1. (11 pts) Let \( f(x) = 1 + \log_2 x \).

(a) (3 pts) Compute \( f(1) \), \( f(2) \), and \( f(4) \). Show your work.

\[
\begin{align*}
  f(1) &= 1 + \log_2 1 = 1 + 0 = 1 \\
  f(2) &= 1 + \log_2 2 = 1 + 1 = 2 \\
  f(4) &= 1 + \log_2 4 = 1 + \log_2 (2^2) = 1 + 2 = 3
\end{align*}
\]

(b) (3 pts) On the axes provided, sketch the graph of \( y = f(x) = 1 + \log_2 x \). INCLUDE THE FUNCTION VALUES COMPUTED IN PART (a).

(c) (2 pts) List all vertical and horizontal asymptotes of the graph in part (b).

- **Vertical:** \( x = 0 \) (y axis)
- **No horizontal asymptotes**
(d) (3 pts) Use your calculator to compute \( f(7) \) (for \( f(x) = 1 + \log_2 x \)) and round your answer to 4 places after the decimal. Show your work.

\[
f(7) = 1 + \log_2 7 = 1 + \frac{\log 7}{\log 2} \quad \text{(base change formula)}
\]

\[
\approx 3.8073549 \\
\approx 3.8074
\]

2. (8 pts) A radioactive isotope has a half-life of 40 minutes. Initially there are 500 g of this isotope. How long does it take for there to be only 1 g left? Round your answer to the nearest minute. Show your work.

\[ A = A_0 e^{rt}, \quad t = \text{time in minutes} \]

half-life 40 minutes implies

\[ A = \frac{1}{2} A_0 \quad \text{if} \quad t = 40 \]

Use this to find \( r \):

\[ \frac{1}{2} A_0 = A_0 e^{r \cdot 40} \]

\[ \frac{1}{2} = e^{40r} \]

\[ -\ln 2 = \ln \frac{1}{2} = \ln(e^{40r}) = 40r \]

\[ \frac{-\ln 2}{40} = r \]

To find \( t \) when \( A = 1 \text{g} \),

\[ A_0 = 500 \text{g} \]

\[ 1 \text{g} = (500 \text{g}) e^{(-\frac{\ln 2}{40})t} \]

\[ \frac{1}{500} = e^{(-\frac{\ln 2}{40})t} \]

\[ -\ln 500 = \ln \frac{1}{500} = \ln(e^{(-\frac{\ln 2}{40})t}) = (-\frac{\ln 2}{40})t \]

\[ t = \frac{\ln 500}{\ln 2} \cdot 40 \approx 358.6314 \text{ min} \]

\[ \approx 359 \text{ min.} \]
3. (8 pts) Short answer. No partial credit.

(a) (2 pts) Convert $3\pi/4$ radians to degrees.
\[
3\pi/4 = \frac{3\pi}{4} \cdot \frac{180^\circ}{\pi} = \frac{3}{4}(180^\circ) = 135^\circ
\]

(b) (2 pts) Compute $\sin \frac{\pi}{3}$. YOUR ANSWER MUST BE EXACT, NOT A DECIMAL APPROXIMATION.
\[
\sin \frac{\pi}{3} = \frac{\text{opp}}{h} = \frac{\sqrt{3}}{2}
\]

(c) (2 pts) Use your calculator to compute $\sec 146^\circ$ correct to 4 decimal places.
\[
-1.2062 \quad \sec 146^\circ = \frac{1}{\cos 146^\circ}
\]
\[
\sim -1.2062179
\]
\[
\sim -1.2062
\]

(d) (2 pts) If an acute angle $\theta$ satisfies $\cos \theta = .4$, compute $\theta$ in degrees. Round to the nearest whole degree.
\[
66^\circ \quad \cos \theta = .4 \quad \theta = \cos^{-1}(.4)
\]
\[
\sim 66.4218^\circ
\]
\[
\sim 66^\circ
\]
4. (8 pts) A surveyor is trying to measure the distance to a distant mountain $M$. She sights $M$ at two points $A$ and $B$, and notes that the angle $\angle MAB$ is a right angle, while the angle $\angle ABM$ measures $88^\circ$. The distance between $A$ and $B$ is 500m. (See the picture below.)

What is the distance from $A$ to $M$? Show your work. Round your answer to the nearest meter.

\[ \tan 88^\circ = \frac{x}{500\text{m}} = \frac{\text{opp}}{\text{adj}} \]

\[ x = (500\text{m}) \tan 88^\circ \]

\[ \sim 14318.127 \text{ m} \]

\[ \sim 14318 \text{ m} \]
5. (5 pts) Compute $\tan \theta$ given the following information: $\cos \theta = \frac{4}{5}$ and $\sin \theta < 0$. Show your work. Your answer should be a rational number.

$$\cos \theta = \frac{4}{5} > 0$$

$$\sin \theta < 0$$

**SohCahToa:**

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r}$$

$$4^2 + y^2 = 5^2$$

$$y^2 = 5^2 - 4^2$$

$$= 25 - 16$$

$$= 9$$

$$y = -3$$

(choose $-3$ in Quad. IV)

$$\sin \theta = \frac{y}{r} = \frac{-3}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{-3}{4} = -\frac{3}{4}$$

6. (5 pts) Solve for $x$. Show your work.

$$4^x + 2^x - 6 = 0.$$

Your answer should be an integer.

$$\left(2^2\right)^x + 2^x - 6 = 0$$

$$\left(2^x\right)^2 + 2^x - 6 = 0$$

Let $u = 2^x$

$$u^2 + u - 6 = 0$$

$$(u + 3)(u - 2) = 0$$

$u + 3 = 0$ or $u - 2 = 0$

$u = -3$ or $u = 2$

$$2^x = -3$$ or $$2^x = 2$$

No solution $\boxed{x = 1}$