1. (17 pts) Consider the function $f(x)$ given by the following graph of $y = f(x)$.

(a) (3 pts) Evaluate $f(-2)$.

$$f(-2) = -1$$

(b) (3 pts) Does $f(x)$ have an inverse function? Why or why not?

No. It fails the horizontal line test.

(c) (3 pts) What are the interval(s) where $f(x)$ is increasing?

$$[-2, 2]$$

(d) (3 pts) Which symmetries does the graph $y = f(x)$ satisfy? $y$-axis symmetry? $x$-axis symmetry? Origin symmetry? Note: This is a hand-drawn graph, so it cannot be perfect. If it seems very close to having a given symmetry, then it does have that symmetry for the purposes of this test.

Origin symmetry only
(c) (5 pts) On the axes provided, sketch the graph of $y = f(-x + 1)$. (The other axes at the bottom of the page may be used for practice or intermediate steps.)

Answer:

Extra $xy$ axes for practice:
2. (10 pts) Consider

\[ F(x) = \frac{x - 3}{2x + 1} \]

(a) (5 pts) Compute the inverse function \( F^{-1}(x) \). Show your work.

\[ y = F(x) = \frac{x - 3}{2x + 1} \]

Switch \( x, y \):

\[ x = \frac{y - 3}{2y + 1} \]

Solve for \( y \):

\[ x(2y + 1) = y - 3 \]

\[ 2xy + x = y - 3 \]

\[ 2xy - y = -x - 3 \]

\[ (2x - 1)y = -x - 3 \]

\[ y = \frac{-x - 3}{2x - 1} = F^{-1}(x) \]

(b) (5 pts) Compute \( (F \circ F)(3) \). Show your work. You may express your answer either as a fraction or as a decimal number.

\[ (F \circ F)(3) = F(F(3)) = F\left( \frac{3 - 3}{2(3) + 1} \right) \]

\[ = F\left( \frac{0}{7} \right) = F(0) \]

\[ = \frac{0 - 3}{2(0) + 1} = \frac{-3}{1} = -3 \]
3. (16 pts)

(a) (4 pts) On the axes provided, sketch the graph of \( y = x^2 \). Clearly label the intercepts.

(b) (4 pts) On the axes provided, sketch the graph of \( y = -\frac{1}{2}x + 1 \). Clearly label the intercepts.

\[
y : \text{intercept : } x = 0 \\
y = -\frac{1}{2}(0) + 1 = 1 \\
\text{so } (0, 1)
\]

\[
x : \text{intercept : } y = 0 \\
0 = y = -\frac{1}{2}x + 1 \\
-1 = -\frac{1}{2}x \\
2 = x \\
(2, 0)
\]
(c) (4 pts) On the axes provided, sketch the graph of

\[ y = g(x) = \begin{cases} 
  x^2 & \text{for } x < 0 \\
  -\frac{1}{2}x + 1 & \text{for } x \geq 2.
\end{cases} \]

(d) (4 pts) Find the domain and the range of the function \( g(x) \) defined above. Put your answers in the boxes provided and in interval notation.

**Domain:** \((-\infty, 0) \cup [2, \infty)\)

Since \( x \) can be either \( x < 0 \) or \( x \geq 2 \),

**Range:** \((-\infty, \infty)\)

Project graph in part (c) to the \( y \)-axis to see range is all \((-\infty, \infty)\).