshall’s algorithm to compute the matrix $M_{R^\infty}$. Show all the intervening steps.

7. Is there an infinite poset with a least element? Either write down an example or prove that this is impossible.
8. Which of the following Hasse diagrams represent lattices?

![Hasse diagrams]

9. For the examples in the previous problem, which represent finite Boolean algebras?

10. Draw the Hasse diagram for the lattice $D_{18}$ consisting of the divisors of 18 with the partial order of divisibility.

11. Given the following Karnaugh map of a Boolean function, write the function as an equivalent Boolean polynomial.

![Karnaugh map]

12. Let $x, y, z$ be Boolean variables. Use the rules of Boolean algebra (or a truth table with Karnaugh map) to simplify the following expression. Your answer should be as simple as possible.

$$(x \land z) \lor (y' \lor (y' \land z)) \lor ((x \land y') \land z')$$
13. If $b = 1011 \in B_4$, write down the minterm $E_b$ in terms of the Boolean variables $x_1, x_2, x_3, x_4$.

14. True/False. Circle T or F. No explanation needed.

(a) T F If $f$ is a one-to-one function from an infinite set $A$ to itself, then $f$ must be onto.
(b) T F If $g$ is a one-to-one function from a finite set $A$ to itself, then $g$ must be onto.
(c) T F If $x$ is a Boolean variable, then $x \lor x = x$.
(d) T F If $y$ is a Boolean variable, then $y \lor I = y$.
(e) T F Let $B = \{0, 1\}$ with the standard partial order and let $A = B \times B \times B$ with the product partial order. Then $A$ is isomorphic as a lattice to $D_{60}$.
(f) T F $D_{85}$ is a Boolean algebra.
(g) T F Every finite lattice has a least element.
(h) T F Every poset has a greatest element.
(i) T F $f(n) = \log_5(n)$ is $O(\lg(n))$.
(j) T F If $g$ is the mod-10 function, then $g(405) = 4$.
(k) T F If $x$ and $y$ are Boolean variables, then $(x \land y)' = x' \lor y'$.
15. Consider the relation $R$ on $A = \{1, 2, 3, 4\}$ given by the matrix

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$ 

Is $R$ a partial order? Why or why not?

16. Consider the poset $(A, \leq)$ given by the following Hasse diagram.

(a) List all the minimal elements of $(A, \leq)$.
(b) List all the upper bounds of $B = \{c, d\} \subseteq A$.
(c) List a pair of incomparable elements of $A$ (if such a pair exists).
(d) Is $(A, \leq)$ a lattice? Why or why not?

17. Let $A = \{*, q, 1\}$, with partial order determined by $q < 1 < *$. Put the following elements of $A \times A \times A$ in lexicographic order:

$$(q, q, 1), \quad (q, 1, *), \quad (*, *, q), \quad (q, q, q), \quad (*, 1, 1), \quad (1, *, q), \quad (q, 1, 1), \quad (*, *, *)$$