Math 473: Practice Problems for Test 1, Fall 2011

Show your work:

1. (a) Compute the Taylor polynomials $P_n(x)$ for $f(x) = \sin x$ and $x_0 = 0$.
   
   (b) Show that the remainder term $R_n(x) = f(x) - P_n(x)$ satisfies
   
   $$|R_n(x)| \leq \frac{|x|^{n+1}}{(n+1)!}.$$ 

   (c) Find the smallest value of $n$ so that $P_n(0.2)$ approximates $\sin(0.2)$ 
   with error less than $10^{-6}$. Show your work.
   
   (d) For the value of $n$ from part (c), compute $P_n(0.2)$ and $\sin(0.2)$. 
   What is the actual (absolute) error?

2. Approximate $\pi$ by $22/7$. What is the absolute error? What is the 
   relative error?

3. Let $f(x) = x^3 - x - 5$, $[a, b] = [0, 2]$. How many iterations of the 
   Bisection Method are required to ensure the approximation $p_n$ is within 
   $10^{-4}$ of a root? Justify your answer. You do not have to compute the 
   approximation.

4. Will the Bisection Method applied to $f(x) = \tan x$ and initial interval 
   $[a, b] = [1, 2]$ converge to a root? Why or why not? To which value, if 
   any, will the Bisection Method converge?

5. If $g(x) = \cos x - 2x$, and $[a_1, b_1] = [0, 1]$, use the Bisection Method to 
   compute $p_3$. Show your work.

6. Consider Newton’s Method with $p_0 = 2$ and $k(x) = x^3 - 6x - 1$. 
   Compute up to $p_4$. Is the method converging to a root?

7. What happens if we apply Newton’s method to $h(x) = \frac{1}{x}$ and $p_0 = 1$? 
   What is $\lim_{n \to \infty} p_n$? Why? (Draw a picture at least to explain your 
   answer.)

8. (Bonus): We consider Newton’s Method on a polynomial $f(x)$ of degree 
   2. Find coefficients of the polynomial so that
   - $f(0) = 2$,
   - for $p_0 = 0$, then $p_1 = 2$ and $p_2$ is undefined.