

## How To Remember Special Values of Sine and Cosine

The following is a special table for remembering the special exact values of the sine and cosine functions in Quadrant I. The key to the following table is just knowing a few simple patterns. The first is to know the important angles in Quadrant I: These are (in degrees)  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $90^\circ$  (note the quadrantal angles  $0^\circ$  and  $90^\circ$  have been included). The second is to recognize the special values of the sine and cosine functions all have the form  $\frac{\sqrt{n}}{2}$  as follows:

$\theta$	$\sin \theta$	$\cos \theta$
$0 = 0^\circ$	$\frac{\sqrt{0}}{2} = \frac{0}{2} = 0$	$\frac{\sqrt{4}}{2} = \frac{2}{2} = 1$
$\frac{\pi}{6} = 30^\circ$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4} = 45^\circ$	$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$
$\frac{\pi}{3} = 60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
$\frac{\pi}{2} = 90^\circ$	$\frac{\sqrt{4}}{2} = \frac{2}{2} = 1$	$\frac{\sqrt{0}}{2} = \frac{0}{2} = 0$

In order to reproduce the above table, you also need to remember that  $\sin \theta$  increases from 0 to 1 in Quadrant I, while  $\cos \theta$  decreases from 1 to 0 in Quadrant I. There are several ways to keep them straight: One way is you can remember the graphs of the sine and cosine function and thereby recall that  $\sin 0 = 0$ ,  $\cos 0 = 1$ . Another way is to recall the unit circle picture of the trigonometric functions. Recall that on the unit circle, at angle  $\theta$ ,  $(x, y) = (\cos \theta, \sin \theta)$ . As  $\theta = 0$  is the positive  $x$  axis—which hits the unit circle at  $(1, 0)$ —we have

$$(1, 0) = (x, y) = (\cos 0, \sin 0), \quad \cos 0 = 1, \quad \sin 0 = 0.$$

The values of angles outside Quadrant I can be computed using reference angles, and the values of the other trigonometric functions can be computed using the reciprocal and quotient identities.

Example: Compute  $\sec \frac{\pi}{4}$  and  $\tan \frac{\pi}{3}$ .

$$\sec \frac{\pi}{4} = \frac{1}{\cos \frac{\pi}{4}} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}, \quad \tan \frac{\pi}{3} = \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \sqrt{3}.$$