1. (11 pts) Let \( f(x) = 1 + \log_3 x \).

(a) (3 pts) Compute \( f(\frac{1}{3}) \), \( f(1) \), and \( f(3) \). Show your work.

\[
\begin{align*}
    f\left(\frac{1}{3}\right) &= 1 + \log_3\left(\frac{1}{3}\right) = 1 + \log_3\left(3^{-1}\right) = 1 + (-1) = 0 \\
    f(1) &= 1 + \log_3 1 = 1 + 0 = 1 \\
    f(3) &= 1 + \log_3 3 = 1 + 1 = 2
\end{align*}
\]

(b) (3 pts) On the axes provided, sketch the graph of \( y = f(x) = 1 + \log_3 x \). INCLUDE THE FUNCTION VALUES COMPUTED IN PART (a).

(c) (2 pts) List all vertical and horizontal asymptotes of the graph in part (b).

\[
\text{Vertical asymptote: } x = 0 \text{ (y axis)} \\
\text{No horizontal asymptote}
\]
(d) (3 pts) Use your calculator to compute \( f(7) \) (for \( f(x) = 1 + \log_3 x \)) and round your answer to 4 places after the decimal. Show your work.

\[
f(7) = 1 + \log_3 7 = 1 + \frac{\log 7}{\log 3} \quad \left( \text{base change formula} \right)
\]

\[
\approx 3.7712437
\]

\[
\approx 3.7712
\]

\[
\approx 2.7712437 \approx 2.7712
\]

2. (8 pts) A radioactive isotope has a half-life of 30 minutes. Initially there are 1000 g of this isotope. How long does it take for there to be only 1 g left? Round your answer to the nearest minute. Show your work.

\[
A = A_0 e^{rt}
\]

Use half-life to compute \( r \): Half-life 30 min means

\[
\frac{1}{2} A_0 = A_0 e^{r(30)}
\]

\[
\frac{1}{2} = e^{30r}
\]

\[
-\ln 2 = \ln (\frac{1}{2}) = \ln (e^{30r}) = 30r
\]

\[
\frac{-\ln 2}{30} = r
\]

Now use \( r \) to compute \( t \) when \( A = 1g \), \( A_0 = 1000g \):

\[
A = A_0 e^{rt}
\]

\[
1g = (1000g) e^{\left(\frac{-\ln 2}{30}\right)t}
\]

\[
\frac{1}{1000} = e^{\left(\frac{-\ln 2}{30}\right)t}
\]

\[
-\ln 1000 = \ln \left(\frac{1}{1000}\right) = \ln \left(e^{\left(\frac{-\ln 2}{30}\right)t}\right) = -\frac{-\ln 2}{30} t
\]

\[
\frac{\ln 1000}{\ln 2} \cdot 30 = t
\]

\[
t \approx 298.9735 \text{ min} \approx 299 \text{ min}
\]
3. (8 pts) Short answer. No partial credit.

(a) (2 pts) Convert $\frac{5\pi}{6}$ radians to degrees.

$$\frac{5\pi}{6} = \frac{5\pi}{6} \cdot \frac{180^\circ}{\pi} = \frac{5}{6} (180^\circ) = 150^\circ$$

(b) (2 pts) Compute $\sin \frac{\pi}{2}$. YOUR ANSWER MUST BE EXACT, NOT A DECIMAL APPROXIMATION.

$$\sin \frac{\pi}{2} = \frac{1}{1} = 1$$

(c) (2 pts) Use your calculator to compute $\cot 141^\circ$ correct to 4 decimal places.

$$\cot 141^\circ = \frac{1}{\tan 141^\circ} \sim -1.2349 \sim -1.2349$$

(d) (2 pts) If an acute angle $\theta$ satisfies $\tan \theta = .3$, compute $\theta$ in degrees. Round to the nearest whole degree.

$$\tan \theta = .3 \quad \theta = \tan^{-1}(.3) \sim 16.699^\circ \sim 17^\circ$$
4. (8 pts) A surveyor is trying to measure the distance to a distant mountain $M$. She sights $M$ at two points $A$ and $B$, and notes that the angle $\angle MAB$ is a right angle, while the angle $\angle ABM$ measures $89^\circ$. The distance between $A$ and $B$ is 600 m. (See the picture below.)

What is the distance from $A$ to $M$? Show your work. Round your answer to the nearest meter.

**Solution:**

\[
\tan 89^\circ = \frac{\text{opp}}{\text{adj}} = \frac{x}{600 \text{ m}}
\]

\[
x = (600 \text{ m}) \tan 89^\circ
\]

\[
\approx 34373.977 \text{ m}
\]

\[
\approx 34374 \text{ m}
\]
5. (5 pts) Compute \( \tan \theta \) given the following information: \( \sin \theta = \frac{4}{5} \) and \( \cos \theta < 0 \). Show your work. Your answer should be a rational number.

\[
\sin \theta = \frac{4}{5} > 0 \\
\cos \theta < 0
\]

\( \text{Quad II} \)

\[
\frac{S}{A} \Rightarrow \text{Quadrant II}
\]

\( \tan \theta < 0 \) in \( \text{Quadrant II} \)

\[
\text{SOH CAH TOA:} \\
\sin \theta = \frac{4}{5} = \frac{\text{opposite}}{\text{hypothesis}} = \frac{y}{r}
\]

\[
\begin{align*}
(x, y) & = (-3, 4) \\
x^2 + y^2 &= 5^2 \\
x^2 + 16 &= 25 \\
x^2 &= 9 \\
x &= -3 \quad \text{(Quad II)} \\
\tan \theta &= \frac{y}{x} = \frac{4}{-3} = -\frac{4}{3}
\end{align*}
\]

6. (5 pts) Solve for \( x \). Show your work.

\( 4^x - 2^x - 12 = 0 \)

Your answer should be an integer.

\[
4^x - 2^x - 12 = 0 \\
(2^2)^x - 2^x - 12 = 0 \\
(2^x)^2 - 2^x - 12 = 0
\]

Let \( u = 2^x \)

\[
\begin{align*}
u^2 - u - 12 &= 0 \\
(u - 4)(u + 3) &= 0 \\
\end{align*}
\]

\[
\begin{align*}
u - 4 &= 0 \quad u + 3 &= 0 \\
u &= 4 \quad u &= -3
\end{align*}
\]

\[
\begin{align*}
2^x &= 4 = 2^2 \\
2^x &= -3 \\
x &= 2
\end{align*}
\]

No solution.